Magnetic Separation News, 1984, Vol. 1, pp. 151–165
0731-3632/84/0103–0151 $25.00/0
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Printed in the United Kingdom

A NEW MATHEMATICAL MODEL FOR THE PERFORMANCE OF AXIAL MAGNETIC FILTERS

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Abstract A mathematical model to describe the macroscopic behaviour of an axial magnetic filter is developed. The model takes into account the long range of absorption forces. Partial mixing of the particles in the fluid is included by introducing the concept of "mixing length". Behaviour of an axial filter under different conditions is numerically simulated. Experiments with colloidal ferrite particles indicate that for an axial filter composed of 12 μm steel wires with a packing factor of 4.8 % the mixing length is of the order of 4 mm.

INTRODUCTION

Magnetic filters are divided into longitudinal, transverse and axial filters according to the mutual orientation of the fibers, the flow velocity v_o, and the magnetic field B_o [1]. All the filters have B_o perpendicular to the fibres. In a longitudinal filter v_o and B_o are parallel, whereas in a transverse filter v_o is perpendicular to B_o and the fibres. An axial filter is rather different, because the flow velocity is parallel to the fibers.

The single wire theory of axial filters has been developed by Birss, Parker, and Gerber[2], by Uchiyama, Kondo, and Takayasu[3], by Birss, Gerber, Parker, and Sheerer[4], and by Gerber[5]. In an ideal flow model the absorption area A_{abs} of an individual fiber is determined by[2,3]

$$\frac{A_{abs}}{a^2} = 4\left(\frac{v_m}{v_o}\right)^{\frac{1}{2}}\left(\frac{L}{a}\right)^{\frac{1}{2}} , \tag{1}$$

where a is the radius and L the length of the fibre, and $v_m = 2J_b J_a b/(9\mu_o \eta_o a)$ is the so called magnetic velocity; here J_b is the magnetic polarization and b the radius of the spherical particles, J_a the magnetic polarization of the fibre, μ_o the permeability and η_o the viscosity of the fluid. The total absorption area of the filter is the sum of the absorption areas of individual fibers. Those particles which enter the filter within the total absorption area are adsorbed onto the fibers. Accordingly, the probability of being absorbed P_{abs} during one pass through the filter is

$$P_{abs} = \frac{4F}{\pi}\left(\frac{v_m}{v_o}\right)^{\frac{1}{2}}\left(\frac{L}{a}\right)^{\frac{1}{2}} , \tag{2a}$$

The output concentration C_{out} of the filter is thus

$$C_{out} = \left[1 - \frac{4F}{\pi}\left(\frac{v_m}{v_o}\right)^{\frac{1}{2}}\left(\frac{L}{a}\right)^{\frac{1}{2}}\right]C_{in} . \tag{2b}$$

Here C_{in} is the input concentration (mass/volume), and F is the packing factor of the fibers inside the filter.

In the laminar flow model of Gerber[5] the absorption area

is of the same order of magnitude, but cannot be expressed in such a simple form as A_{abs} of Eq.(1). In this model $A_{abs} \propto L^{0.45}$, approximately. The laminar flow model has been experimentally verified by Hoffman, Neudel, Schewe, and Reffle[6].

All the single wire calculations, however, loose the physical basis while particles are collected on the fibres. Attempts to describe the performance of a macroscopic filter taking into account the particle build-up have been made, e.g. by Uchiyama, Kurinobu, Takayasu, and Fujii[7]. However, no satisfactory models do exist yet. Also, for practical filters the absorption areas of adjacent fibers overlap, particularly if the filter is long. This reduces the efficiency of the filter as compared with Eq. (2b).

The basic structure and theory of an axial filter are thus seen to be quite different from those of the longitudinal and transverse ones. There is also the fundamental difference that the theoretical description of an axial filter (Eqs. (2a) and (2b)) is based on a completely deterministic capture process, whereas in the description of the other types of magnetic filter the capture is basically a local and random process. Nevertheless, in this paper we want to show that the performance of an axial filter can be theoretically calculated by a method which is very similar to the approach used with the longitudinal and transverse filters.

ABSORPTION OF PARTICLES INSIDE A MAGNETIC FILTER

The magnetic force around a cylindrical fibre decreases with the distance r as $(a/r)^3$. In longitudinal and transverse filter geometries the particles, flowing across the fibre, interact with the fibre within a range of a few wire diameters only. In an axial filter the particles travel along the fibres, interacting with it all the time they are inside the filter. This means that

the rate of absorption at a given point x does not depend just on the concentration at that point but, rather, on the concentration maybe hundreds of fibre diameters upstream from that point. Thus the absorption cannot be considered a local phenomenon as is the case in longitudinal, transverse or random filters.

This means, mathematically, that for non-axial filters we write the absorption rate as[8]

$$\frac{\partial C(x,\tau)}{\partial x} = \alpha(x,\tau)C(x,\tau) \quad , \tag{3}$$

whereas for axial filters we should write

$$\frac{\partial C(x,\tau)}{\partial x} = \beta(x,\tau)C(x - x_m,\tau) \quad . \tag{4}$$

Here x is the distance from the filter inlet,and τ the "filter time" $\tau = t - x/v_o$. The functions $\alpha(x,\tau)$ and $\beta(x,\tau)$ are the absorption probabilities of the particles per unit length of travel within the filter. The parameter x_m describes the average range of the absorption phenomenon in the filter. We call x_m the "mixing length", because it is physically reasonable to assume that inside the filter the range of the absorption phenomenon is longitudinally limited by a gradual mixing of particles in the fluid.

If we apply $L = x - x_m$ to Eq. (2) we obtain

$$C(x) = [1 - k(x - x_0)^{\frac{1}{2}}]C(x_0) \quad , \tag{5}$$

and by differentiating this

$$\frac{\partial C(x)}{\partial x} = - \frac{k}{2(x - x_o)^{\frac{1}{2}}} C(x_o) \quad . \tag{6}$$

Thus we have $\beta(x,\tau) = k/\ 2(x-x_o)^{1/2}$, with $k = (4F/\pi)(v_m/av_o)^{1/2}$ for loosely packed, clean fibres. Putting now $x-x_o = x_m$ we find

$$\frac{\partial C(x)}{\partial x} = - \frac{k}{2x_m^{\frac{1}{2}}} C(x - x_m) \quad , \tag{7}$$

which is of the same form as Eq. (4). We have thus derived an equation for the absorption rate, which also bears some characteristics of the single wire theories. In the first approximation all the time dependence, due to the loading of the filter fibres, is accounted for by the factor k. The concentration of absorbed particles C_{abs} is determined by the condition that as the particles are removed from the fluid they are adsorbed onto the filamentary matrix. For all the filter types, longitudinal, transverse, and axial, we get the same differential equation[8]

$$\frac{\partial C_{abs}(x,\tau)}{\partial \tau} = - v_o \frac{\partial C(x,\tau)}{\partial x} \quad . \tag{8}$$

SOLUTION OF THE EQUATIONS FOR AXIAL FILTERS

First we assume that k and x_m both are constants. To simplify the notation we also put $C_{in} = 1$, so that the solution for $x < x_m$ is as in Eq. (5) with $x_o = 0$,

$$0 < x < x_m: \qquad C(x) = 1 - kx^{\frac{1}{2}} \quad . \tag{9a}$$

For $x > x_m$ the solution of Eq. (7) can be expressed piece-wise:

$$x_m < x < 2x_m: \qquad C(x) = C(x_m) - \frac{k}{2x_m^{\frac{1}{2}}}\left[(x - x_m) - \frac{2}{3}k\,(x - x_m)^{\frac{3}{2}}\right] \tag{9b}$$

$$\begin{aligned} nx_m < x < (n+1)x_m: \; C(x) = {} & C(nx_m) - \frac{k}{2x_m^{\frac{1}{2}}}\{C[(n-1)x_m](x - nx_m) \\ & - \frac{k}{2x_m^{\frac{1}{2}}}\{C[(n-2)x_m]\frac{(x - nx_m)^2}{2!} \quad \ldots \\ & \ldots - \frac{k}{2x_m^{\frac{1}{2}}}\{C(x_m)\frac{(x - nx_m)^{n-1}}{(n-1)!} \\ & - \frac{k}{2x_m^{\frac{1}{2}}}\left[\frac{(x - nx_m)^n}{n!} - \frac{2^n k(x - nx_m)^{n+\frac{1}{2}}}{1\cdot3\cdot5\cdot\ldots\cdot(2n+1)}\right]\}\ldots\}\} \end{aligned}$$

$$= \sum_{j=0}^{n}\left(-\frac{k}{2x_m^{\frac{1}{2}}}\right)^j C[(n-j)x_m]\frac{(x - nx_m)^j}{j!} + \frac{(-k)^{n+1}(x - nx_m)^{n+\frac{1}{2}}}{x_m^{n/2}\cdot1\cdot3\cdot5\cdot\ldots\cdot(2n+1)} \tag{9c}$$

Especially for $x = nx_m$ we obtain

$$C(nx_m) = \sum_{j=0}^{n-1}\left(-\frac{k}{2x_m^{\frac{1}{2}}}\right)^j C[(n-1-j)x_m]\frac{x_m^j}{j!} + \frac{(-k)^n x_m^{n/2}}{1\cdot3\cdot\ldots\cdot(2n-1)} \quad . \tag{10}$$

The meaning of the mathematical formulation is best presented graphically. Fig. 1 depicts the calculated concentration of solids, $C(x)/C_{in}$, inside an axial filter at time $\tau = 0$, as a function of the distance x from the inlet. In the calculation we have assumed that $k = 0.05\ mm^{-1/2}$. We see, for example, that in

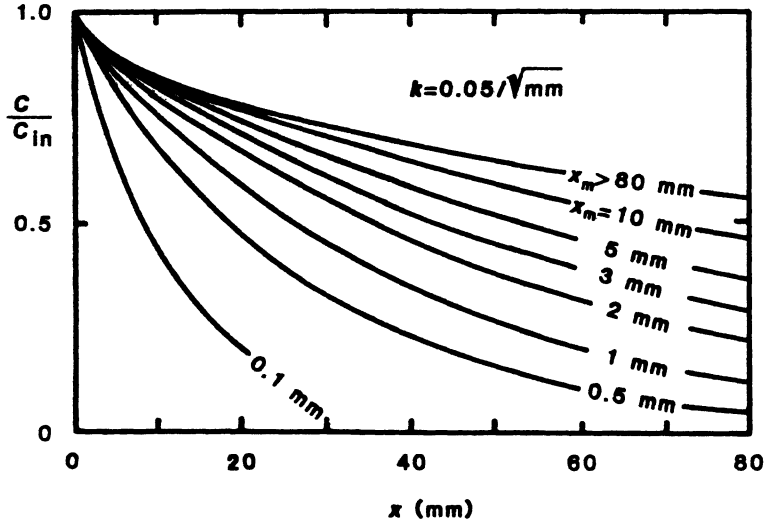

Fig. 1. The calculated concentration of solids (Eq. 10) in an axial filter at time $\tau = 0$ for various values of the mixing length x_m and for $k = 0.05\ mm^{-1/2}$.

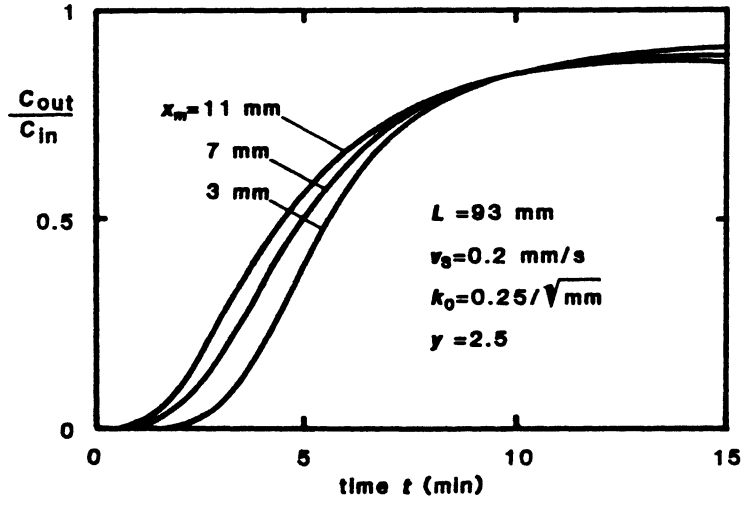

Fig. 2. Calculated effect of x_m on the breakthrough curve of an axial filter.

an 80 mm long ideal filter ($x_m > 80$ mm) we would have $C_{out}/C_{in} = C(80\text{ mm})/C_{in} = 0.56$. Partial mixing of the particles, characterized by the mixing length x_m , is seen to improve the efficiency, the more the shorter x_m is. We may think this to be a consequence of the fact that some of those particles which would otherwise travel through the filter are, in the assumed (random) process of mixing, rendered susceptible of being deposited onto the fibers. The calculated curves of Fig.1 show also that, with the introduction of the mixing length x_m , the filter acquires an exponential $C_{out}/C_{in} \propto \exp(-\alpha L)$ character more typical of the transverse, longitudinal or random filters[8]. We should note (7) is not valid for $x_m = 0$, because this would imply infinitely rapid change of the concentration C(x) with x.

ACCOUNTING FOR THE LOADING OF THE FILTER FIBERS

The deposition of particles onto the fibres, i.e. loading of the filter, will reduce the absorption probability $\beta(x,\tau)$ of Eq. (4). This is taken into account by assuming that k in Eqs. (5)-(7) is a function of the concentration of absorbed particles, $C_{abs}(x,\tau)$. When the filter is partly filled k will vary along the filter. We write $k = k_n$ for $(n-1)x_m < x < nx_m$, i.e. we assume that the absorption probability remains constant for each interval (x_i, x_{i+1}). The concentration is now obtained from the set of equations:

$$0 < x < x_m: \quad C(x) = 1 - k_1 x^{\frac{1}{2}} \tag{11a}$$

$$x_m < x < 2x_m: \quad C(x) = C(x_m) - \frac{k_2}{2x_m^{\frac{1}{2}}}(x - x_m) - \frac{2}{3}k_1 (x - x_m)^{3/2} \qquad (11b)$$

and, eventually,

$$C(nx_m) = \sum_{j=0}^{n-1}\left[\left(-\frac{1}{2x_m^{\frac{1}{2}}}\right)^j \binom{n}{n+1-j} \pi k_i\right) C(n-1-j)x_m \frac{x_m^j}{j!}\right] + \frac{(-1)^n \binom{n}{1} \pi k_i\right) x_m^{n/2}}{1 \cdot 3 \cdot \ldots \cdot (2n-1)} \qquad (11c)$$

At present no theory is available for calculating the function $k = k(C_{abs})$. In the numerical calculations of this paper we have assumed the simple functional dependence

$$k = k_o (1 - C_{abs}/C_s)^y , \qquad (12)$$

where k_o and y are constants, and C_s is the saturation concentration of absorbed particles.

The time evolution of the factors $k_1, \ldots, k_n$ in Eq. (11) can be calculated by employing the differential equation (8). Some calculated breakthrough curves for a filter with L = 93 mm are shown in Figs. 2 - 5. From Fig. 2 we see that the shorter x_m is, the longer the filter remains effective, i.e. the better we can utilize its whole retention capacity. Fig. 3 shows that the filter is effective longer when y is smaller. This is natural, because the absorption probability function $k = k(C_{abs})$ decreases more slowly with C_{abs} when y is small. The effect of k_o, the absorption probability of a clean filter, is shown in Fig. 4. As C_s is constant, the filter is at the beginning more effective when k_o is greater. At $C_{out}/C_{in} = 0.7$ the breakthrough curves cross. The effect of the velocity $v_s = v_o C_{in}/C_s$, with which the

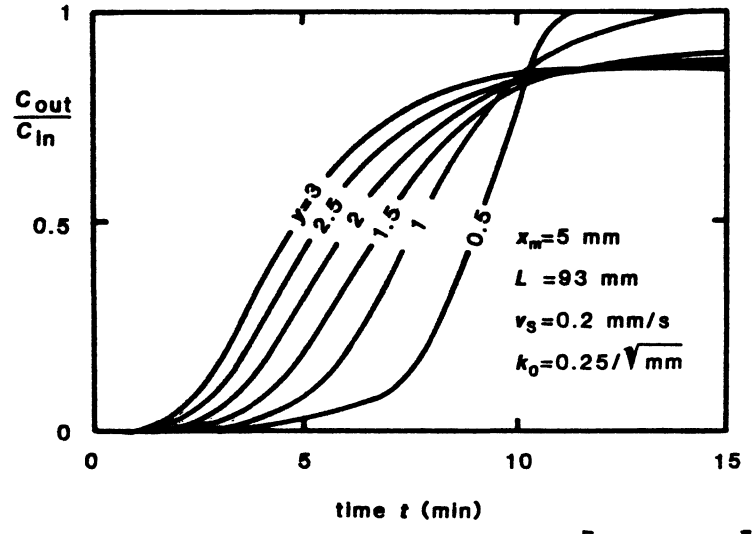

Fig. 3. Calculated effect of the exponent y [cf. Eq.(12)] on the breakthrough curve of an axial filter.

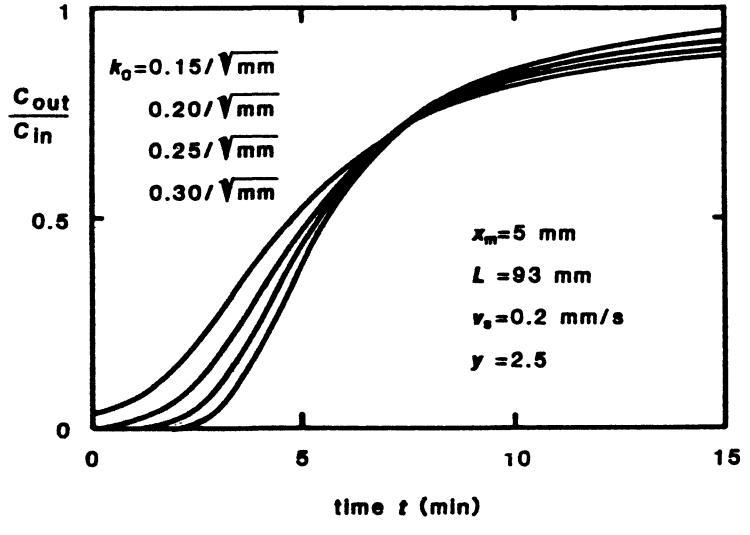

Fig. 4. Calculated effect of the clean filter absorption probability k_0 on the breakthrough curve of an axial filter.

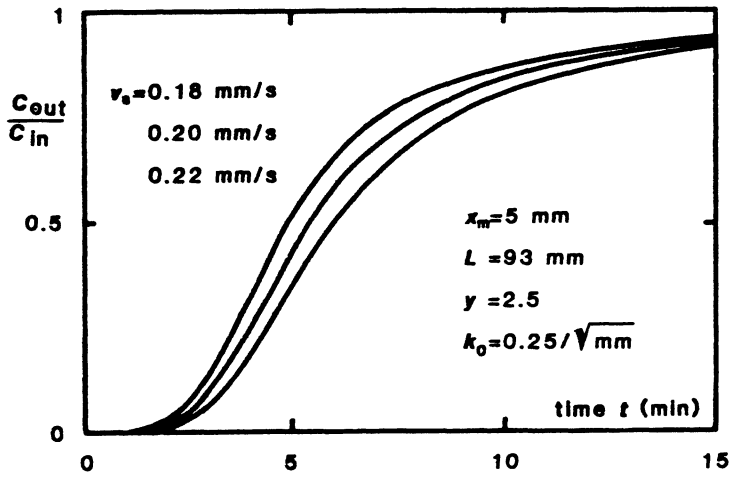

Fig. 5. Calculated effect of the velocity of the saturation front v_s on the breakthrough curve of an axial filter.

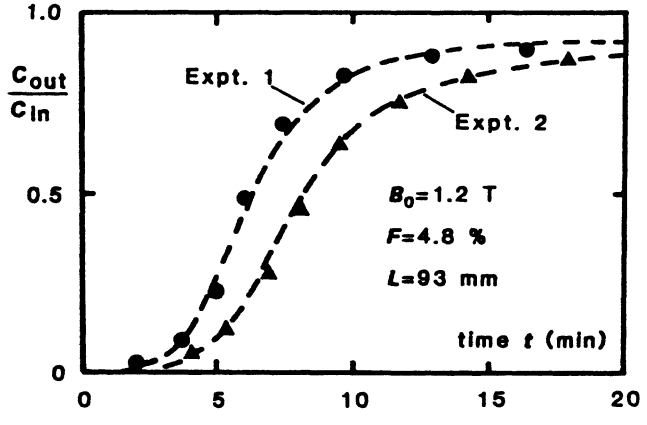

Fig. 6. Output concentration as a function of time in the experiments peformed to test the mathematical model presented. Solid symbols: measurement ; dashed line: calculation.

saturation front advances in the filter, is seen in Fig. 5. As expected, the effect of v_s is the same as the change of time scale.

EXPERIMENTAL TEST OF THE MATHEMATICAL MODEL

Fig. 6 shows results of a preliminary test of the proposed mathematical model. The laboratory experiments were made with a filter composed of a bundle of 12 μm stainless steel fibres[9] for which we measured a saturation polarization J_{as} = 0.34 T. The bundle was packed into a 20 mm diameter and 93 mm long plastic tube so that the packing factor was F = 4.8 %.

To obtain a test suspension, colloidal ferrite particles were mixed in water. From an EM micrograph we determined a particle diameter 2b = 0.34 μm. The measured saturation polarization of the particles was J_{bs} = 0.13 T.

The filter was placed horizontally into the inner bore of our superconducting magnet[10]. The field was B_o = 1.2 T, perpendicular to the steel fibers.

Turbidities of both the input and the output were continuously recorded. Turbidity was then transformed to concentration of solids with the aid of a turbidity-concentration relation measured from separate calibration samples.

The measured saturation concentrations C_s were found to be about three times those measured for longitudinal filters made of the same fibers. An explanation for this might be the mechanical instabilities of the deposition of particles onto the fibers when the flow is perpendicular to the fibers. At the same time, in an axial filter the particles, deposited on the fibers, are also better in the shade of each other.

For the two experiments performed, with different input concentrations C_{in}, we determined the parameters k_o, x_m and y. For the experiment 1, with C_{in}= 203 mg/l and v_o= 2.2 cm/s, fitting to the measured output gives C_s= 23 g/l, x_m=4 mm, y = 1.8, and k_o= 0.25 $mm^{-1/2}$. For the experiment 2, with C_{in}= 174 mg/l and v_o= 2.1 cm/s, we get C_s= 23.6 g/l, x_m= 4 mm , y = 1.8 and k_o= 0.26 $mm^{-1/2}$.

If we calculate k_o from Eq.(2) with $L = x_m$, we obtain k_o = 1.05 $mm^{-1/2}$. The measured k_o was about one fourth of this. An explanation for this might be that some of the particles hitting the fibres do not stick to them. There exists no model for calculating x_m. In our filter, however, the fibres touch each other on several points, and the average distance between the touching points is a few millimeters. The result of the fit, x_m = 4 mm, is thus physically reasonable.

DISCUSSION

We have developed here a mathematical model to account for the performance of axial magnetic filters. The model was , preliminarily, also tested by an experiment. A conclusive test of the model remains to be performed. It could be done by a controlled series of experiments with the filter length L varying. If x_m and k_o were found independent of L, there should be all reason to say that the model is verified.

At first sight, the biggest defect in our theory would seem to be that it does not produce a physically correct picture in the limit $x_m \to 0$. This is, however, of no practical consequence, since in all reasonable cases the mixing length x_m will be an order of magnitude larger than the average separation between the filter fibers. Thus, in practice, we are never close to the limit $x_m = 0$.

ACKNOWLEDGEMENT

The financial support by the Research Council for Technical Sciences, Academy of Finland, and by the Maj and Tor Nessling Foundation is greatfully acknowledged.

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