ON RAYLEIGH WAVES IN A THINLY LAYERED LAMINATED THERMOELASTIC MEDIUM WITH STRESS COUPLES UNDER INITIAL STRESSES*

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ABSTRACT

A study is made of the propagation of Rayleigh waves in a thinly layered laminated thermoelastic medium under deviatoric, hydrostatic, and couple stresses. The frequency equation of the Rayleigh waves is obtained. The phase velocity of the Rayleigh waves depends on the initial stress, deviatoric stress, and the couple stress. The laminated medium is first replaced by an equivalent anisotropic thermoelastic continuum. The corresponding thermoelastic coefficients (after deformation) are derived in terms of initially isotropic thermoelastic coefficients (before deformation) of individual layers. Several particular cases are discussed for the determination of the displacement fields with or without the effect of the couple stress.

Key words: Rayleigh Waves, Incremental Stresses, Thermal Stresses, Couple Stresses.

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1. INTRODUCTION

Based upon the general ideas of incremental deformations, Biot [1-3] first developed an exact solution of the problems of buckling and dynamics of multilayered and laminated structures with stress couples under initial stresses. Subsequently, Tolstoy [4] modified Biot's formulation and obtained simple explicit solutions of the dynamical equations for a prestressed solid under horizontal compression in a gravitational field. He showed how the magnitudes of the effects of two factors, overburden pressure and deviatoric stress (or stress difference) influence seismic wave propagation velocities.

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The laminated thermoelastic medium, which consists of a superposition of thin hard and soft adhering thermoelastic materials, is first approximated by an equivalent anisotropic medium. The corresponding composite incremental coefficients (after deformation) are obtained as functions of the coefficients before deformation and the initial stress components of individual layers. Pal Roy [5] investigated the problem of elastic wave propagation in a thinly layered laminated medium with stress couples under initial stresses. Further, Biot's analysis [2] also included the microelastic properties of the layers by the use of stress couples. This inclusion obviously makes a better approximation.

In his recent papers, Pal Roy [5-7] discussed some studies of elastic wave propagation in a thinly layered laminated medium with stress couples under initial stresses. This work deserves further extension in a thermoelastic medium.

Several authors including Lockett [8], Nowacki and Sokolowski [9] and Biot [3] have made some investigations of the thermoelastic wave propagation in different geometrical configurations. Among them, Biot's work includes the concept of generalized free energy along with a new definition of the dissipation function in terms of the time derivative of an entropy displacement. This work also represents a unified treatment of thermoelastic problems.

The main purpose of this work is to study the propagation of Rayleigh waves in a laminated thermoelastic medium under deviatoric, hydrostatic, and couple stresses. The frequency equation of these waves is derived. It is shown that the phase velocity of the Rayleigh waves depends on the initial stress, deviatoric stress, and the couple stresses. Several particular cases are discussed with or without the effect of the couple stresses.

2. STRESS-STRAIN RELATIONS FOR A THERMOELASTIC LAMINATED MEDIUM

We consider a thermoelastic laminated medium at an absolute temperature T . The medium is composed of thin adhering layers which are alternately hard and soft. The hard and soft materials are both compressible and they occupy fractions α_1 and α_2 of the total thickness. If the rigidity contrast of the layers is not too large, and the layer thickness remains sufficiently small with respect to the wavelength of the deformation field, then such a laminated medium may be approximated by a continuous medium of anisotropic properties. For simplicity and without loss of generality, we assume incremental deformations in the x, y plane only. If we increase the absolute temperature by a small amount θ , the incremental stresses for hard and soft materials can be written following Biot [1-3].

For the hard initially isotropic material, the incremental stresses t_{ij} related to strains e_{ij} are given by

$$(2.1) \quad t_{11}^{(1)} = a_1 e_{xx} + b_1 e_{yy}^{(1)} - \beta_1 \cdot \theta$$

$$(2.2) \quad t_{22} = b_1 e_{xx} + c_1 e_{yy}^{(1)} - \beta_1 \cdot \theta$$

$$(2.3) \quad t_{12} = 2L_1 e_{xy} \quad .$$

Fig. 1: Geometry of the problem.

For the soft material, the stress-strain relations are

$$(2.4) \quad t_{11}^{(2)} = a_2 e_{xx} + b_2 e_{yy}^{(2)} - \beta_2 \cdot \theta$$

$$(2.5) \quad t_{22} = b_2 e_{xx} + c_2 e_{yy}^{(2)} - \beta_2 \cdot \theta$$

$$(2.6) \quad t_{12} = 2L_2 e_{xy}$$

where a_k, b_k, c_k, L_k are the alternative elastic coefficients, $k = 1$ (hard material), $k = 2$ (soft material), and β_r ($r = 1, 2$) are thermal coefficients. Since soft and hard materials are both compressible, the strain component e_{yy} is not the same in each layer. Also we have

$$(2.7) \quad e_{yy} = \alpha_1 e_{yy}^{(1)} + \alpha_2 e_{yy}^{(2)}$$

$$(2.8) \quad t_{11} = \alpha_1 t_{11}^{(1)} + \alpha_2 t_{11}^{(2)}$$

where $\alpha_1 + \alpha_2 = 1$.

When a uniform shear stress t_{12} is applied, the total average shear strain is $e_{xy} = (2L)^{-1} t_{12}$, that is,

$$(2.9) \quad t_{12} = 2L e_{xy}$$

where $L = \left(\frac{\alpha_1}{L_1} + \frac{\alpha_2}{L_2} \right)^{-1}$ is the composite slide modulus. Since we are concerned with a uniform strain without shear, the normal strain component e_{xx} and the normal stress are the same in both materials. Eliminating the four variables, $t_{11}^{(1)}, t_{11}^{(2)}, e_{yy}^{(1)}$ and $e_{yy}^{(2)}$, from equations (2.1)-(2.8), we obtain, for the composite material of anisotropic nature,

$$(2.10) \quad t_{11} = C_{11} e_{xx} + C_{12} e_{yy} - A_1 \cdot \theta$$

$$(2.11) \quad t_{22} = C_{12} e_{xx} + C_{22} e_{yy} - A_2 \cdot \theta$$

where C_{ij} and A_i are the composite thermoelastic coefficients given by

$$(2.12ab) \quad C_{11} = \alpha_1 a_1 + \alpha_2 a_2 - \frac{\alpha_1 \alpha_2 (b_1 - b_2)^2}{\alpha_1 c_2 + \alpha_2 c_1}, \quad C_{22} = \frac{c_1 c_2}{\alpha_2 c_2 + \alpha_2 c_1}$$

$$(2.13) \quad C_{12} = \frac{\alpha_1 b_1 c_2 + \alpha_2 b_2 c_1}{\alpha_1 c_2 + \alpha_2 c_1}$$

$$(2.14) \quad A_1 = \frac{(\beta_1 \alpha_1 + \beta_2 \alpha_2) (\alpha_1 c_2 + \alpha_2 c_1) - (\beta_1 - \beta_2) (b_1 - b_2)}{(\alpha_1 c_2 + \alpha_2 c_1)}$$

$$(2.15) \quad A_2 = \frac{\alpha_1 \beta_1 c_2 + \alpha_2 \beta_2 c_1}{\alpha_1 c_2 + \alpha_2 c_1} .$$

If now we replace A_1 and A_2 by their average value $(1/2)(A_1 + A_2)$, the equations (2.9) - (2.11) become

$$(2.16) \quad t_{11} = C_{11} e_{xx} + C_{12} e_{yy} - A \cdot \theta$$

$$(2.17) \quad t_{22} = C_{12} e_{xx} + C_{22} e_{yy} - A \cdot \theta$$

$$(2.18) \quad t_{12} = 2L e_{xy}$$

where

$$(2.19) \quad A = [(\alpha_1 \beta_1 + \alpha_2 \beta_2) (\alpha_1 c_2 + \alpha_2 c_1) - (\beta_1 - \beta_2) (b_1 - b_2) + (\alpha_1 \beta_1 c_2 + \alpha_2 \beta_2 c_1)] / [2 (\alpha_1 c_2 + \alpha_2 c_1)] .$$

3. THE INITIALLY ISOTROPIC HARD MATERIAL

The state of initial stress introduces anisotropy (induced anisotropy) so that even for an initially isotropic solid defined by two Lamé's constants λ_1 and μ_1 , the number of B_{ij}^1 will always be larger than 2. The initial stress components S_{ij}^k are

$$(3.1a) \quad S_{11}^1 = -P_1 - \rho_1 g y$$

$$S_{22}^1 = -\rho_1 g y$$

where g is the acceleration due to gravity. The elastic coefficients $B_{ij}^{(k)}$ after deformation are obtained in [5] (see also Ref. [1], p. 111).

$$(3.2ab) \quad B_{11}^1 = (2\mu_1 + \lambda_1) \left(1 + \frac{S_{11}^1 - S_{22}^1}{2\mu_1}\right), \quad B_{22}^1 = (2\mu_1 + \lambda_1) \left(1 + \frac{S_{22}^1 - S_{11}^1}{2\mu_1}\right)$$

$$(3.3ab) \quad B_{12}^1 = \lambda_1 - S_{11}^1, \quad Q^1 = \mu_1 + \frac{1}{4}(S_{11}^1 + S_{22}^1)$$

The alternative elastic coefficients a_k , b_k , c_k and L_k are the same in the references [5] and [1], and have the form

$$(3.4ab) \quad a_1 = B_{11}^1 = (\lambda_1 + 2\mu_1) \left(1 - \frac{P_1}{2\mu_1}\right), \quad b_1 = B_{12}^1 - P_1 - \rho_1 g y = \lambda_1$$

$$(3.5ab) \quad c_1 = B_{22}^1 = (2\mu_1 + \lambda_1) \left(1 + \frac{P_1}{2\mu_1}\right), \quad L_1 = \mu_1$$

where P_k denotes the compression.

4. THE INITIALLY ISOTROPIC SOFT MATERIAL

Following the above exactly, the alternative elastic coefficients a_2 , b_2 , c_2 and L_2 are obtained as (Ref. [5] and [1], p. 61)

$$(4.1ab) \quad a_2 = (\lambda_2 + \mu_2) \left(1 - \frac{P_2}{2\mu_2}\right), \quad b_2 = \lambda_2$$

$$(4.2ab) \quad c_2 = (\lambda_2 + \mu_2) \left(1 + \frac{P_2}{2\mu_2}\right), \quad L_2 = \mu_2.$$

5. THE COMPOSITE ANISOTROPIC MATERIAL

From equations (2.12ab) - (2.15), (3.4ab) - (3.5ab), and (4.1ab) - (4.2ab), we derive the following:

$$(5.1) \quad C_{11} = \alpha_1(\lambda_1 + 2\mu_1) \left(1 - \frac{P_1}{2\mu_1}\right) + \alpha_2(\lambda_2 + 2\mu_2) \left(1 - \frac{P_2}{2\mu_2}\right) - \frac{\alpha_1\alpha_2}{K}(\lambda_1 - \lambda_2)^2$$

$$(5.2) \quad C_{12} = \frac{1}{K}[(\lambda_2 + 2\mu_2) \left(1 + \frac{P_2}{2\mu_2}\right) \alpha_1 \lambda_1 + (2\mu_1 + \lambda_1) \left(1 + \frac{P_1}{2\mu_1}\right) \alpha_2 \lambda_2]$$

$$(5.3) \quad C_{22} = \frac{1}{K} [(\lambda_1 + 2\mu_1) (\lambda_2 + 2\mu_2) (1 + \frac{P_1}{2\mu_1}) (1 + \frac{P_2}{2\mu_2})]$$

$$(5.4) \quad L = 1 / (\frac{\alpha_1}{\mu_1} + \frac{\alpha_2}{\mu_2})$$

$$(5.5) \quad A = \frac{1}{2} (\alpha_1 \beta_1 + \alpha_2 \beta_2) - \frac{1}{2K} [(\beta_1 - \beta_2) (\lambda_1 - \lambda_2) + \alpha_1 \beta_1 (\lambda_2 + 2\mu_2) (1 + \frac{P_2}{2\mu_2}) + \alpha_2 \beta_2 (\lambda_1 + 2\mu_1) (1 + \frac{P_1}{2\mu_1})] .$$

$$(5.6) \quad K = \alpha_1 (\lambda_2 + 2\mu_2) (1 + \frac{P_2}{2\mu_2}) + \alpha_2 (\lambda_1 + 2\mu_1) (1 + \frac{P_1}{2\mu_1}) .$$

6. COUPLE STRESS ANALOGY

Following Biot [2], the couple stress of the composite material is

$$(6.1) \quad M = b \frac{\partial^2 v}{\partial x^2}$$

where the couple stress coefficient, b , is

$$(6.2) \quad b = \frac{1}{3} h^2 \alpha_1 \alpha_2 \frac{(L_1 - L_2)}{(\alpha_1 L_1 + \alpha_2 L_2)} (M_1 \alpha_1^2 - M_2 \alpha_2^2) ,$$

and the total thickness h' is the sum of the thicknesses h_1 and h_2 of the layers so that

$$(6.3) \quad h' = h_1 + h_2$$

and

$$(6.4) \quad M_i = \frac{1}{4c_i} (a_i c_i - b_i^2) , \quad i = 1, 2 .$$

7. BASIC GOVERNING EQUATIONS

The equations of equilibrium in terms of alternative stress components t_{ij} are given by (Ref. [1], p. 61 and 264)

$$(7.1) \quad \frac{\partial t_{11}}{\partial x} + \frac{\partial t_{12}}{\partial y} + \left(\frac{P}{4} + \frac{\rho g y}{2}\right) \frac{\partial^2 v}{\partial x \partial y} + \left(\frac{P}{4} - \frac{\rho g y}{2}\right) \frac{\partial^2 u}{\partial y^2} = \rho \frac{\partial^2 u}{\partial t^2}$$

$$(7.2) \quad \begin{aligned} & \frac{\partial t_{12}}{\partial x} + \frac{\partial t_{22}}{\partial y} + \frac{1}{2} \left(\frac{P}{2} + \rho g y\right) \frac{\partial^2 u}{\partial x \partial y} - \frac{1}{2} \left(\frac{3}{2} P + \rho g y\right) \frac{\partial^2 v}{\partial x^2} + g \rho \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \\ & = \rho \frac{\partial^2 v}{\partial t^2} + b \frac{\partial^4 v}{\partial x^4} \end{aligned}$$

where u and v are the displacement fields, ρ is the density, and P is the average compression, while the effect of the couple stress of the composite medium has been incorporated in these equations.

Tolstoy [4] has shown, by taking a typical value of shear wave velocity $c_s \simeq 4 \times 10^3 \text{ ms}^{-1}$, that the effect of gravity (g) is approaching 1% of the inertial terms for periods $T \geq 255$. If then one assumes $(P/\mu) \geq 2 \times 10^{-2}$, the buoyancy term corrections will be appreciably smaller than those of prestress for periods of half a minute or less. As such, in dealing with typical seismic body wave frequencies, we shall therefore neglect the g (buoyancy) terms in equation (7.2) in comparison with the prestress terms. (We, however, keep the hydrostatic pressure terms in equations (7.1) and (7.2).)

When we use relations (2.9) - (2.11) and drop the buoyancy term in (7.2), equations (7.1) - (7.2) become

$$(7.3) \quad C_{11} \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 v}{\partial x \partial y} + D \frac{\partial^2 u}{\partial y^2} = \rho \frac{\partial^2 u}{\partial t^2} + A \frac{\partial \theta}{\partial x}$$

$$(7.4) \quad C_{22} \frac{\partial^2 v}{\partial y^2} + B \frac{\partial^2 u}{\partial x \partial y} + (D - P) \frac{\partial^2 v}{\partial x^2} = \rho \frac{\partial^2 v}{\partial t^2} + A \frac{\partial \theta}{\partial y} + b \frac{\partial^4 v}{\partial x^4}$$

where

$$(7.5ab) \quad B = C_{12} + L + \frac{1}{4}P + \frac{1}{2}\rho gy, \quad D = L + \frac{1}{4}P - \frac{1}{2}\rho gy.$$

Equations (7.3) - (7.4) are the modified forms of the equations previously obtained by Pal Roy [5]. The equation of heat conduction is obtained from Biot [3] in the form

$$(7.6a) \quad \nabla^2 \theta - \frac{a}{\kappa} \frac{\partial \theta}{\partial t} - \frac{TA}{\kappa} \frac{\partial e}{\partial t} = 0$$

where a is the specific heat of the unit volume in the absence of deformation, κ is the coefficient of heat conduction, T is the absolute temperature for the state of zero stress-strain, and

$$(7.6b) \quad e = e_{xx} + e_{yy} \equiv u_x + v_y.$$

We next express the displacement fields u and v in terms of the classical potentials ϕ and ψ as

$$(7.7ab) \quad u = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x}.$$

Thus equation (7.6) can be written in terms of ϕ and θ as

$$(7.8) \quad \nabla^2 \theta - \frac{1}{\xi} \frac{\partial \theta}{\partial t} - \eta \nabla^2 \left(\frac{\partial \phi}{\partial t} \right) = 0$$

where $\xi = \frac{\kappa}{a}$ and $\eta = \frac{TA}{\kappa}$.

In view of (7.7ab), equations (7.3) - (7.4) assume the form

$$(7.9) \quad \frac{\partial^2 \phi}{\partial x^2} + \frac{(B+D)}{C_{11}} \frac{\partial^2 \phi}{\partial y^2} = \frac{\rho}{C_{11}} \frac{\partial^2 \phi}{\partial t^2} + \frac{A \cdot \theta}{C_{11}}$$

$$(7.10) \quad \frac{\partial^2 \phi}{\partial x^2} + \frac{C_{22}}{(B+D-P)} \frac{\partial^2 \phi}{\partial y^2} = \frac{\rho}{(B+D-P)} \frac{\partial^2 \phi}{\partial t^2} + \frac{b}{(B+D-P)} \frac{\partial^4 \phi}{\partial x^4} + \frac{A \cdot \theta}{(B+D-P)}$$

$$(7.11) \quad \frac{\partial^2 \psi}{\partial x^2} + \frac{(C_{22} - B)}{(D - P)} \frac{\partial^2 \psi}{\partial y^2} = \frac{\rho}{(D - P)} \frac{\partial^2 \psi}{\partial x^2} + \frac{b}{(D - P)} \frac{\partial^4 \psi}{\partial x^4}$$

$$(7.12) \quad \frac{\partial^2 \psi}{\partial x^2} + \frac{D}{(C_{11} - B)} \frac{\partial^2 \psi}{\partial y^2} = \frac{\rho}{(C_{11} - B)} \frac{\partial^2 \psi}{\partial t^2}.$$

Plane wave solutions. We consider compressional and distortional waves along the x direction only. Eliminating θ from equations (7.8) and (7.9), we obtain (7.13) and retain (7.11) so that

$$(7.13) \quad \left(\nabla^2 - \frac{1}{\xi} \frac{\partial^2}{\partial t^2} \right) \left(\frac{\partial^2}{\partial x^2} + \frac{B+D}{C_{11}} \frac{\partial^2}{\partial y^2} - \frac{\rho}{C_{11}} \frac{\partial^2}{\partial t^2} \right) \phi - \eta m \nabla^2 \left(\frac{\partial \phi}{\partial t} \right) = 0$$

$$(7.14) \quad \frac{\partial^2 \psi}{\partial x^2} + \frac{C_{22} - B}{D - p} \frac{\partial^2 \psi}{\partial y^2} = \frac{\rho}{D - p} \frac{\partial^2 \psi}{\partial t^2} + \frac{b}{D - p} \frac{\partial^4 \psi}{\partial x^4}$$

For a plane harmonic wave propagating in the x direction, we seek solutions in the form

$$\phi(x, y, t) = \Phi(y) \exp[i\omega(t - x/c)]$$

(7.15ab)

$$\psi(x, y, t) = \Psi(y) \exp[i\omega(t - x/c)]$$

where c is the phase velocity of the Rayleigh waves in the thermoelastic medium under initial stresses.

Putting equations (7.15ab) into equations (7.13) - (7.14), we obtain

$$(7.16) \quad [(D^2 - \alpha^2 - q)(D^2 - \alpha_1^2 + \sigma^2) - q\varepsilon(D^2 - \alpha^2)]\Phi(y) = 0$$

$$(7.17) \quad (D^2 - \alpha_2^2 + \tau^2)\Psi(y) = 0$$

where $D \equiv \frac{d}{dy}$, $q = \frac{i\omega}{\xi}$, $\alpha^2 = \omega^2/c^2$,
 $\alpha_1^2 \equiv \alpha^2 C_{11}/(B + D)$, $\sigma^2 = \rho\omega^2/(B + D)$, $\varepsilon = m\eta\xi C_{11}/(B + D)$,
 $\alpha_2^2 = \alpha^2(D - P)/(C_{22} - B)$, $\tau^2 = \frac{\rho\omega^2}{C_{22} - B} \left(1 - \frac{b\alpha^2}{\rho c^2}\right).$

The solutions of equations (7.16) - (7.17), satisfying the boundary conditions in the form of the corresponding stresses decaying exponentially with depth, have the form:

$$\Phi(y) = A_1 e^{-\lambda_1 y} + B_1 e^{-\lambda_2 y} \quad (7.18ab)$$

$$\Psi(y) = C_1 e^{-\gamma y}$$

where λ_1 and λ_2 are the roots of the biquadratic equation

$$(7.19) \quad \lambda^4 - \lambda^2(\alpha^2 + \alpha_1^2 + q_1 - \sigma^2) + (\alpha^2 - q)(\alpha_1^2 - \sigma^2) + q\epsilon\alpha^2 = 0$$

and

$$(7.20) \quad q_1 = q(1 + \epsilon)$$

$$(7.21) \quad \gamma = (\alpha_2^2 - \tau^2)^{1/2} = (\alpha^2 e^2 - \tau^2)^{1/2}$$

$$(7.22) \quad e^2 = (D - p)/(C_{22} - B).$$

From equations (7.15ab) and (7.18ab), we obtain

$$\phi = (A_1 e^{-\lambda_1 y} + B_1 e^{-\lambda_2 y}) \exp[i\omega(t - x/c)] \quad (7.23ab)$$

$$\psi = C_1 e^{-\gamma y} \exp[i\omega(t - x/c)]$$

and θ can be found from (7.9) in the form

$$(7.24) \quad \theta = \frac{1}{m} e^{i\omega(t - x/c)} [\eta_1 A_1 e^{-\lambda_1 y} + \eta_2 B_1 e^{-\lambda_2 y}],$$

where

$$(7.25ab) \quad \eta_1 = d^2(\lambda_1^2 - \alpha_1^2 + \sigma^2), \quad \eta_2 = d^2(\lambda_2^2 - \alpha_1^2 + \sigma^2)$$

and

$$(7.26) \quad d^2 = (B + D)/C_{11}.$$

8. THE FREQUENCY EQUATION FOR THE RAYLEIGH WAVES

We set the boundary conditions on the plane $y = 0$ as (Biot [1] and Nowacki [10])

$$(8.1abc) \quad e_{xy} = 0, \quad t_{22} = 0, \quad \frac{\partial \theta}{\partial y} + h\theta = 0$$

where h is the ratio of the coefficients of heat transfer to the thermal conductivity.

We next use (2.9) - (2.11), (7.7ab), and (7.23ab) - (7.24) to rewrite the boundary conditions (8.1abc) in the form

$$(8.2) \quad 2\alpha\lambda_1 A_1 + 2\alpha\lambda_2 B_1 + i(\gamma^2 + \alpha^2)C_1 = 0$$

$$(8.3) \quad (\lambda_1^2 C_{22} - C_{12}\alpha^2 - \frac{A\eta_1}{m}) A_1 + (\lambda_2^2 C_{22} - \alpha^2 C_{12} - \frac{A\eta_2}{m}) B_1 + \gamma k (C_{22} - C_{12}) C_1 = 0$$

$$(8.4) \quad A_1 \eta_1 (\lambda_1 - h) + B_1 \eta_2 (\lambda_2 - h) = 0.$$

Eliminating A_1 , B_1 and C_1 from these results, we get

$$(8.5) \quad \begin{vmatrix} 2\alpha\lambda_1 & 2\alpha\lambda_2 & \gamma^2 + \alpha^2 \\ \lambda_1^2 C_{22} - C_{12}\alpha^2 - \frac{A\eta_1}{m} & \lambda_2^2 C_{22} - \alpha^2 C_{12} - \frac{A\eta_2}{m} & \gamma\alpha (C_{22} - C_{12}) \\ \eta_1 (\lambda_1 - h) & \eta_2 (\lambda_2 - h) & 0 \end{vmatrix} = 0$$

Expanding this determinant, we obtain the frequency equation for the Rayleigh waves

$$(8.6) \quad \begin{aligned} & \eta_1 (\lambda_1 - h) [2\gamma\lambda_2 \alpha^2 (C_{22} - C_{12}) - (\gamma^2 + \alpha^2) (\lambda_2^2 C_{22} - \alpha^2 C_{12} - \frac{A\eta_2}{m})] \\ & + \eta_2 (\lambda_2 - h) [(\gamma^2 + \alpha^2) (\lambda_1^2 C_{12} - C_{12}\alpha^2 - \frac{A\eta_1}{m}) \\ & - 2\alpha^2 \gamma \lambda_1 (C_{22} - C_{12})] = 0. \end{aligned}$$

It is clear from equation (8.6) that the phase velocity of the Rayleigh waves depends on the initial stress, deviatoric stress, and couple stress.

9. SOME PARTICULAR CASES OF INTEREST

We assume a convection condition (8.1c) for the temperature on the boundary. In the particular case $h = 0$, we deal with thermal insulation; while for $h \rightarrow \infty$, the temperature vanishes on the boundary. In the particular case of the uncoupled problem ($\varepsilon = 0$), we have

$$(9.1ab) \quad \lambda_1 = (\alpha^2 + q)^{\frac{1}{2}}, \quad \lambda_2 = \left(\frac{\alpha^2}{d^2} - \sigma^2 \right)^{\frac{1}{2}}$$

$$(9.2ab) \quad \eta_1 = (d^2 - 1)\alpha^2 + d^2(\sigma^2 + q), \quad \eta_2 = 0.$$

Equation (8.6) now becomes

$$(9.3) \quad 2 \sqrt{\frac{1}{d^2} - \frac{\sigma^2}{\alpha^2}} \sqrt{e^2 - \frac{\tau^2}{\alpha^2}} (C_{22} - C_{12}) - (1 + e^2 - \frac{\tau^2}{\alpha^2}) \left[\left(\frac{1}{d^2} - \frac{\sigma^2}{\alpha^2} \right) C_{22} - C_{12} \right] = 0,$$

when there is no couple stress ($b = 0$) and equation (9.3) reduces to the form

$$(9.4) \quad 2 \left\{ \left(\frac{1}{d^2} - \frac{X^2}{\mathcal{L}^2} \right) (e^2 - X^2) \right\}^{\frac{1}{2}} \left(1 - \frac{C_{12}}{C_{22}} \right) - \left(1 + e^2 - X^2 \right) \left(\frac{1}{d^2} - \frac{X^2}{\mathcal{L}^2} - \frac{C_{12}}{C_{22}} \right) = 0,$$

where

$$(9.5) \quad X = \frac{c}{c_1}, \quad c_1^2 = \frac{C_{22} - B}{\rho} = \frac{C_{22} - C_{12} - L - \frac{P}{4}}{\rho}, \quad \mathcal{L}^2 = \frac{\tau^2}{\sigma^2}.$$

We consider a thinly layered laminated thermoelastic medium of 50 cm. in thickness. The hard and soft materials occupy respectively 30 cm. and 20 cm. of the total. The following dimensions are taken into account:

$$\lambda_1 = \mu_1 = 6 \times 10^{11} \text{ dynes/cm}^2, \quad \beta_1 = 4 \times 10^{11} \text{ dynes/cm}^2$$

$$\lambda_2 = \mu_2 = 3 \times 10^{11} \text{ dynes/cm}^2, \quad \beta_2 = 2.5 \times 10^{11} \text{ dynes/cm}^2$$

$$\alpha_1 = \frac{3}{5}, \quad \alpha_2 = \frac{2}{5}, \quad \frac{P_1}{2\mu_1} = 0.48, \quad \frac{P_2}{2\mu_2} = 0.32.$$

Then from formulas (5.4), (5.6), and (6.4) and the above results we obtain Tables 1 and 2.

Table 1

$10^{11} \text{ dynes/cm}^2$			
L	K	M_1	M_2
4.2857143	17.784	2.0021622	1.3406061

Table 2 Non-Dimensional Values of Constants

C_{11}/L	C_{12}/L	C_{22}/L	A/L	b/L	$P/2L$	m
1.8532599	0.9805668	4.1523886	0.00533738	14.766541	0.9856	0.00288

We have, from equation (9.4),

$$(9.6) \quad X^8 - 8.1808195 X^6 + 19.231638 X^4 - 12.643484 X^2 + 9.3293485 = 0.$$

The corresponding equation when the effect of couple stress is taken into account becomes

$$(9.7) \quad X^8 - 8.1869549 X^6 + 19.274874 X^4 - 12.725369 X^2 + 9.346593 = 0.$$

The equation (9.6) has a real root 1.8698823 (when $b = 0$). The corresponding root of equation (9.7) is 1.8697807 (when $b \neq 0$). This shows the measure of the structure's couple stress effect on the velocity of Rayleigh wave propagation. Assuming the effect of hydrostatic pressure is small, i.e. at a depth of y cm. $M = \frac{\rho g y}{L}$ is small, we can express the functions ϕ and ψ .

(a) In the absence of couple stress ($b = 0$), we find

$$(9.8) \quad \phi = B_1 \exp[-i\alpha(0.2530411 - 1.9891009M)y + i\omega(t - x/c)]$$

$$\begin{aligned} \psi = B_1 & (-0.2143526 + 0.7331583M) \times \\ (9.9) \quad & \exp[-i\alpha(1.8333055 + 0.1224613M)y + i\omega(t - x/c)] \end{aligned}$$

(b) In the presence of couple stress ($b \neq 0$), we obtain

$$(9.10) \quad \phi = B_1 \exp[-i\alpha y(0.2530411 - 1.9891009M) + i\omega(t - x/c)]$$

$$\begin{aligned} (9.11) \quad \psi = B_1 & (-0.2009407 + 0.6738628M - 0.2005658x^2 + 0.5684386M\alpha^2) \times \\ & x \exp[-i\alpha y(1.9556071 - 1.9520222\alpha^2 - 1.013815M\alpha^2) + i\omega(t - x/c)] \end{aligned}$$

The displacement components can be easily be obtained from equations (7.7ab), (9.7) - (9.8) and (9.9) - (9.11).

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