A CATASTROPHIC YIELD THEORY OF SHEAR BAND FORMATION IN α -BRASS

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SUMMARY

This paper emphasises the role that dislocation pile-ups have in promoting instability that leads to shear banding, and the Dillamore formalism is used to demonstrate their importance. The model is capable of being applied to both laminar structures and particle hardened alloys. By incorporating the idea of a critical length in which shear band branching can dissipate shear imposed from neighbouring grains, it is possible to estimate the rolling strain at which shear banding changes from being grain scale to sample scale.

SYMBOLS

β	Shear Band Angle
m	Strain Rate Sensitivity
N	Mobile Dislocation Density
M	Taylor Factor
M'	Effective Taylor Factor
w	Grain Thickness
b	Burger Vector
n	Work Hardening Index
n_p	Number of Dislocation in the Pile-Up
n_{pi}	Number of Pile-Up Dislocation of the i^{th} Slip System
γ_i	Shear Strain of the i^{th} Slip System
ξ_i	Angle between i^{th} Slip System and RD
$\hat{R}$	Unit Vector of the Rolling Directing
$\hat{n}_i$	Unit Vector of the normal of the i^{th} Slip System
t	Shear Band Thickness
Λ	Dislocation Avalanche Factor
l_b	Critical Shear Band Branching Length

- d Initial Grain Size
r Rolling Reduction

1. INTRODUCTION

There is a considerable body of work concerning shear bands in FCC metals based upon the ideas that (i) a strong texture is necessary, (ii) the material has a laminar microstructure and (iii) that shear bands are a form of localised instability caused by geometric softening when the instability condition is reached¹ i.e. when

$$\frac{1}{\sigma} \frac{d\sigma}{d\epsilon} = \frac{n}{\epsilon} + \frac{m}{\epsilon} \frac{d\epsilon}{d\epsilon} + \frac{1+n+m}{M} \frac{dM}{d\epsilon} - \frac{m}{N} \frac{dN}{d\epsilon} \leq 0 \quad (1)$$

According to eqt (1) the geometrical softening makes $(1/M \, dM/d\epsilon)$ go negative, hence shear bands result. This explanation was considered by Lee et.al.,² to be incapable of explaining the formation of macroscopic shear bands formed in α brass which had been heavily rolled in one direction, and then re-rolled by ~5-10% in the transverse direction. Instead, they considered the naturally negative term in $dN/d\epsilon$, which Dillamore et.al.¹ had associated with yield point behaviour. In this case the dislocations are forced to glide in structures determined by the earlier rolling deformation, and these gliding dislocations render the pre-existing structures unstable. The shear bands are macroscopic, i.e. they cross the material from surface to surface, because crystals which normally would accommodate the shear shape changes of their neighbours by slip³, are no longer able to do so because of the rotation of the material through 90°. The instability condition is determined in this case by the yield point term which was shown, for a dislocation pile-up, to be given by

$$\frac{m}{N} \frac{dN}{d\epsilon} = \frac{mM \sin \beta}{Nwb}$$

This re-rolling experiment has led to a reconsideration of the observed transition from grain scale to macroscopic shear banding in conventional rolling which occurs in α -brass at rolling strains of $\geq 90\%$ ⁴. It has also been found, using a reformulation of the term in $dN/d\epsilon$, together with the process of shear band branching, to give a unified theory of shear banding at both the grain and sample scale which is also capable of including laminar and particle hardened materials.

2. THE DISLOCATION PILE-UP MODEL

Consider a finely twinned volume with a potential shear band at an angle β to the twin lamellae, as shown in Fig. 1a. Dislocations pile-up against the twin boundaries and, at a critical stress the piled up dislocations break through and propagate as a shear band using polyslip. The density of dislocations freed is given by

$$dN = \frac{dn_p}{t_w} \sin \beta \quad (2), \quad \text{and the strain by } d\epsilon = \frac{\sum |\gamma_i|}{M} \quad (3)$$

where the summation is done over the active slip systems in the shear band. Now the shear strain on the i th system γ_i is given by

$$\gamma_i = \frac{n_{pi} b}{t \sin \xi_i} \frac{\sin \beta}{\sin \beta}$$

writing $\sin \xi_i$ as $|\hat{R} \cdot \hat{n}_i|$ and rearranging gives

$$n_{pi} = \left(\frac{t |\hat{R} \cdot \hat{n}_i|}{b \sin \beta} \right) \gamma_i$$

$$\text{Now } n_p = \sum n_{pi} = \frac{t}{b \sin \beta} \sum |\hat{R} \cdot \hat{n}_i \gamma_i| \quad (4)$$

$$\text{i.e. } dN = \frac{1}{bw} \sum |\hat{R} \cdot \hat{n}_i \gamma_i| \quad (5)$$

hence the yield point function in equation 1 is

$$\frac{m}{N} \frac{dN}{d\varepsilon} = \frac{mM}{Nwb} \frac{\sum |\hat{R} \cdot \hat{n}_i \gamma_i|}{\sum |\gamma_i|}$$

$$\text{If we define } \Lambda = M \frac{\sum |\hat{R} \cdot \hat{n}_i \gamma_i|}{\sum |\gamma_i|}$$

$$\text{then } \frac{m}{N} \frac{dN}{d\varepsilon} = \frac{m}{Nwb} \Lambda \quad (6)$$

and Λ is called the dislocation avalanche factor; the larger Λ , the more likely is instability to arise. Λ is computed for the twin related orientations $(111)[\bar{1}12]$ and $(111)[11\bar{2}]$ for positive shear bands in Fig. 2, and also plotted are the effective Taylor factors¹. It is important to note that Λ has relatively large values over the range of angles allowed for which shearing requires minimum energy i.e. minimum M' . The orientation $\{111\}\langle 110 \rangle$ is of importance in the brass texture and Λ and M' for both $(111)[\bar{1}10]$ and $(111)[11\bar{0}]$ are shown in Fig. 3. Only one curve is necessary due to the symmetry of this orientation, and because the curve in Λ is rather flat over the angular range where M' is minimum, i.e. from $\sim 30^\circ - 60^\circ$, hence instability can occur over a wide angular range.

3. PROPAGATION ANGLES

The value of Λ is not considered to determine of the angle of shearing in this formulation. It seems intuitively more reasonable to consider the effective Taylor factor M' , and such other factors as reduced slip conditions⁵ and geometric softening¹ as angle determinants, but the values of Λ along with $1/M \, dM/d\varepsilon$ must be such as to trigger instability, i.e. the instability criterion, eqt 1, must be met. In this sense Λ and the propagation angles are related. Clearly Λ is near to maximum at 35° in $(111)[\bar{1}12]$, M' is minimum, and only two slip systems are needed for shear at $\sim 35^\circ$, Fig. 2a, and these condition are not met for a positive shear band making $\sim 60^\circ$ to the rolling direction in its twin, Fig. 2b.

4. PARTICLE HARDENED SYSTEMS

Although the model has been developed for laminar structures, in principle any system capable of producing pile-ups should be covered by equation 1. It is well known that precipitate hardened aluminium produces well developed shear bands on rolling, while commercially pure material does not, and it necessary to associate shear banding with the particles. The plane strain conditions favour precipitate cutting when the interparticle spacing is small rather than looping or voiding, hence the term in $dN/d\varepsilon$ is of great importance.

5. MOBILE DISLOCATION DENSITY

It is obviously difficult to estimate the mobile dislocation density with great accuracy, hence it is not possible to predict the strain at which instability occurs. However, if we take reasonable values for the parameters it is possible to estimate the mobile dislocation density. Assume that for a material with $n = 0.2$ and $m = 0.1$, and that the material become unstable when $\varepsilon = 0.7$. For Dillamore's average texture, $1+n+m/M \, dM/d\varepsilon = 0.65$ and $n/\varepsilon = 0.29$, thus $m\Lambda/Nwb \approx 1$ when equation (1) has a value of zero. Hence $N \approx 3 \times 10^{13} \, \text{m}^{-2}$. This figure has to be compared with the total dislocation density for annealed and heavily cold worked materials,

commonly given as 10^{12} and 10^{16} m^{-2} respectively. This derived value for the density of mobile dislocation when $\varepsilon = 0.7$ is not considered unreasonable.

6. SHEAR BAND BRANCHING AND THE TRANSITION TO MACROSCOPIC SHEARING

It has been established that some orientations are prone to shear banding, notably $\{111\}\langle uvw \rangle$ in α brass, but there are significant volume fractions in the Goss and Brass orientations, whose behaviour was not known. Recent work using the crystallographic etch developed by Kohlhoff et.al.,⁵ for copper and copper based alloys has allowed the behaviour of the orientations Goss and Brass to be known³. Brass-oriented crystals are of two kinds, one showing a uniform structure, the other a framework structure very similar to those previously described as arising in cross rolled brass⁶. The uniform structure dissipates shear by slip but grains with the framework structure are sheared. This branching is also common in Goss oriented grains, and branching prevents propagation of shear by dissipating the strain³. An example of shear band branching is shown in Fig. 4. These observations imply that there is a critical length ℓ_b which must exist in a grain if the imposed shear band strain is to be accommodated. It is thus clear that if a branching grain is not thick enough to provide the branching length, the undissipated shear must be passed to neighbouring grains and, under such conditions of small projected grain thickness in the longitudinal section, greatly increases the probability of 'macroscopic' shearing. Now, it is clear that as rolling strain increases grain thickness decreases and hence shear banding is likely to propagate from grain to grain as both branching grains as well as crystallographically soft grains become thinner. Hence, if we assume that both crystallographically soft and branching grains become ineffective at absorbing neighbouring shears at the same thickness, then macroscopic shear banding occurs when

$$\ell_b \sin \beta > d (1-r) \quad (7)$$

where d = initial grain size
 r = reduction

The transition from single grain shear banding to macroscopic shear banding thus depends on initial grain size and rolling reduction.

Considering α -brass with an initial grain size of $30 \mu\text{m}$, and shear band angles of $\sim 35^\circ$, experimental observation using the Kohlhoff etch shows that ℓ_b is of the order of a few microns. From equation (7) the strain at which macroscopic shear bands form is of the order of 90%, which is in accordance with the early work of Duggan et.al.,⁴.

7. REFERENCES

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Fig. 1 a) Geometry of a finely twinned grain with a potential shear band. b) Dislocation pile up on the i -th slip system at a twin boundary.

Fig. 2. Dislocation avalanching factor (Λ) and effective Taylor factor (M') for a positive shear band in a) $(111)[-1-1 2]$, b) $(111)[1 1-2]$ crystal.

Fig. 3. Dislocation avalanche factor (Λ) and effective Taylor factor (M') for a positive shear band in $\{111\} \langle 110 \rangle$ crystals.

Fig. 4. TEM showing a shear band propagating its strain into a homogeneously deforming grain by branching of localised coarse slip bands.