USING THE MEAN FIELD MODEL TO ANALYZE THE INFLUENCE OF TEXTURE ON THE HYSTERESIS BEHAVIOUR OF SILICON STEELS

T. KOZINA AND J.A. SZPUNAR
Department of Mining and Metallurgical Engineering
McGill University, Montreal,
H3A 2A7 Canada

ABSTRACT

A critical study of the Jiles and Atherton Mean Field Model was done, to determine the validity of the model, a tool in describing and understanding the magnetization process in textured silicon steels.

Hysteresis loops were generated using an Epstein apparatus in various directions with respect to rolling and for various external magnetic fields. Various techniques of generating the loops from the model and analyzing the experimental results were proposed. These techniques were then used to obtain the model parameters.

An analysis of the experimental data using this model lets us conclude that the model gives a close description of texture influence on hysteresis behaviour and predicts the variation of the pinning parameter k which agrees with our understanding of the role of texture in changing the parameter. We have observed that the highest value of this parameter coincides with the angles at which it is most difficult to magnetize the specimen.

TEXTURE AND ANISOTROPY OF MAGNETIC PROPERTIES OF SILICON STEEL

Magnetic steels used to build transformers and generators have anisotropic magnetic properties. The power loss and permeability are important in the applications of these materials, and are strongly influenced by texture.

Controlling texture is especially important in oriented silicon steels. The power loss can be reduced by increasing the strength of the [110]<001> texture, and also by reducing the sheet thickness, increasing the silicon content and influencing the nucleation of reversible domains.

Texture influences the magnetization process in steels both through the domain wall movement and the rotation of the direction of magnetization. The alignment of an easy magnetic direction along the direction of the applied magnetic field, generates, through the movement of the 180 degree domain walls, a large change in magnetization. The total energy required for rotating the magnetic moments towards the direction of magnetization is also influenced by texture.

Various attempts were made in order to correlate the texture with the anisotropy of important properties like the power losses and permeability. The influence of texture on these properties is calculated by averaging the properties over the orientation distribution function. The methods of averaging have been discussed in previous papers. The calculated anisotropy of properties are compared in these papers to experimental data obtained at saturation. For the magnetic field strength lower than saturation, the theory of averaging the properties over the texture function would be much more complex, since not only the grain orientation, but also magnetostatic and magnetoelastic interactions between grains, grain shape and other factors contribute to the complexity of the magnetization process in steels. Explanations of the magnetization curves and magnetic hysteresis in textured silicon steels is therefore a difficult and complex problem which can barely be solved, even with a drastic simplification of the magnetization process in textured steels.

THE MEAN FIELD MODEL AND THE MEAN FIELD HYSTERESIS EQUATION

The complexity of magnetization processes in polycrystalline materials, where parameters such as strain, texture and grain size affect the ferromagnetic domain movement and rotational processes is so great, that often only simple models can be used to interpret the magnetization process. We would like to discuss a model developed by Jiles & Atherton and later used by Szpunar & Szpunar to analyze the hysteresis curves of deformed and textured steel specimens respectively.

One of the most simple models of the magnetization process is the Langevin model, where magnetization changes are described using the Langevin function as a function of an applied external field. The theory assumes a random distribution of magnetic moments and a condition of reversibility. The Langevin model was
proposed for paramagnetic materials and therefore does not describe hysteresis.

Before introducing the Langevin model, it is convenient to introduce the following shorthand notation:
\[
L^0(x) \text{ is the Langevin function and is defined as:}
\]
\[
L^0(x) = \coth(x) - (1/x^2)
\]

We shall now define successive derivatives of the Langevin functions as follows:
\[
L^0(x) = \coth(x) - (1/x^2)
\]
\[
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\]

If we define \( M \) at saturation as \( M_s \), then the Langevin equation for magnetization is:
\[
M = M_s \left( L^0(\Omega/A) \right)
\]
where \( A \) is constant depending on the magnetic properties of the material. In fact \( A = k_0 T/m \) where \( m \) is the magnetic moment, \( T \) is the temperature in Kelvin and \( k_0 \) is the Boltzmann’s constant (1.38 x 10^{-23} J/K).

Let us now modify the Langevin model in two ways by assuming that the magnetization is affected by the magnetization of neighbouring grains as well as the applied external field and by assuming that the magnetization phenomenon also involves irreversible steps.

Let us discuss and justify these two assumptions: the magnetic domains are not only affected by the external field strength, but also, directly or indirectly, by the magnetism of nearby magnetic domains. Furthermore, neighbouring domains are physically coupled through the domains walls. Thus, the resultant magnetization is not only a function of the applied field strength but also of the bulk magnetization of the specimen. In other words the resultant magnetization is a function of a modified field instead. If we want the simplest relationship describing the modified external field, then it will be of the form \( C_1 M + C_2 H \) where \( C_1 \) and \( C_2 \) are two constants.

Domain movement and rotation involve an energy loss. Let us define a general resistance \( R \) that impedes magnetization, always contrary to the change in magnetization. If we want a very simple relationship then we let this resistance be proportional to the incremental magnetic susceptibility. The resistance term then becomes:
\[
R = C_1 \frac{dM}{dH}
\]

Thus we get a modified Langevin model of the form:
\[
M = M_s \left( L^0(C_1 M + C_2 H) - C_2 \frac{dM}{dH} \right)
\]

Such an equation could be very useful to us for studying magnetic materials. Jules and Atherton proposed such an equation assuming that the energy, which is supplied to the specimen during a magnetization process, is split into two parts. One part of the energy is dissipated against pinning and the other part is responsible for reversible changes in magnetization. This energy balance can be expressed by:
\[
\int L^0(\alpha M + H) \, dB_e = \int M \, dF_e + \delta / k \, dM
\]

By successive integrating by parts, and setting limits for a hysteresis loop, the equation can be rewritten into a non-differential equation.

Letting \( (\alpha M + H) / \alpha = F_M \) and \( (\alpha M + H) / \alpha = F_{m}, \) we obtain the mean field hysteresis equation:
\[
M = M_s \left( L^0_F(F_M) + (k_0 \delta a) * (L^0_P(F_m)) + 2 * (k_0 \delta a) * (L^0_{P'}(F_m) - L^0_{P''}(F_m)) + \ldots \right)
\]

where the higher terms can be neglected because they are small. This equation was then fitted to the experimental data and the parameters of the model obtained.

THE FITTING PROCEDURE

Semi-numerical technique has been developed to analyze the experimental hysteresis loops in various
textured materials by Szpunar and Szpunar. An approximate solution of the hysteresis equation was obtained from which analytical formula relating parameters of the model to the coercive force, the remanence and the initial susceptibility were derived. In order to obtain the best fit, the parameters $k$ and $\alpha$ were derived analytically from the equation for the susceptibility and the coercive force. Finally only one parameter has been fitted. There is a need however to develop a new technique which will be used routinely to analyze many different samples. Still another advantage of having a fully numerical technique is being able to add more parameter terms in the future to accomodate improved models.

In the program the following five incoming data points were used: $M_s$, the susceptibility at 3 oersteds, $a$, $H_c$ and the coercivity. The program developed is fully automatic, and the criteria for fitting is specified by the user. The secant method was used throughout to solve all equations. The result of the program is a data file which can be plotted if desired. The $M_s$ was fixed since it a main parameter of the mean field model as well as being directly measurable. Specifying $M_s$ greatly simplified the solution of the fitting problem. Furthermore, we wish to use the secant method wherever possible to speed up program execution, and the secant method requires fixed points.

![Hysteresis loops](image)

Figure 1  Hysteresis loops measured (---) and calculated (- - - -) for direction $\alpha = 0^\circ$, for various maximum field strength.

The two points chosen for the secant method were the endpoint and the pseudo-coercivity. Three points were not used because the mean field model equation does not guarantee a three point solution. In this technique, a pair of secant loops were used, to solve for $k$ and $\alpha$ simultaneously. $\alpha$ was found by calculating the error in fitting and searching for the corresponding minimum error condition. In the proposed improved mean field model, where $M_s$ is allowed to vary from experimental data, $M_s$ and $\alpha$ were found by calculating the error in fitting. Various combinations of the fitting parameters were attempted. First we assumed that $M_s$ must be taken from data. Although the computer program worked successfully, the fittings were often poor. To improve the fittings, we tried varying $M_s$. Such an assumption has resulted in a much better fitting. This was expected, as the model then had one extra parameter.
One may argue that $M_s$ should remain constant for the same material, because at infinite field strength, all magnetic moments of atoms align along the direction of the field. However, we would like to argue that it takes a very large field strength to achieve close-to-saturation levels, and in fact even at field strengths of 100 Oersteds, differences in $M_s$ are still in the order of 40%. Furthermore, we are only discussing results up to 5 Oersteds.

![Hysteresis loops measured and calculated for rolling direction $\alpha = 40$, for various maximum field strengths](image)

**EXPERIMENTAL RESULTS AND ANALYSIS**

The measurements of magnetic hysteresis were carried out on silicon steel using an Epstein apparatus at various angles with respect to the rolling direction for the maximum field strength of 1 Oe., 3 Oe. and 5 Oe.

The hysteresis curves were obtained in 10 degree intervals. The texture of the steel was measured using x-ray diffraction, to obtain three pole figures (110), (200), (211), from which the crystal orientation distribution function ODF was determined. The analysis of the ODF shows that there is a strong GOSS type orientation present having at most 30 random units.
HYSTERESIS BEHAVIOUR OF SILICON STEELS

Exemplary results of the 1, 3, and 5 Oe. fittings are presented in Figures 1, 2, 3. Such experimental results are allowing us to analyze not only the texture influence on the pinning constant and other parameters of the model, but also to investigate the influence of field and magnetization on these constants. The results of the analysis are given in Figures 4 and 5 in the form of relationships between the parameters of the model and the field strength as well as the direction angles from RD. We will now analyze the data obtained and discuss the results in order to assess just how much the model can be used to provide useful information about magnetization processes in textured silicon steel.

The pinning constant, $k$, illustrated in Figure 4 is most strongly affected by the existing texture. In order to explain the changes of $k$ with direction we must stress that in a broad sense this parameter is a restraining force which inhibits changes in magnetization and may not necessarily be related to domain wall movement. The results obtained show that the pinning constant for three different field strengths is smallest along the rolling direction and highest along 60 to 70 degrees from the rolling direction. The results obtained correspond to the variation of the hysteresis loss in the Goss oriented silicon steels, where losses are lowest at 0 degrees from the rolling direction and highest at 60 degrees from the rolling direction. The parameters $a$ and $\alpha$ fluctuate quite significantly. The high sensitivity of these parameters to changes in angle from the rolling direction does not have a physical meaning but simply illustrates the fact that the shape of the hysteresis curve is a function of $\alpha (M + H)/a$. Thus the ratio ($a/\alpha$) could be responsible for a change in the shape of the hysteresis curve.

The maximum field strength affects the variation of $k$ with the field strength only at 1 Oe. and the differences between the values at 1 Oe. and those obtained for 3 Oe. and 5 Oe. are related to the difficulty in estimating the saturation from measurements of magnetization at such low fields. For field strengths at 3 and 5 Oe., and we believe higher as well, the pinning constant is not affected by the field strength, and therefore one can consider that it is truly a constant. Such constant values characterize the specimen investigated in a concise way, and there is no strong need to introduce the variation of $k$ with the field strength and magnetization, as was previously suggested by Jiles and Atherton.

Figure 3  Hysteresis loops measured and calculated for rolling direction $\alpha = 80$, for various maximum field strengths

<table>
<thead>
<tr>
<th>Field Strength, Oe.</th>
<th>Magnitude, Emu/cm$^3$</th>
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<tbody>
<tr>
<td>1</td>
<td></td>
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<td>3</td>
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<td>5</td>
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Field Strength, Oe.
CONCLUSIONS

A completely numerical method of fitting the model to experimental data was proposed and implemented. A general program was written which obtains the best fitting using a least square method. A better fitting was obtained when $M_r$ was a fitting parameter. The parameter $k$, representing the pinning effect, changes with direction along which the hysteresis loops were measured. The changes observed can be justified by texture observed in silicon steel. Accordingly, since the pinning constant represents resistance to magnetization, its value is the smallest along the rolling direction (this corresponds to easy magnetization direction) and is the highest between 60 and 70 degrees from RD where hysteresis losses in silicon steels are usually the highest. Also, the $k$ constant does not vary a lot with field strength and magnetization directions. Saying this, we have to, however, admit that at low field strengths at 60 to 70 degrees from the rolling, $k$ values are much higher for $H_0 = 5$ Oe than at 30 Oe or 1 Oe.

REFERENCES