EVALUATION OF THE PARAMETERS OF TEXTURE COMPONENTS ON THE BASIS OF A DISCRETE FORM OF ORIENTATION DISTRIBUTION

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INTRODUCTION

A method is proposed which enables to evaluate the parameters of the texture components (i.e. volume fraction and the scattering width around the ideal orientation) on the basis of orientation distribution given in a discrete form. The presented method is a continuation and generalisation of the evaluation method applied so far, in which the coefficients of a series expansion of the orientation distribution function (ODF) are used. The old method may be now utilized as a preliminary stage in the determination of the parameter values.

TEXTURE COMPONENT

As a texture component we mean the orientation \( g_i = (hkl)[uvw] \) together with its surroundings, in which there occurs increased orientation density. A simple description of a texture component in the position \( g_i \) is obtained by assuming that all the orientations \( g \) from the surroundings \( g_i \) defined by the rotation of a crystallite with the orientation \( g_i \) around an arbitrary rotation axis \( \bar{a} \) by the same rotation angle \( \omega_c \) are equally probable

\[
f_i(g) = f(\{\bar{a}, \omega_c\}) = \text{const.}; \quad \omega = |g_i^{-1}g|.
\]

If there exists a symmetry of the crystallographic lattice and a statistical symmetry of the sample of the examined material, all the symmetrically equivalent orientations should be considered.

The \( f_i(g) \) function defined that way is dependent only on the rotation angle \( \omega \)

\[
f_i(g) = S_i(\omega).
\]
The normalisation coefficient of the function $S_i(\omega)$, called the scattering function, corresponds by definition to the volume fraction $W_i$ of the $i$-th component in the texture

$$\int f_i(g)dg = V_i/V = W_i,$$  \hspace{1cm} (3)

where $V$ - sample volume, $V_i$ - volume of crystallites attributed to the $i$-th component.

The function $S_i(\omega)$ can be described by means of Gauss distribution

$$S_i(\omega) = S_{0i} \exp(-\omega^2/\omega_{0i}^2)$$  \hspace{1cm} (4)

(after H.J. Bunge$^3$) or

$$S_i'(\omega) = S_{0i} \exp[s_{0i} \cos(\omega)]; s_{0i} = \ln 2/[2\sin^2(b_{01}/4)]$$  \hspace{1cm} (5)

(after S. Matthies$^4$).

In the relationships (3-5) $W_i$ denotes the volume fraction of the component $i$, and $\omega_{0i}$ (or $b_{01}$) characterizes the scattering of the orientations around the position $g_i$. The parameters $W_i$ and $\omega_{0i}$ ($b_{01}$) represent the quantitative characteristic of the component $i$.

**MODEL ODF**

In a general case, in the texture, there may occur several preferred orientations $\{g_i; i=1, I\}$ surrounded by the scattering areas of high orientation densities. By adopting a mathematical description for the orientation distribution in each of these areas a model representation of the ODF $f_{mod}(g)$ is obtained

$$f_{mod}(g) = \sum_{i=1}^{I} f_i(g) + r; \hspace{1cm} r + \sum_{i=1}^{I} W_i = 1,$$  \hspace{1cm} (6)

where $r$ represents the fraction of randomly distributed crystallites. The values of the parameters $\{W_i, \omega_{0i}; i=1, I\}$ and $r$ can be determined by a mean square approximation of the model distribution $f_{mod}(g)$ to the distribution obtained on the basis of experimental data $f_{exp}(g)$

$$\int [f_{exp}(g)-f_{mod}(g)]^2dg = \min.$$  \hspace{1cm} (7)

Approximation of the function $f_{exp}(g)$ by means of the function $f_{mod}(g)$ can be performed by using the expansion of each of the functions into a series of generalized spherical functions. Estimation of the
value of $W_i$ and $\omega_{0i}$ becomes then reduced to the approximation of the coefficients of the series expansion of $f_{\exp}(g)$: $C_{1i}^\mu$ by means of the coefficients of the function $f_{\text{mod}}(g)$:

$$C_{1i}^\mu(r, \{W_i, \omega_{0i}, g_i; i=1,I\})^{1,2,5,6}.$$ 

Besides the methods of the ODF determination based on its series expansion, the so-called discrete methods of the ODF determination from pole figures are more and more frequently employed at present. These methods supply a set of the $f_{\exp}(g_j)$ values at the points $g_j$ in the orientation space $(j = 1,J)$. Evaluation of the parameters of the texture components may be then performed directly on the basis of relation (7) in which the integration is replaced by summing with the weight factors $w(g_j)$

$$\sum_{j=1}^{J} [f_{\exp}(g_j) - f_{\text{mod}}(g_j)]^2 w(g_j) = \text{min.} \tag{8}$$

Such procedure allows to avoid the truncation error connected with breaking the ODF series expansion at $l_{\text{max}}$, where $l_{\text{max}} \ll \infty$ ($l$ - series expansion order). Also, mostly when using the ODF series expansion we have at our disposal only the coefficients $C_{1i}^\mu$ for even value of $l$ ($l = 2l'$), thus incomplete information about ODF. The discrete methods, on the other hand, provide more reliable data about the orientation distribution by eliminating the effect of disturbances, the so-called "ghost phenomena".

RESULTS

As an example of the application of discrete data the relation (8) has been used to determine the parameters of texture components in a silver sample cold-rolled to 98.4% reduction. The ODF has been reproduced by the ADC method. The results of fitting have been compared with the values obtained by the method utilizing the coefficients of the ODF series expansion (Table 1). It is observed that for the main texture components both methods yield similar values of the parameters, but considerable differences occur for the minor components and the fraction of randomly distributed crystallites. The differences are due to high relative errors of the low orientation densities when the series expansion method is used. Figure 1a shows sections of the ODFs obtained by discrete method, while figure 1b - those calculated and fitted by the series expansion method.
Fig. 1 Sections through the ODF of a 98.4% cold-rolled silver.

a) The discrete method has been applied: \( f_e \) - experimental ODF, \( f_m \) - model ODF calculated with component parameters from Table 1.

b) The series expansion method has been used: \( f_e \) - experimental ODF \( (l_{\max} = 22) \) including only 1-even terms, \( f_m \) - model ODF including only 1-even terms, \( f_{m_r} \) - model \( m_r \) ODF including 1-even and 1-odd terms (component parameters from Table 1.).
Table 1. Comparison of the values of the texture component parameters

<table>
<thead>
<tr>
<th>Component</th>
<th>$\phi_1$</th>
<th>$\phi$</th>
<th>$\phi_2$</th>
<th>W(%)</th>
<th>$\omega_0^{(0)}$</th>
<th>W(%)</th>
<th>$\omega_0^{(0)}$</th>
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<tbody>
<tr>
<td>${110}&lt;112&gt;$</td>
<td>35.0</td>
<td>45.0</td>
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<td>50.4</td>
<td>7.4</td>
<td>47.2</td>
<td>8.4</td>
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<td>45.0</td>
<td>0.0</td>
<td>8.0</td>
<td>8.2</td>
<td>6.0</td>
<td>9.6</td>
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<tr>
<td>$\sim{110}&lt;115&gt;$</td>
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<td>45.0</td>
<td>0.0</td>
<td>17.3</td>
<td>8.4</td>
<td>7.0</td>
<td>8.3</td>
</tr>
<tr>
<td>$\sim{627}&lt;496&gt;$</td>
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<td>40.0</td>
<td>70.0</td>
<td>24.2</td>
<td>8.8</td>
<td>23.5</td>
<td>8.1</td>
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</tbody>
</table>

SUMMARY

The new method has been applied to determine the parameters of texture components. It allows to characterize the texture by a few numbers only which define the position of the main components and the values of their parameters (volume fractions and scattering widths) as well as the fraction of randomly distributed crystallites. The discrete representation of the ODF provides more reliable data for approximation and the parameter values are then also fitted more precisely.

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