Journal of Applied Mathematics and Stochastic Analysis
6, Number 4, Winter 1993
SHORT REPORTS AND COMMUNICATIONS

TIME-DEPENDENT SOLUTION OF THE FOREGROUND-BACKGROUND PROCESSOR-SHARING QUEUE

Submitted for publication to *Problems of Information Transmission*, a journal of Russian Academy of Sciences

SERGEI F. YASHKOV

Institute for Problems of Information Transmission
Russian Academy of Sciences
19, Yermolovoy Street,
101447 Moscow GSP-4, Russia
e-mail: yashkov@ippi.msk.su

This work deals with time-dependent analysis of a random process associated with the number of customers and their service times spent in the M/GI/1 queueing system under the foreground-background processor-sharing (FBPS) discipline. The FBPS discipline assumes that only the set of customers with the least amount of attained service share the server in the processor-sharing fashion.

In [3], we found the distribution of queue-length at time t for this system in terms of the Laplace transform of the corresponding generating function. (It was assumed there that the system was empty at $t = 0$). Using the same analytical method (which is also described in [1]), we obtain the following generalization of the main result of [3]: the time-dependent distribution of the random counting measure representing the transient state of the FBPS queue in all details. In other words, we derived the distribution of the number of customers each of which attained service $a < y$ at time t, in terms of the Laplace transform.

This new result, that has been presented in [1] as Theorem 4.6, leads to an additional insight into the transient behavior of the queue-length process. This also leads to some other new results, including heavy traffic limit theorems, even for the case of an oversaturated system (e.g., in the spirit of formula (6) from [2], which is a law of large numbers for the queue-length process in a related oversaturated processor-sharing queue).

References

[1] Yashkov, S.F. Mathematical problems in the theory of shared-processor systems, *J. of Soviet Mathematics*, 58, No. 2 , 101-147, 1992.

[2] Yashkov, S.F. On a heavy traffic limit theorem for the M/G/1processor-sharing queue, *Commun. Statist. - Stochastic Models*, 9, No. 3, 467-471, 1993.

[3] Yashkov, S.F. Time-dependent analysis of the foreground-background processor-sharing system, *Commun. Statist. - Stochastic Models* (submitted).

