TIME-DEPENDENT SOLUTION OF THE FOREGROUND-BACKGROUND PROCESSOR-SHARING QUEUE

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This work deals with time-dependent analysis of a random process associated with the number of customers and their service times spent in the M/GI/1 queueing system under the foreground-background processor-sharing (FBPS) discipline. The FBPS discipline assumes that only the set of customers with the least amount of attained service share the server in the processor-sharing fashion.

In [3], we found the distribution of queue-length at time $t$ for this system in terms of the Laplace transform of the corresponding generating function. (It was assumed there that the system was empty at $t = 0$). Using the same analytical method (which is also described in [1]), we obtain the following generalization of the main result of [3]: the time-dependent distribution of the random counting measure representing the transient state of the FBPS queue in all details. In other words, we derived the distribution of the number of customers each of which attained service $a < y$ at time $t$, in terms of the Laplace transform.

This new result, that has been presented in [1] as Theorem 4.6, leads to an additional insight into the transient behavior of the queue-length process. This also leads to some other new results, including heavy traffic limit theorems, even for the case of an over-saturated system (e.g., in the spirit of formula (6) from [2], which is a law of large numbers for the queue-length process in a related oversaturated processor-sharing queue).

References
