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THE DISTRIBUTION OF MAGNETIC SUSCEPTIBILITY IN CRUSHED ORES

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Abstract: Particles of the crushed products form a dispersive system in which the continuous phase (waste rock) constitutes a matrix for inclusions of dispersed phases (metal–bearing minerals). In the process of crushing multicomponent ores, the inclusions of dispersed phases are bound in random to the consecutive particles of the crushed product. Value of of the magnetic susceptibility of particles depends on the number of inclusions of magnetic minerals. Starting from the assumption that the number of inclusions in the particles of narrow size fraction has the Poisson distribution, the author has derived an equation of the distribution function of the magnetic susceptibility of particles of narrow size fraction. The distribution function is expressed by the Pearson function. This paper presents a detailed form of this function for weakly and strongly magnetic ores. Parameters of the distribution function are connected to the values characterising a given sample.

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INTRODUCTION

Magnetic separation is based on the application of the difference in magnetic properties of useful mineral and waste rock. Magnetic susceptibility belongs to the group of physical properties whose mean value determines a product into which a particle is attributed in the process of enrichment. Therefore, the washability of magnetic ores is determined by the distribution of magnetic susceptibility of particles of the crushed product.

The mean volume magnetic susceptibility χ of a particle is defined as follows [1]:

$$\chi = \frac{\mu}{H\,V} \tag{1}$$

where H is the intensity of the magnetic field, μ is the total magnetic moment of a particle and V is the volume of a particle. Hence, the value of the particle volume magnetic susceptibility is determined by the volume concentration of the mineral components that have magnetic properties.

Particles of the products of the crushing, depending on their sizes and size of inclusions of useful minerals, will be either pure particles of the waste rock, or a useful mineral, or particles of intergrowths. As a consequence, magnetic properties of respective particles will be different.

This paper derives a general equation of the distribution function of the magnetic susceptibility of a particle in a crushed ore. The dispersive particle model [2] has been assumed to be a starting point. In analogy to the liquid dispersive systems, this model differentiates a continuous (carrier) phase which constitutes the matrix for inclusions of the disperse phase. The continuous phase is constituted by the minerals of the waste rock, while the dispersed phases are constituted by the metal–bearing minerals.

In the process of crushing the multicomponent ores the inclusions of the dispersed phases are connected at random with respective particles. The value of the magnetic susceptibility depends on the number of inclusions. Therefore, the magnetic susceptibility of a particle is a random variable.

Let X be a random variable determining the particle magnetic susceptibility. Consequently, for the continuous and discrete distributions of magnetic susceptibility, the equation of the distribution function can be written as follows [3]:

$$F(x) = P(X < x) = \int_{x_{min}}^{x} f(z)dz \tag{2a}$$

$$F(x) = P(X < x) = \sum_{x_i < x} \gamma(X = x_i) \tag{2b}$$

where $f(x)$ is the probability density function of the occurrence of particles of the magnetic susceptibility x, while γ $(X = x_i)$ is the yield of a fraction for which $X = x_i$.

Applying the formula (2b) and accepting the assumption concerning the distribution of the number of inclusions of the dispersed phase in the particles of the product of the crushing, an equation of the distribution function of the magnetic susceptibility in weakly magnetic, strongly magnetic and mixed ores has been derived.

THE DISTRIBUTION FUNCTION OF MAGNETIC SUSCEPTIBILITY

For the sake of clarity, it has been assumed that the feed in constituted by a narrow–sized fraction of the crushed product of the volume of a single particle V, with one single–size dispersed phase.

The number of inclusions in respective particles is a random variable. It has been assumed that the probability of s inclusions in a particle of the volume V is determined by Poisson distribution [3]:

$$P(s|V) = \frac{(bV)^s}{s!} e^{-bV} \tag{3}$$

where b is the average number of inclusions per unit volume. The product bV denotes the average number of inclusions in a particle of volume V.

If there are N particles in a given size fraction, the product $N\ P(s|\ V) = N_s$ represents the number of particles in which the number of inclusions equals s. Therefore, the expression

$$\frac{N_s}{N} = P(s|V) \tag{4}$$

represents a fraction of the general number of particles, or a volume yield of particles in which the number of inclusions is s.

According to definition (2b), equation of the distribution function of the number of inclusions is expressed by the formula:

$$F(s|V) = \sum_{n=0}^{s} P(n|V) = e^{-bV} \sum_{n=0}^{s} \frac{(bV)^n}{n!} \tag{5}$$

After applying the special functions, the expression (5) assumes the form [4]:

$$F(s|V) = \frac{\Gamma(bV;\ 1+s)}{\Gamma(1+s)} = I(bV;\ 1+s) = \frac{1}{\Gamma(1+s)} \int_{bV}^{\infty} e^{-t} t^{s} dt \tag{6}$$

where $\Gamma(1+s)$ is the gamma–function, $\Gamma(bV;\ 1+s)$ is the incomplete gamma–function and $I(bV;\ 1+s)$ is the Pearson's function.

For the two–phase system (one dispersed phase) in a weakly–magnetic ore, the particle volume susceptibility is equal to [5]:

$$\chi = \chi_o + (\chi_k - \chi_o) \frac{s v_o}{V} \tag{7}$$

where v_o represents the volume of a single inclusion, χ_o is the magnetic susceptibility of the continuous phase, χ_k is the magnetic susceptibility of the dispersed phase. The fraction $sv_o/V = \lambda$ is the volume content of the dispersed phase in a particle.

For eq. (7) the number of inclusions in the particle is:

$$s = \frac{\chi - \chi_o}{\chi_k - \chi_o} \cdot \frac{V}{v_o} = a\lambda \tag{8}$$

where $a = V/v_o$.

Analogically,

$$bV \frac{v_0}{V} = \Lambda_k \tag{9}$$

or, respectively:

$$bV = a\Lambda_k \tag{10}$$

where Λ_k is the average volumetric content of the dispersed phase in the feed.

Inserting eqs. (8) and (10) into formula (6), we obtain:

$$F(\chi|V) = I(a\Lambda_k;\ 1+a\frac{\chi - \chi_0}{\chi_k - \chi_0}) \tag{11}$$

The expression (11) represents the equation of the distribution function of the volume magnetic susceptibility in weakly–magnetic ores, under the assumption of monodispersivity of both the dispersed phase and the feed.

Let $f_2(v)$ be the probability density function of distribution of the volume of a single inclusion, where $v \in (v_{min}, v_{max})$. Then, the product $\lambda f_2(v)dv = \lambda(v)$ represents the volumetric content of inclusions of the range of the volume of a single inclusion $(v, v + dv)$, in a particle of volume V, while λ denotes the total volume content of the disperse phase in a single particle.

The expression $ds = V/v\lambda(v)$ is the number of inclusions of the range of volume of a single inclusion $(v, v + dv)$. Therefore, for $V > v_{max}$:

$$s = V\lambda\int_{v_{min}}^{v_{max}} \frac{1}{v} f_2(v)dv = VC\lambda = VC\frac{\chi - \chi_0}{\chi_k - \chi_0} \tag{12}$$

where

$$C = \int_{v_{min}}^{v_{max}} \frac{1}{v} f_2(v)dv$$

The value of the constant C depends on the distribution of inclusion sizes in a given particle and may vary from one particle to another. To make the problem simpler, it was assumed that the distribution of the inclusion sizes $f_2(v)$ is the same in all particles of the monodispersive feed.

Analogically, as in formula (12):

$$bV = CV\Lambda_k(V) \tag{13}$$

When compared with eqs. (12) and (13) for polydispersive dispersed phase, the equation of the distribution function of the volume magnetic susceptibility is as follows:

$$F(\chi|V) = I\,[CV\Lambda_k(V);\ 1 + CV\,\frac{\chi - \chi_o}{\chi_k - \chi_o}] \tag{14}$$

If the particle size distribution of the feed is described by the probability density function $f_1\ (V)$ then the equation of the distribution function of the magnetic susceptibility for the two–phase system will be expressed by the formula:

$$F(\chi) = \int_{V_{min}}^{V_{max}} F(\chi|V)\, f_1(V)\, dV \tag{15}$$

Therefore, the general equation of the distribution function of the magnetic susceptibility in the a product of crushing of weakly–magnetic ores has the following form:

$$F(\chi) = \int_{V_{min}}^{V_{max}} I[CV\Lambda_k(V);\ 1 + (\chi - \chi_o)\,\frac{CV}{\chi_k - \chi_o}\,]\ f_1(V)\ dV \tag{16}$$

If $\Lambda_k(V)$ = const., and the product CV is constant, then eq. (16) can be simplified to the form:

$$F(\chi) = I\,[\,K_1;\ 1 + K_2(\chi - \chi_o)] \tag{17}$$

where

$$K_1 = CV\Lambda_k = \text{const} \tag{18a}$$

$$K_2 = \frac{CV}{\chi_k - \chi_o} \tag{18b}$$

In such a case the distribution of the volume magnetic susceptibility does not depend on the particle size distribution of the feed, and the distribution function has two parameters, K_1 and K_2.

In the case of strongly magnetic ore, such as the magnetite ore, the volume magnetic susceptibility is proportional to the square of the magnetite content [6]. We can thus write:

$$\frac{\chi - \chi_o}{\chi_k - \chi_o} = \lambda^2 \tag{19}$$

For the two–phase system and the polydispersive distribution of the inclusion sizes:

$$s = (\chi - \chi_o)^{\frac{1}{2}} \frac{CV}{(\chi_k - \chi_o)^{\frac{1}{2}}} = K_2 (\chi - \chi_o)^{\frac{1}{2}} \tag{20}$$

The equation of the distribution function of magnetic susceptibility in a crushed strongly–magnetic ore is therefore given by:

$$F(\chi) = I\,[K_1;\ 1 + K_2 (\chi - \chi_o)^{\frac{1}{2}}] \tag{21}$$

where

$$K_1 = CV\Lambda_k \tag{22a}$$

$$K_2 = \frac{CV}{(\chi_k - \chi_o)^{\frac{1}{2}}} \tag{22b}$$

In the case of mixed ores, such as the ilmenite–magnetite ore, which contain both weakly–magnetic minerals, such as ilmenite, and strongly–magnetic components, such as magnetite, the equation of the distribution function is the sum of the

distribution functions expressed by eqs. (17) and (21), with respective weight factors:

$$F(\chi) = \gamma_1 F_1(\chi) + \gamma_2 F_2(\chi) H(\chi - \chi_f) \tag{23}$$

where:

$F_1(\chi)$ is the distribution function of the volume magnetic susceptibility in the ilmenite fractions, expressed by eq. (17)

$F_2(\chi)$ is the distribution function of the volume magnetic susceptibility in the magnetite fractions, expressed by eq. (21)

γ_1 is the total yield of the non–magnetic fraction and the ilmenite fractions

γ_2 is the yield of the magnetite fractions ($\gamma_1 + \gamma_2 = 1$)

$H(\chi - \chi_f)$ is the Heaviside function [7]

χ_f is the division value of the magnetic susceptibility between the ilmenite (weakly–magnetic) factions and the magnetite (strongly magnetic) fractions.

Equations (17) and (21) will be used for approximation of the experimental dependencies.

EXPERIMENTAL

The obtained dependencies were verified experimentally using oxidised Zn–Pb and magnetite–ilmenite ores. Magnetic fractionation of the Zn–Pb ore in the –1+0.1 mm size fraction, and the ilmenite ore in the –0.5+0.05 mm size fraction was carried out in a disc separator. Fractionation of the magnetite part of the ilmenite–magnetite ore was performed in the –0.5+0.05 mm size fraction by means of a laboratory permanent magnet with adjustable distance from the surface of separation. Magnetic susceptibilities of each narrow magnetic fraction were measured by means of a magnetic balance.

RESULTS AND DISCUSSION

Distribution of Magnetic Susceptibility in the Oxidised Zn–Pb Ore

Figure 1 shows the distribution function of the magnetic susceptibility in the oxidised Zn–Pb ore. This ore contains one weakly–magnetic phase, namely goethite. Distribution of the goethite content determines distribution of the magnetic susceptibility of the particles of the ore.

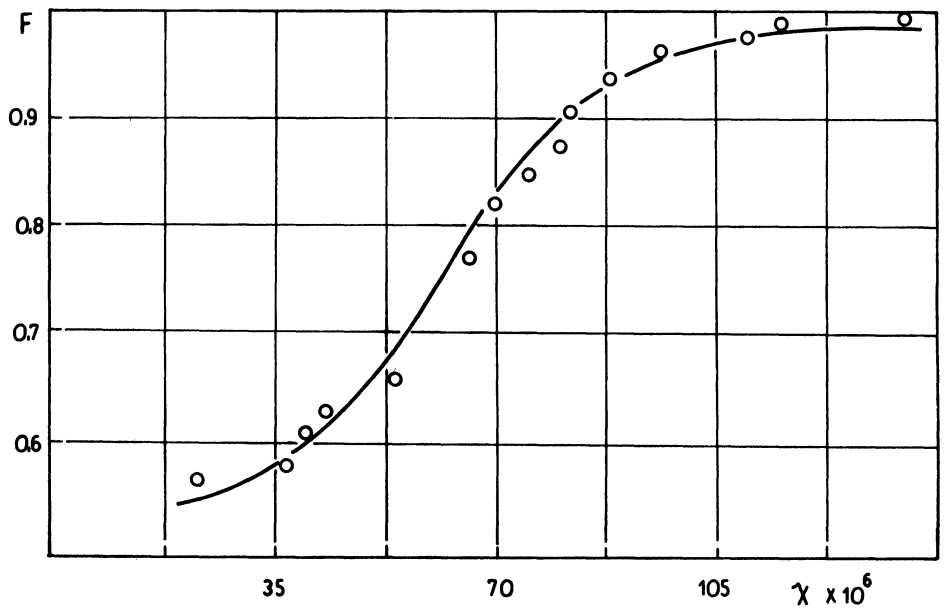

Fig. 1 The cumulative distribution function of the magnetic susceptibility in the oxidised Zn–Pb ore

Parameters K_1 and K_2 in the model dependences were determined by means of tables of the Pearson's function [8], by the step–by–step method, reaching such a situation in which, for a given value of K_1, the parameter K_2 reaches identical values at all experimental points (relative difference between the maximum and minimum values of the parameter K_2 not larger than 10%). An average value of K_2 is assumed in the model dependences.

The model equation of the distribution function of the volume magnetic susceptibility is as follows:

$$F(\chi) = 0.433\ I[7.6;\ 1 + 0.143\times10^{6}(\chi - 14.5\times10^{-6})] + 0.55 \qquad (24)$$
$$R = 99.32\%$$

R represents the value of the curvilinear correlation coefficient.

As can be seen from Fig. 1 and from the value of the correlation coefficient, compatibility of experimental data with the model dependence (continuous curve) is very good.

The volume magnetic susceptibility of the continuous phase is equal to $\chi_0 = 14.5\times10^{-6}$ (cgs units) while the magnetic susceptibility of goethite is $\chi_k = 147.2\times10^{-6}$ (cgs) [5].

Knowing the values of the parameters K_1 and K_2, and by applying equations (18a) and (18b), it is possible to calculate the volume magnetic susceptibility of the dispersed phase (goethite) for the two–phase system which is the oxidised Zn–Pb ore (from the point of view of its magnetic properties). We thus obtain, from eqs. (18a) and (18b):

$$\chi_k = \chi_0 + \frac{K_1}{K_2 \Lambda_k} \qquad (25)$$

For $\chi_0 = 14.5\times10^{-6}$ (cgs), $K_1 = 7.6\ cm^3$, $K_2 = 0.143\times10^{-6}\ cm^3$, $\Lambda_k = 0.38$, we obtain, from eq. (25): $\chi_k = 154\times10^{-6}$ (cgs). Compatibility of this value with that obtained from direct measurements is thus satisfactory.

It can thus be said that for the two–phase system, approximation of the value of the magnetic susceptibility of the dispersed phase, from the parameters of the distribution function, gives a result that is convergent with the value obtained for a pure component.

Distribution of Magnetic Susceptibility in the Ilmenite–Magnetite Ore

There are two dispersed phases in the ilmenite–magnetite ore, namely the weakly–magnetic, ilmenite phase, and the strongly–magnetic, magnetite phase.

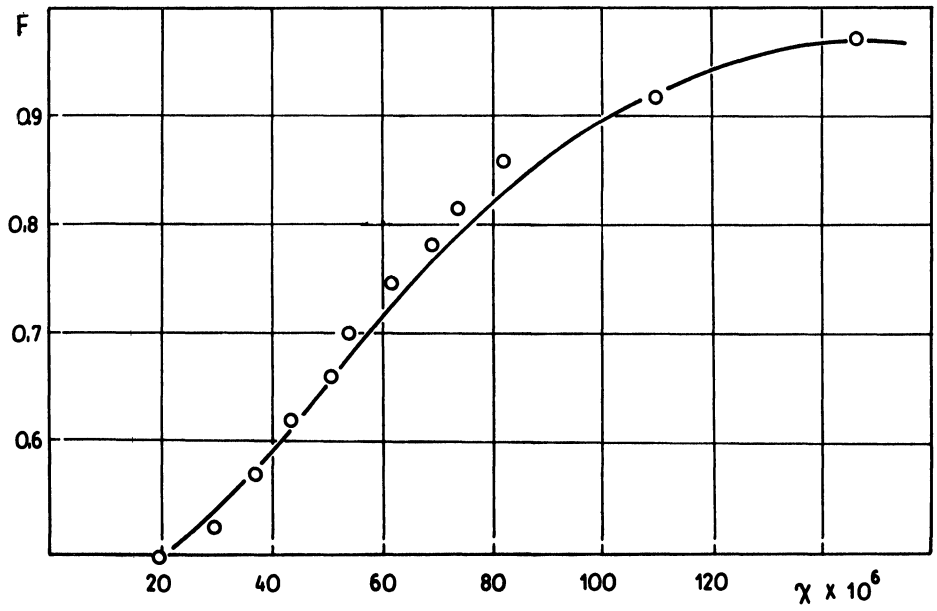

Fig. 2 The cumulative distribution function of magnetic susceptibility in the ilmenite ore

Figures 2 and 3 depict distribution functions of magnetic susceptibility for the ilmenite and magnetite fractions, respectively. The system can be treated in a two–phase way, in each part separately, i.e. the ilmenite and magnetite fractions. Although ilmenite occurs in the magnetite part, its magnetic susceptibility is three orders of magnitude lower than the susceptibility of magnetite. The contribution of ilmenite to the particle magnetic susceptibility can thus be neglected.

The equations of the distribution functions of the volume magnetic susceptibility are as follows:

(a) ilmenite fractions:

$$F(\chi) = 0.56\ I[3.4;\ 1+0.05\times10^{6}(\chi-4\times10^{-6})] + 0.42 \qquad (26)$$
$$R = 99.39\%$$

(b) magnetite fractions:

$$F(\chi) = 1.33\ I[1.5;\ 1+0.5\times10^{3/2}(\chi-0.25\times10^{-3})^{\frac{1}{2}}] - 0.3 \qquad (27)$$
$$R = 98.93\%$$

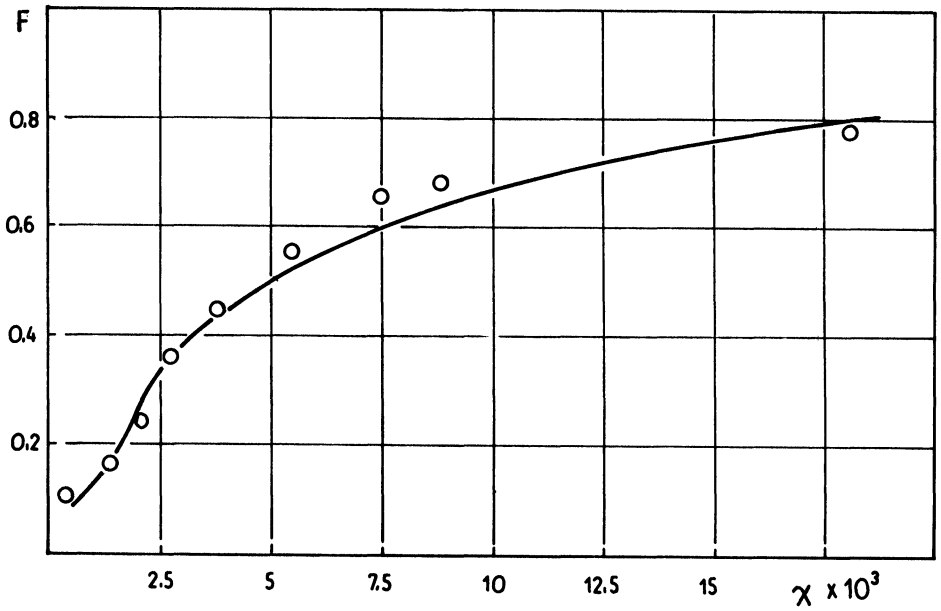

Fig. 3 The cumulative distribution function of magnetic susceptibility in the magnetite ore

Dependence (26) was calculated after separating the strongly magnetic fraction from the ore sample in a drum separator.

It can be seen from the figures and from the correlation coefficients that compatibility of the model with experimental data is very good. By applying formula (25) it is possible to calculate the volume magnetic susceptibility of

ilmenite. For $\chi_0 = 4\times10^{-6}$ (cgs) [5], $K_1 = 3.4$ cm^3, $K_2 = 0.05\times10^6$ cm^3, $\Lambda_k = 0.132$, the volume magnetic susceptibility of ilmenite is $\chi_k = 519.2\times10^{-6}$ (cgs unit). The value of the magnetic susceptibility of ilmenite, obtained by direct measurements, is $\chi_k = 541.9\times10^{-6}$ (cgs) [5]. The agreement of both values is thus satisfactory.

The volume magnetic susceptibility of magnetite can be calculated, in the fashion similar to that for the weakly magnetic ores, from the parameters of the distribution function of the magnetic susceptibility. We can obtain, from eqs. (22a) and (22b):

$$\chi_k = \chi_0 + \left[\frac{K_1}{K_2\Lambda_k}\right]^2 \tag{28}$$

For $\chi_0 = 0.25\times10^{-3}$ (cgs), $K_1 = 1.5$ cm^3, $K_2 = 0.5\times10^{3/2}$ cm^3, $\Lambda_k = 0.24$, we obtain $\chi_k = 156.2\times10^{-3}$ (cgs). The value obtained from direct measurements is $\chi_k = 151\times10^{-3}$ (cgs) [3]. The difference between the values obtained by two different methods is minimum.

The ilmenite–magnetite ore can be treated as a three–phase system consisting of the non–magnetic fraction as the continuous phase, together with ilmenite and magnetite as two dispersed phases.

The total yield of the non–magnetic and the ilmenite fractions is $\gamma_1 = 0.3$, while the yield of the magnetite fraction is $\gamma_2 = 0.7$. In this case the distribution function of the volume magnetic susceptibility will be expressed by the formula:

$$\begin{aligned} F(\chi) = \quad & 0.3\,\{0.56\,I[3.4;\,1+0.05\times10^6(\chi-4\times10^{-6})] + 0.42\} + \\ & 0.7\,\{1.33\,I[1.5;\,1+0.5\times10^{3/2}(\chi-0.25\times10^{-3})^{\frac{1}{2}}] - 0.3\} \times \\ & H(\chi - 0.25\times10^{-3}) \end{aligned} \tag{29}$$

Figure. 4 show the distribution function of the volume magnetic susceptibility for the full range of the particle magnetic susceptibility in the ilmenite–magnetite ore.

Such a shape of the distribution function as shown in fig. 4 is obtained in the case when two dispersed phases do not occur together in the precipitated fractions. Although ilmenite occurs in the magnetite part of the ore, owing to much lower

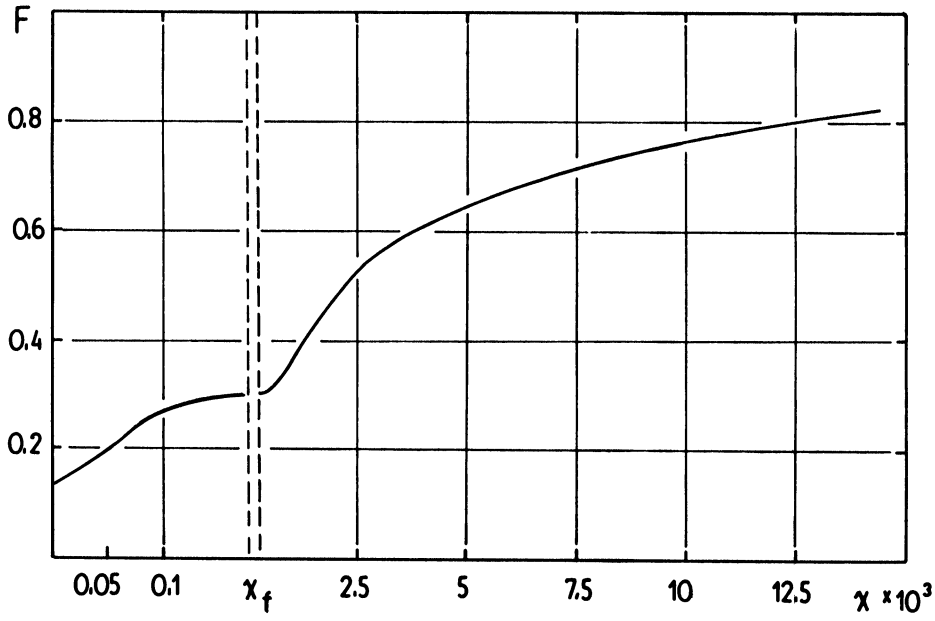

Fig. 4 The cumulative distribution function for the full range of the magnetic susceptibility in the ilmenite–magnetite ore.

magnetic susceptibility of ilmenite it does not affect substantially the value of the particle magnetic susceptibility.

If there were two phases of close values of magnetic susceptibility in the strongly magnetic fraction it would be impossible to separate, in a magnetic separator, particles of different concentrations of the respective phases. Analogical comment can be made about the part of dependence (29) describing the weakly–magnetic phase. In this case the values of the magnetic susceptibility determined from eqs. (25) and (28) are the mean values for all dispersed phases in a given group.

In conclusion, the following comment should be made. Experimental equations of the distribution function of the magnetic susceptibility contain two additional parameters. Consequently, the general equation of the distribution function in weakly–magnetic ores will be as follows:

$$F(\chi) = C_1 I [K_1; 1 + K_2 (\chi - \chi_o)] + C_2 \quad (30)$$

where the constants C_1 and C_2 are the scale parameters. If the constant C_2 is positive it can be interpreted as a yield of a pure liberated continuous phase because $I(\chi = \chi_o) = 0$, and $F(\chi = \chi_o) = C_2$.

The degree of liberation L_c in the two–phase system is:

$$L_c = \frac{C_2}{1 - \Lambda_k} \quad (31)$$

On the other hand, when C_2 is negative it can be said that there are no liberated particles of the continuous phase in the tested sample of the crushed material. From the condition:

$$C_1 I[K_1; 1 + K_2(\chi - \chi_o)] = C_2 \quad (32)$$

it is possible to calculate the magnetic susceptibility of the poorest intergrowths, and from the dependence $\lambda(\chi)$ their phase composition.

Analogical remarks concern also the distribution function of magnetic susceptibility in strongly magnetic ores.

CONCLUSIONS

The distribution function of the volume magnetic susceptibility in the crushed product of ores is expressed by Pearson's function. The detailed form of Pearson's function is determined by the shape of dependence of the particle magnetic susceptibility on the content of the dispersed phase which is different for the weakly–magnetic and strongly–magnetic ores.

The parameters of the distribution function, K_1 and K_2, are connected with the values characterising the tested ore sample, such as the size of a particle, the average content of the dispersed phase in the feed, magnetic susceptibility of the continuous phase and magnetic susceptibility of the dispersed phase.

By means of fitting the experimental data to the model dependence, the values of of the distribution parameters K_1 and K_2 are determined. In the case of the two–phase system (one dispersed phase), it is possible to calculate the value of the volume magnetic susceptibility of the dispersed phase when these parameters (K_1 and K_2) are known.

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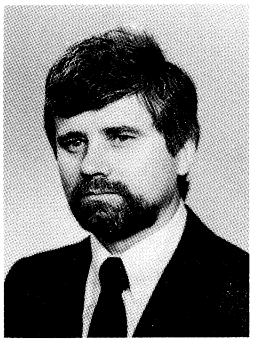

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