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THEORETICAL MODEL OF SEPARATION OF FINE PARTICLES IN A ROTATING FERROFLUID

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Abstract: The application of centrifugal forces to a ferrofluid with finely grained particles located in a non–homogeneous magnetic field is the basis of the theoretical model of separation outlined in this paper. An analytical solution of the equation of the particle motion, its application under certain conditions, as well as the graphic representation of the results are presented.

INTRODUCTION

Methods of magnetohydrostatic separation, widely developed [1], also in the Institute of Geotechnics [2 to 4], are not very efficient for separation of finely grained particles smaller than 70 μm. Problems of separation are caused by viscosity and surface tension of the ferrofluid, as well as by the throughput of the device. It appears that the application of the centrifugal force to the ferrofluid located in non–homogeneous magnetic field can increase the efficiency of separation of fine particles.

It follows from the results of magnetohydrostatic separation that a non–homogeneous magnetic field acting on the ferrofluid can change the density of the fluid. Depending on the value of the gradient of the magnetic field, it is possible to obtain various values of the density of the ferrofluid. The separation process thus depends on the mutual relationship between the buoyant and gravity forces acting on a particle located in the ferrofluid. These results can be applied to the non–static case where the ferrofluid is rotating together with fine non–magnetic particles.

A THEORETICAL MODEL

The theoretical model presented here is based on the application of centrifugal force to a particle moving in the rotating ferrofluid located in the non–homogeneous magnetic field. Let us investigate the motion of the particle. Let ρ_p and m denote the density and the mass, respectively, of the particle. Density of the ferrofluid is denoted by ρ_f and ω is the angular velocity of the rotating particle and of the ferrofluid. It will be assumed that the particle is spherical.

The motion of the particle can be described by differential equations including all acting forces, while the forces of interaction between the particles are being neglected. The solution of the problem becomes more transparent if the cylindrical co–ordinate system is introduced. It is assumed that the origin of the co–ordinate system is located at the centre of the top level of ferrofluid. The z axis is identical to the axis of the vessel and r, φ and z are the cylindrical co–ordinates of the particle.

It follows from the symmetry of the system that only the tangential component of the velocity of ferrofluid differs from zero and depends on the radial co–ordinate of the point under investigation. As a result of the applied non–homogeneous magnetic field, the influence of the magnetomotive force on the volume of ferrofluid is observed. This force causes changes in the density of ferrofluid. Thus the "effective" density of ferrofluid depends on the gradient of the magnetic field as well as on the centrifugal acceleration.

Let us assume that the intensity of the applied magnetic field is dependent on the radial co–ordinate of the point under consideration. It means that the changes in the density of ferrofluid can occur only in the radial direction. Design of the magnetic field for which its gradient varies linearly with the radial co–ordinate was investigated in [5].

After separating the co–ordinates thus introduced, the most important motion, in the radial direction, is investigated. All forces acting on a non–magnetic particle in ferrofluid and entering the equation of motion are as follows: viscous force, centrifugal force, force of buoyancy related to the centrifugal force, and magnetic buoyant force. The equation of motion can be written:

$$m\frac{d^2r}{dt^2}+6\pi\eta a\frac{dr}{dt}-m\omega^2\left(1-\frac{\rho_f}{\rho_p}\right)r+VM_s\mu_0\frac{\partial H}{\partial r}=0 \tag{1}$$

$$K_1=\frac{6\pi\eta a}{m}$$

$$K_2=\omega^2\left(1-\frac{\rho_f}{\rho_p}\right)$$

$$K_3=\frac{M_s\mu_0\frac{\partial H}{\partial r}}{\rho_p},$$

where

μ_0 is the permeability of vacuum
M_S is the saturation magnetisation of ferrofluid
η is the viscosity of ferrofluid
$\partial H/\partial r$ is the partial derivative of the magnetic field intensity with respect to r
a is the radius of the particle
ρ_p is the density of the particle
ρ_f is the density of the ferrofluid
ω is the angular velocity and f is the frequency.

Equation (1) can be written in the form:

$$\frac{d^2r}{dt^2}+K_1\frac{dr}{dt}-K_2r+K_3=0 \tag{2}$$

Equation (2) is the second–order non–homogeneous differential equation. Its solution can be found as a sum of the general solution of the homogeneous equation, and of a particular solution of the non–homogeneous equation. Assuming that at $t=0$ $r=r_0$ and $v_r=0$, the solution is then given by equation (3):

$$r = \frac{r_0 - \frac{K_3}{K_2}}{h_2 - h_1}\left[h_2 e^{h_1 t} - h_1 e^{h_2 t}\right] + \frac{K_3}{K_2} \tag{3}$$

where

$$h_1 = \frac{-K_1}{2} + \frac{\sqrt{K_1^2 + 4K_2}}{2}$$

$$h_2 = \frac{-K_1}{2} - \frac{\sqrt{K_1^2 + 4K_2}}{2}$$

From the practical point of view, it is important to assume the vertical motion of ferrofluid suspension with the flowrate of $Q = 1.5\times10^{-7}$ m^3s^{-1}. The dependence of the vertical co–ordinate on time can be expressed as:

$$z = \frac{Q}{\pi R^2} t \tag{4}$$

where R is the radius of the vessel.

THE APPLICATION OF THE RESULTS AND DISCUSSION

As an application of the solution of the equation of motion, the separation process under certain conditions will be modelled. Analysis of the separating forces depending on density, particle size distribution, as well as on the angular velocity can be used for graphical representation of the motion of a particle. For this reason, the following values have been accepted:

$\rho_f = 1000$ kgm^{-3}
ρ_p from 3×10^3 to 10^4 kgm^{-3}
$\eta = 2.4\times10^{-3}$ Nsm^{-2}
$M_s = 2\times10^4$ Am^{-1}
$r_0 = 2\times10^{-3}$ m
$\partial H/\partial r = 10^6$ Am^{-2}
a from 10^{-5} to 6×10^{-5} m
$f = 10$ or 20 s^{-1}
$R = 2\times10^{-2}$ m

Under these assumptions graphic representations shown in Figures 1 to 4 have been obtained. It follows from these representations that the most important parameter which can influence the process of separation is the frequency of rotation. For small values of the frequency no separation is possible. At the frequency of 10 s^{-1} the separation process is efficient for particles of the density $\rho_p = 10\ 000\ kgm^{-3}$. For lower densities , as well as for particle sizes to 40 μm, higher frequencies are more efficient, as can be seen in Figures 1 to 3.

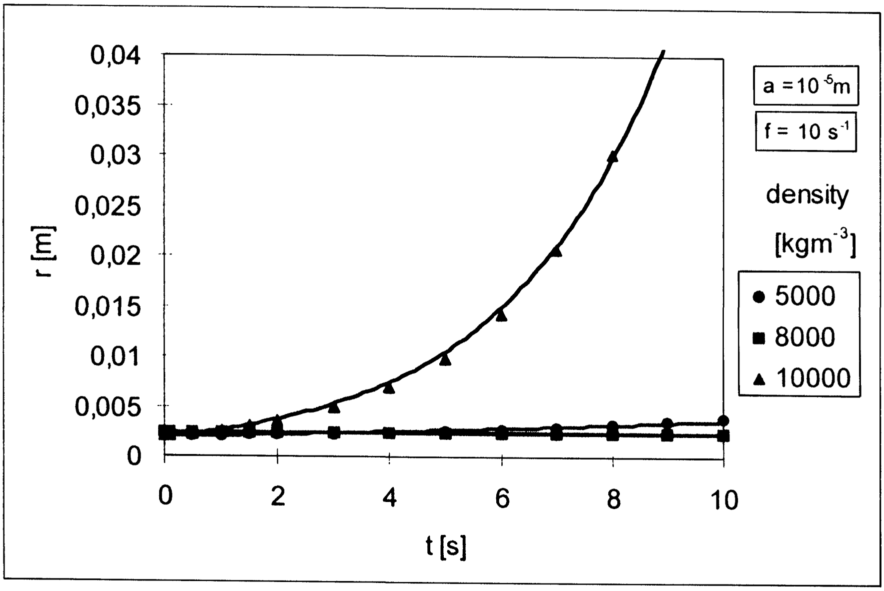

Fig. 1 The effect of the densities of particles on the separation process

If also the vertical motion of the ferrofluid suspension is assumed, the location of a particle in the vessel during the separation process can be determined. The dependence of the location of a particle on its density is shown in Figure 4.

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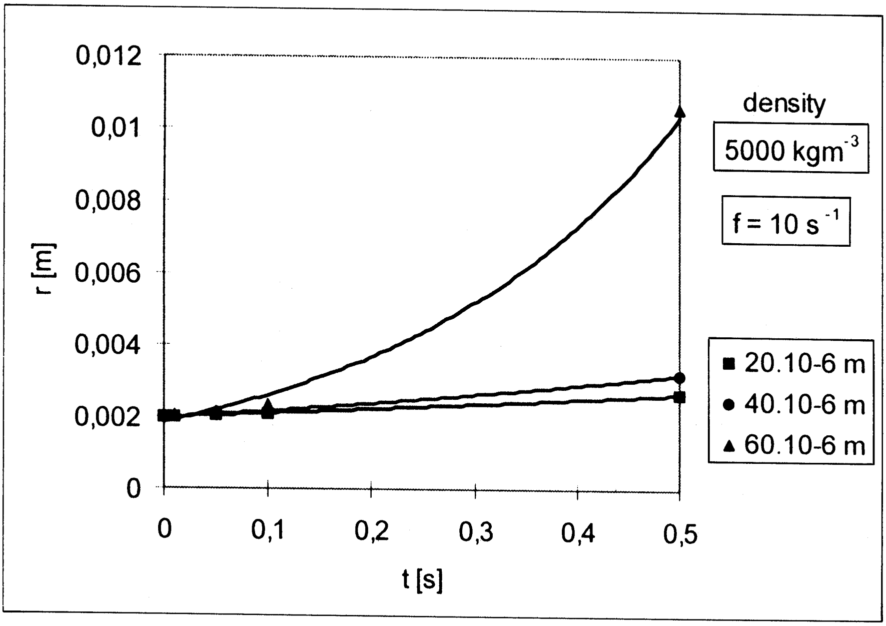

Fig. 2 The effect of particle size on the separation process

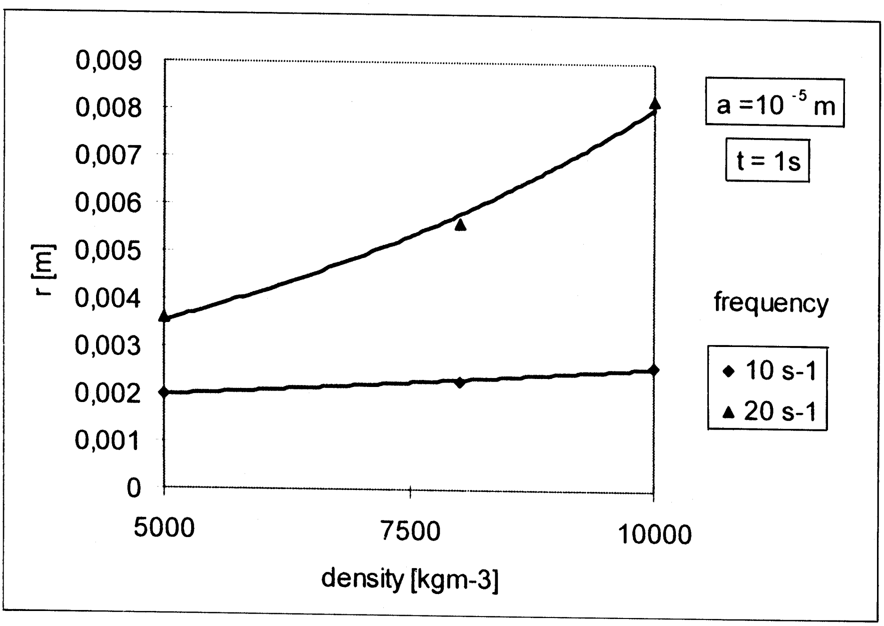

Fig. 3 The effect of frequency on the separation process under certain conditions.

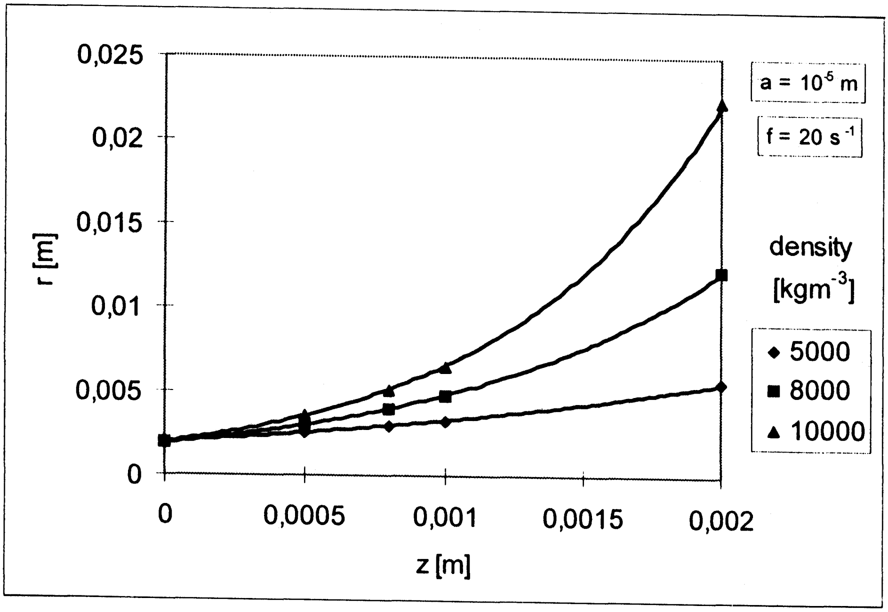

Fig. 4 The effect of the density of particles on their location in the vessel during the separation process

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