

PANEL DISCUSSION ON UNITS IN MAGNETISM*

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An evening panel discussion on magnetic units, attended by 150 participants, was held at the 1994 Joint MMM–Intermag Conference in Albuquerque, New Mexico, USA. The session was organized by C.D. Graham, Jr., and moderated by R.B. Goldfarb. The panel members were asked to describe the use of magnetic units in their countries, and to make appropriate comments and recommendations. In addition to units, several panelists talked about distinction between magnetic induction B and magnetic field strength H , and the conversion of equations. After the panelists' opening statements, the floor was opened for questions and discussion from the audience. Below are the panelists' summaries of their remarks. By agreement with authors, this article is not subject to copyright.

C.D. GRAHAM, JR.

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I would like to consider the units for magnetic susceptibility. Susceptibility is defined as the slope (usually the initial slope) of a plot of magnetization vs. field. The magnetization may be expressed as moment per unit volume M , or moment per unit mass σ . When dealing with small samples, or with a range of temperatures, the sample mass is usually better known than its volume, so σ is very commonly employed. In CGS magnetic units, we have an unofficial, but widely used unit of magnetic moment m , called the emu, so that volume susceptibility is:

$$\chi_v = M/H = m/VH \quad [\text{emu.cm}^{-3}.\text{Oe}^{-1}]$$

*This article was originally published in the IEEE Magnetics Society Newsletter, Vol. 31, No. 4, October 1994

and mass susceptibility is:

$$\chi_m = \sigma/H = m/wH \quad [\text{emu.g}^{-1}.\text{Oe}^{-1}]$$

where w is the sample mass. These units are clear and explicit.

In SI units, the parallel structure between volume and mass susceptibility is lost. Magnetic moment is in $[\text{A.m}^2]$, and field is in $[\text{A.m}^{-1}]$. The volume susceptibility χ_v is in $[\text{A.m}^2.\text{m}^{-3}.\text{A}^{-1}.\text{m}]$, which is dimensionless. Mass susceptibility χ_m is in $[\text{A.m}^2.\text{kg}^{-1}.\text{A}^{-1}.\text{m}] = [\text{m}^3.\text{kg}^{-1}]$, which is reciprocal density. It would help slightly to express field in teslas; then χ_v would be in $[\text{A.m}^2.\text{m}^{-3}.\text{T}^{-1}] = [\text{A.m}^{-1}.\text{T}^{-1}]$ and χ_m would be in $[\text{A.m}^2.\text{kg}^{-1}.\text{T}^{-1}]$. But the volume susceptibility unit and the mass susceptibility unit would still not be parallel in construction.

An SI equivalent of the emu is needed. I suggest the creation of the *sim* (SI moment): $1 \text{ sim} = 1 \text{ A/m}^2$. (This is in a sense similar to the SI unit of pressure, where one pascal equals one newton per square meter). It would also be helpful to have a single name for the SI unit of field, to replace the ampere per meter. There is no magnetic unit named for a Japanese scientist, despite the many contributions made to magnetism by Japan. Why not the *honda*, named for Kotaro Honda, a distinguished scientist and engineer? The symbol would be H_o , since H is already used for the henry. Using the *sim* and *honda*, we have χ_v $[\text{sim}.\text{m}^{-3}.\text{Ho}^{-1}]$ and χ_m $[\text{sim}.\text{kg}^{-1}.\text{Ho}^{-1}]$. Volume susceptibility remains dimensionless, of course, but the units discreetly conceal this unpleasant fact and therefore avoid the current messy and confusing situation.

Alternatively, if field is expressed in teslas, χ_v is in $[\text{sim}.\text{m}^{-3}.\text{T}^{-1}]$ and χ_m is in $[\text{sim}.\text{kg}^{-1}.\text{T}^{-1}]$. In any case, experimental and theoretical values of susceptibility must have their units clearly stated; "susceptibility (SI):" or "CGS susceptibility" is inaccurate.

SOSHIN CHIKAZUMI

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The SI unit of magnetic field, $[\text{A.m}^{-1}]$, is too small, so numerical values measured in this unit are too large. For instance, the field produced by a superconducting

magnet may be 8–16 MA.m⁻¹, the record value of a pulsed magnetic field is 450 MA.m⁻¹, and the exchange field in iron is 0.8 GA.m⁻¹.

Moreover, the CGS unit of magnetic field, the oersted, is popular, and the irrational conversion factor $4\pi/10^3$ from [A.m⁻¹] to oersted is troublesome. Thus, I would like to present a justification for the use of tesla instead of the ampere per meter.

In MKSA units, in the E–H analogy, the fundamental formula relating the flux density or magnetic induction B , intensity of magnetization I , and magnetic field H is given by

$$B = I + \mu_0 H$$

where μ_0 is the permeability of vacuum and has the value $4\pi \times 10^{-7}$ H.m⁻¹. In this connection, I propose the use of $\mu_0 H$ [T], which is to be called the "induction field", instead of H [A.m⁻¹], to describe the magnetic field strength. The conversion factor from teslas to oersted is 10^4 , which contains no π , and the unit tesla is practical. In our examples above, the induction fields produced by superconducting magnets are 10–20 T, the record pulsed induction field is 560 T, and the exchange field in iron is 1000 T. In order to describe weak magnetic fields, such as the Earth's field, we can use prefixes such as 30 μ T. (Since the unit [A.m⁻¹] has a composite structure, the use of prefixes makes the name of the unit long and complex, as in "mega–ampere per meter". If ampere per meter were replaced by a single name, such as that of a famous magnetician, the situation might be improved. We do not have this problem with prefixes for tesla, because this unit consists of a single word.

The advantage of using $\mu_0 H$ in place of H is that the definitions of permeability and susceptibility are greatly improved: Permeability is defined by

$$\mu = B/\mu_0 H$$

and magnetic susceptibility is defined by

$$\chi = I/\mu_0 H.$$

Thus defined, μ and χ are dimensionless and are equal to the relative permeability and relative susceptibility, respectively. (Here, μ is the same as the CGS value, while χ is 4π times the CGS value).

In conjunction with this proposal, I would like to make the following warning: *Never use B for magnetic field!* Even if one proposed to use the tesla, which is the unit for B , to describe the magnetic field strength, one should not confuse the concepts of H and B . Some textbooks on electromagnetic theory give the formula

$$\nabla \times \mathbf{B} = \mathbf{i}$$

where \mathbf{i} is the current density. This formula is correct only in a region without magnetic materials, or only if \mathbf{i} includes the intrinsic currents which cause the magnetization. If \mathbf{i} is a measurable current density, Ampere's theorem gives

$$\nabla \times \mathbf{H} = \mathbf{i}.$$

Moreover, many people believe that the real field existing in magnetic materials is B , not H . This depends on the experiment. Since the Hall effect for magnetic materials is a function of both H and I , B is the relevant quantity. (However, the Hall voltage is not a unique function of B ; the current senses a field which is a complex function of H and I). However, the force or torque exerted on the magnetization must be described by H , not B . This is because a term that is proportional to I forms the internal force. The situation is similar to the dynamic problem of solving for the path of a projectile. Even if the real force acting in the body includes the universal gravitational force between the different parts of the body, it is not considered because it is an internal force.

ROBERT STREET

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The conversion to SI units of measurement in Australia was achieved through the efforts of the Metric Conversion Board (MCB). Its objective was to facilitate the introduction of SI units as the only legal units of measurement in use for trade in all the states and territories of Australia. The first industry to benefit from the introduction of metric units of mass and length was the horse racing industry. This was a deliberate policy initiated by the Chairman of the Board, who held the view that everything was trivial compared with the conversion of the horse racing fraternity to SI units.

In the changeover from the previously existing confusion known as the Imperial System of units, the MCB worked closely with the National Standards Commission, a statutory body responsible for legislation concerning units of

weights and measures in use for trade. In my view, two things were primarily responsible for the undoubted success and the smooth transition to a radically new philosophy of measurement achieved by the Metric Conversion Board. The first was the enormous influence exerted by the late Alan Harper, a physicist from the National Measurements Laboratory of the Commonwealth Scientific and Industrial Research Organization (CSIRO). He held key positions on both the Board and the Commission. The second factor was a determination by the government that SI units of measurement were to be the only legal units in use for trade. This provided a powerful incentive for all sections of the community to learn and operate the new system of units.

Consideration was given to the specification of units to be used in electromagnetism. Commercial incentives arising from needs to specify quantitative information on the properties of magnetic materials did not exist. As we know, international conventions on magnetic units offer no clear guidelines. The question was too difficult to resolve, but CSIRO did adopt a policy which provides for the use of SI units, including SI magnetic units, in their publications.

It was not possible to make such a clear-cut decision in the universities and other research oriented institutes. SI units are almost universally adopted in the teaching programs of undergraduates. However, in graduate work there is the familiar schizophrenic approach in the choice of electromagnetic units. The most usual defence of this state of affairs is that the majority of international literature in magnetism uses the CGS system of units. At the present time there are no overwhelming advantages to be gained in adopting SI units to the exclusion of CGS. There will be a continuing need to be literate (and numerate) in both systems.

However, there are many advantages in moving to the universal adoption of a common system of units. In my opinion the SI is the only sensible candidate worthy consideration. After this opening skirmish at the MMM-Intermag Conference in Albuquerque, I would propose that discussions continue at an international level by electronic mail. The objective should be to produce a modified SI (including the introduction of desirable names of units, consideration of quantities such as an acceptable code for use in publications and trade in magnetic products internationally. My pet wishes are for a consistent constitutive

equation, either $B = \mu_0(H + M)$ or $B = \mu_0H + J$, and also for a name to be given to the unit of magnetic moment.

It is only when numerical answers are required to questions such as: how much energy? or how large a force? are units and relations between systems of units necessary. The latter steps are inevitable when magnetic materials are to be bought, sold, used and compared. Why not agree to adopt a consistent system of units in reporting the properties of magnetic materials, bearing in mind that the numerical results required have to be in established units (joules, newtons, etc.)? Hence, we should aim at those adjustments of SI that improve the convenience of its use in describing the quantitative properties of magnetic materials.

ANTHONY ARROTT

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In discussing the equations of magnetism in both Gaussian units and SI, I believe that there is a greater understanding that comes from being bilingual. With regard to units, the late William Fuller Brown, the founder of micromagnetism, wrote in his "Tutorial Paper on Dimensions and Units" (*IEEE Trans. Magn.*, vol. 20, pp. 112–117, January 1984), "If this seems a bit arbitrary and confusing, bear in mind two principles: first, dimensions are the invention of man, and man is at liberty to assign them in any way he pleases, as long as he is consistent throughout any one interrelated set of calculations. Second, international committees arrive at their decisions by the same irrational procedures as do various IEEE committees that you have served on." To writers on the subject, Brown advised "At all costs avoid conversion tables: with them, you never know whether to multiply or divide". On the other hand relations such as $1 \text{ T} \Rightarrow 10^4 \text{ G}$ and $1 \text{ G} \Rightarrow 10^{-4} \text{ T}$ are unambiguous.

While much has been published about conversions of units, it is not as common to find direct, line by line, comparisons of the equations of magnetism in the two systems of units, as is carried out in the appendix of Richard Becker's *Electromagnetic Fields and Interactions*, edited by F. Sauter, translated by A.W. Knudsen (Blackie, London, 1964). The rules for converting the equations are found in the basic reference, *Symbols, Units, Nomenclature and Fundamental Constants*

in *Physics*, prepared by E. Richard Cohen and Pierre Giacomo for the International Union of Pure and Applied Physics, *Physica*, vol. 146A, pp. 1–68, November 1987. For each quantity in an equation in SI units, it is necessary to apply one set of rules to obtain the equation in Gaussian units. Even after the rules are applied, it is necessary to invoke the identity $\mu_0\epsilon_0c^2 = 1$ to remove any left over $\mu_0\epsilon_0$. In going from equations in the Gaussian system to SI, one completes the conversion by using this identity to remove the velocity of light c .

If one uses starred variables for the Gaussian system and unstarred variables for the SI, the categories of conversions are:

$$(4\pi\epsilon_0)^{\frac{1}{2}} = E^*/E \text{ (electric field)} = V^*/V \text{ (potential)} \\ = Q/Q^* \text{ (charge)} = I/I^* \text{ (current)} = P/P^* \text{ (polarization)}$$

$$(4\pi/\epsilon_0)^{\frac{1}{2}} = D^*/D \text{ (electric flux density)}$$

$$4\pi\epsilon_0 = C/C^* \text{ (capacitance)} = R^*/R \text{ (resistance)} = L^*/L \text{ (inductance)}$$

$$(4\pi/\mu_0)^{\frac{1}{2}} = \phi^*/\phi \text{ (magnetic flux)} = B^*/B \text{ (magnetic flux density)} = \\ = M/M^* \text{ (magnetization)} = \gamma/\gamma^* \text{ (gyromagnetic ratio)} = \\ = A^*/A \text{ (vector potential)}$$

$$(4\pi\mu_0)^{\frac{1}{2}} = H^*/H \text{ (magnetic field strength)}$$

$$4\pi = \chi_e/\chi_e^* \text{ (electric susceptibility)} = \chi/\chi^* \text{ (magnetic susceptibility)}$$

Dimensionless quantities, other than susceptibilities, convert directly. Also, the mechanical quantities convert directly:

$$1 = x^*/x \text{ (length)} = t^*/t \text{ (time)} = v^*/v \text{ (velocity)} = m^*/m \text{ (mass)} = \\ = F^*/F \text{ (force)} = U^*/U \text{ (energy)} = T^*/T \text{ (torque)} = P^*/P \text{ (power)} = \\ = S^*/S \text{ (Poynting vector)}$$

Memorization of such a table would be a daunting task. I have written a chapter devoted to the comparison of equations in SI and Gaussian units in a forthcoming book on *Ultrathin Magnetic Structures*, edited by B. Heinrich and J.A.C. Bland (Springer–Verlag, Berlin, 1994).

J. M. D. COEY

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The current practice regarding units in magnetism is a mess. Advertisements for instruments for magnetic measurements in *Physics Today* exemplify the situation. Large fields are given in teslas, small ones in gaussses or oersteds. Magnetic moments and susceptibility are given in emu. The tesla is the only SI unit that seems to have caught on, as often with the symbol H as with B . However, it is common practice everywhere to use SI in undergraduate teaching, which has the powerful advantage that concepts and calculations from one area of the subject can be related to those in another—electricity and magnetism for example. The advantages of a coherent unit system in science and engineering far outweigh minor drawbacks such as the magnitude of μ_0 or the need to employ subscripts on the same symbols used for different quantities (for example, J for polarization, current density and exchange constant). Nevertheless, CGS units remain widespread in research; more than 80% of the papers in the joint MMM–Intermag Conference use them. Hence the sentiment is that SI is okay for kids, but real scientists use GCS. One might have expected that as older professors retire, and as younger ones exposed to SI as undergraduate students and teachers take over, we would see the gradual adoption of the coherent units system across physics and engineering. This does not seem to be happening. Progress, if any, in the past 20 years has been at a snail's pace. Like domestic metrication in the U.S., converting magnetism to SI seems to have ground to halt.

Why bother to try to change many people's habits of a lifetime? Is it not acceptable to advocate bilingualism and let everyone do as they please? The reasons for adopting SI in magnetism are the following: advantage of coherence and transparency achieved by using the same units as the rest of science; equations readily checked for dimensional correctness; straightforward calculations without the need to remember conversion factors, which are often misapplied; confidence in results of simple calculations based on the formulas learned at college.

But there is also an urgent reason why it is not in the long-term interests of the magnetism community to persist in the present shambles. Public concern is growing, especially in the U.S. and Germany, about possible harmful effects of weak electromagnetic fields produced by power lines, domestic appliances, video

display units, etc. This may be the "green" issue of the decade, costing billions of dollars. The experience of the nuclear industry should be instructive. The public may no longer accept bland assurances from the experts that levels are negligibly small. They may wish to check the elementary calculations for themselves, or buy a teslameter and make their own measurements. It should not require a Ph.D. expert to tell where a 4π or a factor of 10^4 needs to be popped in, or explain why equations like $\mathbf{E} = \mathbf{v} \times \mathbf{B}$ do not mean what they say. If the elementary physics cannot be made quantitatively transparent, it risks being discredited.

How to proceed? Change for all current practitioners should be made as painless as possible. This may be achieved by exploiting the current acceptability of the tesla as a unit of field, given the symbol B_0 in free space. Its use for smaller fields can be promoted through the use of [mT] and [μ T]. Magnetic moment, from the energy relation $W = -\mathbf{m} \cdot \mathbf{B}_0$, is measured in [$\text{J} \cdot \text{T}^{-1}$]. Magnetic polarization is also measured in teslas. $B = \mu_0 H + J$ and $B = \mu_0(H + M)$ can coexist. For practical measurements the numerical equivalence of mass magnetization σ in [$\text{emu} \cdot \text{g}^{-1}$] and [$\text{J} \cdot \text{T}^{-1} \cdot \text{kg}^{-1}$] is a useful reference point. Mass and volume susceptibilities χ_m and χ are in [$\text{J} \cdot \text{T}^{-2} \cdot \text{kg}^{-1}$] and [$\text{J} \cdot \text{T}^{-2} \cdot \text{m}^{-3}$]. Admittedly, $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m} \cdot \text{A}^{-1}$ must be committed to memory, but its magnitude makes it easy to spot if it has been left out of one side or the other of an equation. Equivalent units of M are [$\text{J} \cdot \text{T}^{-1} \cdot \text{m}^{-3}$] or [$\text{A} \cdot \text{m}^{-1}$], the same as for H .

Here is an SI tool kit, a summary of what I find I need to function effectively in SI:

$$B[\text{T}] = \mu_0(H[\text{A} \cdot \text{m}^{-1}] + M[\text{A} \cdot \text{m}^{-1}])$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m} \cdot \text{A}^{-1}$$

$$1\mu_B = 9.27 \times 10^{-24} \text{ J} \cdot \text{T}^{-1}$$

$$W[\text{J}] = -\mathbf{m}[\text{J} \cdot \text{T}^{-1}] \cdot \mathbf{B}_0[\text{T}]$$

$$H = nI [\text{A} \cdot \text{m}^{-1}] \text{ (solenoid)}$$

$$m = IA [\text{A} \cdot \text{m}^2] = [\text{J} \cdot \text{T}^{-1}] \text{ (current loop)}$$

$$M[\text{A} \cdot \text{m}^{-1}] = M[\text{J} \cdot \text{T}^{-1} \cdot \text{m}^{-3}] = m[\text{J} \cdot \text{T}^{-1}] / V[\text{m}^{-3}]$$

$$w[\text{J} \cdot \text{m}^{-3}] = \frac{1}{2} B[\text{T}] \cdot H[\text{J} \cdot \text{T}^{-1} \cdot \text{m}^{-3}]$$

$$\chi_m = \sigma / B_0 [\text{J} \cdot \text{T}^{-2} \cdot \text{kg}^{-1}]$$

$$\chi = M / B_0 [\text{J} \cdot \text{T}^{-2} \cdot \text{m}^{-3}]$$

Personally, my entire student and professional experience was in CGS until I became a university professor. I made the switch.

MIKE R. J. GIBBS

University of Sheffield

A major issue that must be recognized is that, certainly in the U.K. and mainland Europe, the new generation of scientists coming through schools and universities is being trained exclusively in the SI system. It must be a retrograde step to ask them to work in older unit system or even to learn cumbersome transformations. The U.K. Institute of Physics Magnetism Group has had a working party looking at the issue of units in magnetism. Its members are J. Crangle, C.D. Graham, Jr., M.R. J. Gibbs, S. Brunt and P.T. Squire.

Our contribution to the debate is that the H field should be replaced by the free space induction B_0 . There is a problem in magnetic circuits containing an air gap, where H and B in the gap can be in opposite directions. Great care would be necessary to distinguish the inductions in the gap from the magnetic material and the free poles. We then turned our attention to the representation of the effect of the free space induction on a magnetically polarizable material. Our main concern as a working party centered around whether or not magnetic susceptibility should be dimensionless. There really is no *a priori* reason why it should be. If magnetization is used, the volume susceptibility is written as M/B_0 , which is not dimensionless. We would prefer to use polarization J , whence the susceptibility is J/B_0 , which is dimensionless.

We noted that saturation induction B_s may be a recognized label for a material, but strictly it is not uniquely valued. The $B-B_0$ loop always has a high-field slope of unity. We therefore consider the use of saturation polarization J_s as an attractive alternative. J_s is uniquely defined, and a $J-B_0$ plot would show saturation.

What this amounts to is a development of the Kennelly model, giving us a defining equation $B = B_0 + J$.

RON B. GOLDFARB

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One of the main problems with the CGS Gaussian and EMU systems is the reluctance of researchers to express magnetic moment in units of $[\text{erg}\cdot\text{G}^{-1}]$ or $[\text{erg}\cdot\text{Oe}^{-1}]$. Instead, they use the designation "emu", which is not a unit at all, but simply an indicator of electromagnetic units. Thus, rather than expressing magnetic moment per unit volume, that is, volume magnetization, as $[\text{erg}\cdot\text{G}^{-1}\cdot\text{cm}^{-3}]$, they use $[\text{emu}\cdot\text{cm}^{-3}]$; and rather than expressing volume susceptibility as dimensionless, simplified from $[\text{erg}\cdot\text{G}^{-2}\cdot\text{cm}^{-3}]$, many researchers write $[\text{emu}\cdot\text{cm}^{-3}\cdot\text{Oe}^{-1}]$ or $[\text{emu}\cdot\text{cm}^{-3}]$.

Another problem with CGS is that, in electricity, the Gaussian unit of current is the statampere, and the EMU of current is the abampere. When electrical and magnetic quantities are combined, care is required. Many researchers working in CGS are reluctant to abandon the ampere and resort to writing equations in mixed units, typically expressing current in amperes, distance in centimetres, magnetization in gauss, and magnetic field strength in oersteds. Equations with such combinations that do not balance dimensionally can cause trouble when they are used in further derivations.

Yet another problem with CGS is the ambiguity between the unit for M $[\text{erg}\cdot\text{G}^{-1}\cdot\text{cm}^{-3}]$ and $4\pi M$ [G]. Dimensionally, they are equivalent (this can be seen by substituting $[\text{cm}^2\cdot\text{g}\cdot\text{s}^{-2}]$ for [erg] and $[\text{cm}^{-1/2}\cdot\text{g}^{1/2}\cdot\text{s}^{-1}]$ for [G]), but numerically, the quantities differ by the factor 4π .

SI (derived from the MKSA system) unifies magnetic units with the practical electrical units ampere and volt. The dimensional balance of equations is always apparent, if one remembers that $[\text{T}] = [\text{Wb}\cdot\text{m}^{-2}]$, $[\text{Wb}] = [\text{V}\cdot\text{s}]$, and that $[\text{A}\cdot\text{m}^2]$, the unit of magnetic moment, is the same as $[\text{J}\cdot\text{T}^{-1}]$.

One ambiguity of SI is that quantities appearing in both $B = \mu_0(H + M)$ and $B = \mu_0 H + J$ are recognized. Two quantities associated with these equations are magnetic moment m $[\text{A}\cdot\text{m}^2]$ and magnetic dipole moment j $[\text{Wb}\cdot\text{m}]$. The ambiguity is not a problem as long as we explicitly give the names of the quantities (magnetization or magnetic polarization, for example) and indicate their

units. Permeability is always defined as B/H , with units $[\text{H}\cdot\text{m}^{-1}]$, and relative permeability is defined as $B/\mu_0 H$ (dimensionless). Volume susceptibility in SI is dimensionless and may be obtained as M/H or $J/\mu_0 H$. As used by the International Organization for Standardization (ISO), volume susceptibility is never defined as M/B , J/B , $M/\mu_0 H$, or J/H . A difficulty that arises in reporting volume susceptibility is that it is also dimensionless in CGS, but its value differs by a factor of 4π . To compensate for the lack of specific units, I recommend that "(SI)" or "(CGS)" follow numerical values of volume susceptibility and that these designations be included on the axis labels of figures.

Further discussion on units can be found in my article, "Magnetic Units and Material specification", in *Concise Encyclopedia of Magnetic and Superconducting Materials*, edited by J. Evetts (Pergamon, Oxford, 1992).

Keywords: SI units, GCS units, magnetic susceptibility, magnetic field, magnetic induction, magnetization