MODEL CALCULATIONS OF THE RECRYSTALLIZATION TEXTURE FORMATION IN α -IRON

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Model calculations of primary recrystallization textures in bcc metals were carried out on the basis of a model which considers oriented nucleation as well as oriented growth. The influence of the input parameters, as deformation texture, nucleation distribution and growth law were investigated systematically. It is shown that the inhomogeneity of the deformed microstructure has to be taken into account. With this essential assumption the main features of the recrystallization texture formation can be explained by a growth selection process according to a $27^\circ \langle 110 \rangle$ and $84^\circ \langle 110 \rangle$ orientation relationship.

KEY WORDS: Primary recrystallization, simulation, bcc metals, growth selection, oriented nucleation, $27^\circ \langle 110 \rangle$ -orientation relationship, $84^\circ \langle 110 \rangle$ -orientation relationship.

1. INTRODUCTION

The rolling and recrystallization textures of bcc metals are usually described by two fibre components (v. Schlippenbach, Emren, Lücke 1986): The partial α -fibre with a $\langle 110 \rangle$ direction parallel to the rolling direction contains the orientations $\{001\}\langle 110 \rangle$, $\{112\}\langle 110 \rangle$ and $\{111\}\langle 110 \rangle$. The second fibre component γ has a $\langle 111 \rangle$ -axis parallel to the normal direction and includes the orientations $\{111\}\langle 110 \rangle$ (which actually belongs to both fibres) and $\{111\}\langle 112 \rangle$. With increasing deformation the rolling texture changes in a characteristic way, considering the main components: the orientations $\{001\}\langle 110 \rangle$ to $\{111\}\langle 110 \rangle$ increase steadily up to high deformation degrees, while the orientation intensity of the component $\{111\}\langle 112 \rangle$ shows a strong increase up to a deformation degree of about 50% but remains constant with further rolling. At high deformation degrees maxima are observed in the orientations $\{112\}\langle 110 \rangle$ or $\{111\}\langle 110 \rangle$.

During early stages of the recrystallization process the ODF shows a certain increase of the orientations $\{001\}\langle 110 \rangle$ to $\{112\}\langle 110 \rangle$. Simultaneously the orientation density along the $\{111\}$ -fibre decreases slightly. Later some drastical changes occur. The orientations $\{001\}\langle 110 \rangle$ to $\{112\}\langle 110 \rangle$ decrease, while the orientation $\{111\}\langle 110 \rangle$ keeps a constant orientation density and the orientation $\{111\}\langle 112 \rangle$ becomes stronger. After complete recrystallization the texture consists of a strong $\{111\}$ fibre texture with a maximum in the orientation $\{111\}\langle 112 \rangle$ (Emren, v. Schlippenbach, Lücke 1986).

These observations were confirmed by several other investigations, e.g. Kern and Bunge (1984) and Kern, Grewen and Bunge (1984). Sometimes, especially for rolling degrees from 50 to 70%, and coarse grained material, the occurrence of a $\{110\}\langle 001 \rangle$

Goss component could be observed in the recrystallized state (Hölscher, Raabe and Lücke 1991, Plutka and Hougardy 1991, v. Schlippenbach and Lücke 1986).

Investigations of Schläfer and Bunge (1974) showed some slight differences: in the deformation texture a shift of the $\{111\}\langle 112 \rangle$ component to the $\{11,11,8\}\langle 4,4,11 \rangle$ orientation was found. Nevertheless the development of the rolling texture with the rolling degree was in principle the same. During recrystallization the textures in the specimens with a low deformation degree were weakened. The recrystallization texture in specimens with high rolling degrees showed a strong $\{111\}$ -fibre and a weak $\{113\}$ fibre component (Bunge, Schleusener and Schläfer 1974).

Although the recrystallization process in bcc metals obviously leads to some typical texture changes, there exists no undisputed statement about the mechanisms controlling this process. The aim of our model calculations was to find some answers to the question which physical mechanisms may play a role in the texture formation process. This was done by formulating a mathematical model that considers oriented nucleation and oriented growth. We tried to adjust the simulated results to the experimental ones by varying the input parameters in sensible limits.

2. THE MATHEMATICAL MODEL FOR HOMOGENEOUS RECRYSTALLIZATION

An extensive description of the model was given in an earlier paper by Bunge and Plege (1987). In this model oriented nucleation as well as oriented growth are taken into account. The texture of the fully recrystallized state can be calculated with the ODF of the nuclei $f^N(g)$ and their mean linear growth rate $\bar{W}(g)$:

$$f^R(g) = \text{const.} \cdot f^N(g) \cdot \bar{W}(g)^3 \quad (1)$$

The central problem of the simulations is the calculation of the average growth rate $\bar{W}(g)$. As simplification we assume that all nuclei start to grow at the same time and stop growing when 100% recrystallized volume fraction is reached. The average growth rate $\bar{W}(g^R)$ of nuclei with the orientation g^R can be calculated from the deformation texture $f^D(g^D)$ and the local growth rate which is a function of the orientation dependent driving force $P(g^D)$ and the mobility $m(\Delta g)$ of the moving grain boundary. The latter one depends on the orientation difference Δg between the growing grain g^R and matrix grain g^D .

$$\frac{1}{\bar{W}(g^R)} = \oint \frac{f^D(g^D)}{P(g^D) \cdot m(\Delta g)} dg^D \quad (2)$$

This equation can be considered as the mathematical formulation of the "compromise" texture formation. That means that the distance between nuclei is large compared with the grain size, so that each nucleus has to grow into grains of all orientations present in the deformation texture.

Thus we have several parameters influencing the results:

- The deformation texture $f^D(g)$
This parameter is generally well known from pole figure measurements. Comprehensive investigations showed that a typical texture development occurs during the rolling process (chapter 1). In this paper we introduced model

deformation textures, idealized in the form of a superposition of i ideal orientations g_i with a Gaussian spread $\omega_{D,i}$, considering their volume fractions v_i .

$$f^D(g^D) = \sum_i v_i \cdot f^{Gauss}(g)_{g_i, \omega_{D,i}} \quad (3)$$

In this way we have the possibility to examine the effect of small modifications of the deformation texture on the simulated results and to connect these with experimental observations.

- The nuclei distribution function $f^N(g)$.

We have to assume that it is not possible to investigate the nucleation process itself. When a small grain is found in the deformed matrix, in general some growth must already have taken place. It is not possible to decide whether the configuration found at that time corresponds to the initial one or whether the growth process has already changed the characteristics of the nuclei distribution.

For first calculations the ODF of nuclei was assumed to be random ($f^N(g) = 1$). In this way we consider the selective growth process alone to be responsible for the recrystallization texture formation. In order to take oriented nucleation into consideration we can either introduce a measured nuclei distribution (e.g. experimental results of single orientation measurements) or a simulated one, taking specific nucleation mechanisms into account.

In several investigations of the recrystallization process in α -iron it was stated that nuclei originate from certain microstructural configurations in the deformed material and that they have the same orientation distribution as the deformed matrix ($f^N(g) = f^D(g)$). Possible nucleation mechanisms are subgrain coalescence in transition bands (Hu, 1962, Dillamore *et al.*, 1972), subgrain growth (Smith and Dillamore 1970, Every and Hatherly 1974) or strain induced boundary migration (Inokuti and Doherty 1977). In most of these papers it was mentioned, that the deformed material shows an orientation dependent distribution of the stored energy connected with a very inhomogeneous microstructure. Thus deformed grains with different microstructures can show a different nucleation behaviour (chapter 4).

In a paper by Böttcher *et al.* (1991) the orientation distribution of very small recrystallized grains in RGO steel, 60% cold rolled was determined by the EBSP method. Because of the very high resolution of this method it was possible to determine the orientation of very small grains ($\geq 0.3\mu\text{m}$) as well as that from the neighbouring matrix. The maximum of the nuclei distribution was also found in the main component of the deformation texture, but besides that a rather high fraction of randomly oriented nuclei was found.

These observations were considered in our model calculations by modeling a nuclei distribution:

$$f^N(g) = (1 - v) + v \cdot f^D(g) \quad (4)$$

Thus we have two limiting cases: With $v = 0$ we consider random nucleation, with $v = 1$ we assume that the nuclei distribution is identical to the deformation texture.

- The local growth law (driving force and mobility)

The first calculations were carried out on the assumption that the driving force is orientation independent ($P(g) \equiv 1$). Thus the local growth rate coincides with the grain boundary mobility $W(\Delta g) = m(\Delta g)$.

For the calculation of the mean growth rate $\bar{W}(g)$ according to equation 2 the local growth law was approximated as a superposition of model distributions extended into the three dimensional misorientation space. Each of these i model distributions, weighted with a factor v_i , takes on a maximum at a certain misorientation (Δg_o) and has an isotropic Gaussian spread ω_G . It is assumed that orientations with a large orientation difference from the maximum have a small growth rate W_o , which we set virtually on a fraction of 1/10th of the maximum:

$$W(\Delta g) = W_o + \sum_i v_i \cdot W^{Gauss}(\Delta g)_{\Delta g_o, i, \omega_G, i} \quad (5)$$

The central values in this local growth law are chosen according to results from growth selection experiments with single crystals of Silicon Iron by Ibe and Lücke (1966). They found, that about 90% of the fast growing grains had a $27^\circ \langle 110 \rangle$, the other 10% a $84^\circ \langle 110 \rangle$ orientation relationship to the matrix grains.

As example the local growth rates as function of the misorientation between growing grains and deformed matrix with these two orientation relationships as maxima are shown in Figure 1. The calculations were either carried out with one of these two model functions (with varying spread ω_G), or with a superposition of both.

The calculations were carried out using the series expansion method (Bunge 1982), extended to a degree $L_{max} = 22$. Therefore it was not sensible to consider values less than 8° as spread of the Gaussian distribution functions.

Figure 1 Sections $\varphi_2 = 0^\circ$ to 45° of the local growth rate $W(\Delta g)$ as a function of the misorientation between growing grains and deformed matrix according to equation 5 with $\Delta g_o, \omega_G = 12^\circ$, and linebreak $W_o = 1/10 \cdot W(\Delta g)_{max}$
a) $\Delta g_o = 27^\circ \langle 110 \rangle$,
b) $\Delta g_o = 84^\circ \langle 110 \rangle$.

3. RESULTS OF NUMERICAL CALCULATIONS

3a. Simulations with single components

All results in this paper are presented by the ODF sections $\varphi_1 = 0^\circ, 45^\circ$ and 90° . Here the most important texture components of rolling and recrystallization textures in bcc metals can be found. As an example Figure 2 shows an experimental deformation and recrystallization texture of ARMCO Iron, cold rolled to a final reduction of 90%. The most important deformation and recrystallization texture components are identified in Figure 2c.

The first calculations were made with deformation textures consisting of ideal components according to the main components of the rolling texture, that are $\{001\}\langle 110 \rangle$, $\{112\}\langle 110 \rangle$, $\{111\}\langle 110 \rangle$, $\{111\}\langle 112 \rangle$ and $\{11,11,8\}\langle 4,4,11 \rangle$. For all these model deformation textures the spread ω_d was kept constant at 12° . Earlier calculations with varying spreads of the deformation texture components in fcc metals illustrated a strong influence of this parameter on the strength of the calculated growth rates (Köhler *et al.*, 1992). The same effect was observed with the starting deformation textures considered in this paper. Another parameter which has a strong influence on the intensity of the calculated texture (but not on the position of the maxima and minima), is the "background" value W_0 in the local growth law. This parameter was introduced into equation 5 to prevent a division by zero in equation 2. Unfortunately we have no knowledge about quantitative data of this parameter (which corresponds to the growth rate of nuclei with a large orientation difference from fast growing grains), so that we had to choose a plausible value.

So we have several possibilities to influence the maxima of our calculated results. Since we do not start with real deformation textures, but with simulated ones, it is not useful to compare the absolute intensities of the calculated results with experimental recrystallization textures. The aim of our calculations – in the present stage – is to explain the main features of the recrystallization texture development. Therefore we do not mention the absolute intensities of the maxima and minima in the examples presented in this paper.

The distribution of nuclei was assumed to be random, thus the ODF of the recrystallization texture is proportional to $\bar{W}(g)^3$. According to the results from growth selection experiments by Ibe and Lücke (1966) the maximum of the local growth law

Figure 2 Sections $\varphi_1 = 0^\circ, 45^\circ$ and 90° of a rolling and recrystallization texture of α -Iron

a) ARMCO Iron, 90% cold rolled

b) ARMCO Iron 90% cold rolled and annealed

c) Ideal orientations:

A: $\{001\}\langle 110 \rangle$,

B: $\{112\}\langle 110 \rangle$,

C: $\{111\}\langle 110 \rangle$,

D: $\{111\}\langle 112 \rangle$,

E: $\{11,11,8\}\langle 4,4,11 \rangle$,

G: $\{110\}\langle 001 \rangle$,

α : skeleton line of the partial $\langle 110 \rangle$ fibre,

γ : skeleton line of the $\{111\}$ fibre.

was chosen corresponding to a $27^\circ\langle 110 \rangle$ orientation relationship, extended into the three dimensional misorientation space according to a Gaussian distribution. The spread ω_G of this model distribution (Figure 1a) was varied in a range from 8° to 20° .

The results obtained with these input parameters are shown in Figure 3. For all five starting deformation textures the variation of the spread ω_G of the local growth law has a very strong influence on the results. Since in some cases components appear, which are not visible in the ODF sections shown in Figure 3, all growth components are listed in Table 1.

The deformation texture component $\{001\}\langle 110 \rangle$ gives rise to a maximum growth rate in the orientation range $\{0^\circ, 0^\circ\text{--}30^\circ, 45^\circ\}$, depending on the spread ω_G . Experiments showed that these orientations disappear during the recrystallization process. On the other hand orientations of the $\{111\}$ fibre, which are very strong in the recrystallization texture have a very low growth rate into this deformation texture component (Figure 3a).

The starting textures $\{112\}\langle 110 \rangle$ and $\{111\}\langle 110 \rangle$ generate a growth component near the $\{111\}\langle 112 \rangle$ orientation, where the maximum of the experimental recrystallization texture is found. The strength of this component depends on the spread values of the local growth law (Figures 3b and 3c). For both deformation textures some other growth components appear which have no experimental evidence as recrystallization texture components (Table 1).

Table 1 Orientations of maximum growth rate for the different main components of the deformation texture, obtained with a $27^\circ\langle 110 \rangle$ growth law and different spread values.

$f^D(g)$ ϕ_1 Φ ϕ_2	$\omega_G = 8^\circ$ ϕ_1 Φ ϕ_2	$\omega_G = 12^\circ$ ϕ_1 Φ ϕ_2	$\omega_G = 16^\circ$ ϕ_1 Φ ϕ_2	$\omega_G = 20^\circ$ ϕ_1 Φ ϕ_2
$\{110\}\langle 110 \rangle$ $0^\circ, \quad 0^\circ, \quad 45^\circ$	$0^\circ, \quad 23^\circ, \quad 45^\circ$ $90^\circ, \quad 23^\circ, \quad 45^\circ$ $45^\circ, \quad 16^\circ, \quad 16^\circ$ $45^\circ, \quad 16^\circ, \quad 74^\circ$	$0^\circ, \quad 19^\circ, \quad 45^\circ$ $90^\circ, \quad 19^\circ, \quad 45^\circ$ $45^\circ, \quad 14^\circ, \quad 14^\circ$ $45^\circ, \quad 14^\circ, \quad 76^\circ$	$0^\circ, \quad 12^\circ, \quad 45^\circ$ $90^\circ, \quad 12^\circ, \quad 45^\circ$ $45^\circ, \quad 9^\circ, \quad 9^\circ$ $45^\circ, \quad 9^\circ, \quad 81^\circ$	$0^\circ, \quad 0^\circ, \quad 45^\circ$
$\{112\}\langle 110 \rangle$ $0^\circ, \quad 35^\circ, \quad 45^\circ$	$35^\circ, \quad 45^\circ, \quad 0^\circ$ $90^\circ, \quad 61^\circ, \quad 45^\circ$ $0^\circ, \quad 0^\circ, \quad 45^\circ$	major: $35^\circ, \quad 45^\circ, \quad 0^\circ$ minor: $0^\circ, \quad 0^\circ, \quad 45^\circ$ $90^\circ, \quad 61^\circ, \quad 45^\circ$	major: $35^\circ, \quad 45^\circ, \quad 0^\circ$ minor: $0^\circ, \quad 0^\circ, \quad 45^\circ$ $90^\circ, \quad 61^\circ, \quad 45^\circ$	major: $35^\circ, \quad 45^\circ, \quad 0^\circ$ minor: $0^\circ, \quad 0^\circ, \quad 45^\circ$ $90^\circ, \quad 61^\circ, \quad 45^\circ$
$\{111\}\langle 110 \rangle$ $0^\circ, \quad 55^\circ, \quad 45^\circ$	$90^\circ, \quad 55^\circ, \quad 45^\circ$ $0^\circ, \quad 90^\circ, \quad 45^\circ$ $30^\circ, \quad 45^\circ, \quad 0^\circ$	major: $90^\circ, \quad 55^\circ, \quad 45^\circ$ minor: $0^\circ, \quad 90^\circ, \quad 45^\circ$ $30^\circ, \quad 45^\circ, \quad 0^\circ$	major: $90^\circ, \quad 55^\circ, \quad 45^\circ$ minor: $0^\circ, \quad 90^\circ, \quad 45^\circ$ $30^\circ, \quad 45^\circ, \quad 0^\circ$	major: $90^\circ, \quad 55^\circ, \quad 45^\circ$ minor: $0^\circ, \quad 90^\circ, \quad 45^\circ$ $30^\circ, \quad 45^\circ, \quad 0^\circ$
$\{111\}\langle 112 \rangle$ $90^\circ, \quad 55^\circ, \quad 45^\circ$	$0^\circ, \quad 55^\circ, \quad 45^\circ$ $0^\circ, \quad 45^\circ, \quad 0^\circ$ $60^\circ, \quad 45^\circ, \quad 0^\circ$	major: $0^\circ, \quad 55^\circ, \quad 45^\circ$ minor: $0^\circ, \quad 45^\circ, \quad 0^\circ$ $60^\circ, \quad 45^\circ, \quad 0^\circ$	major: $0^\circ, \quad 55^\circ, \quad 45^\circ$ minor: $0^\circ, \quad 45^\circ, \quad 0^\circ$ $60^\circ, \quad 45^\circ, \quad 0^\circ$	major: $0^\circ, \quad 55^\circ, \quad 45^\circ$ minor: $0^\circ, \quad 45^\circ, \quad 0^\circ$ $60^\circ, \quad 45^\circ, \quad 0^\circ$
$\{11,11,8\}\langle 4,4,11 \rangle$ $90^\circ, \quad 63^\circ, \quad 45^\circ$	major: $0^\circ, \quad 45^\circ, \quad 0^\circ$ minor: $0^\circ, \quad 52^\circ, \quad 45^\circ$	major: $0^\circ, \quad 45^\circ, \quad 0^\circ$ minor: $0^\circ, \quad 52^\circ, \quad 45^\circ$	major: $0^\circ, \quad 45^\circ, \quad 0^\circ$ minor: $0^\circ, \quad 52^\circ, \quad 45^\circ$	major: $0^\circ, \quad 45^\circ, \quad 0^\circ$ minor: $0^\circ, \quad 52^\circ, \quad 45^\circ$

Figure 3 Sections $\varphi_1 = 0^\circ, 45^\circ$ and 90° of the starting deformation texture and the mean growth rate $\bar{W}(g)^3$ calculated with a $27^\circ\langle 110 \rangle$ orientation relationship and the spread values ω_G in the local growth law.

a) deformation textures: $\{001\}\langle 110 \rangle$.

Figure 3 (continued)
b) deformation texture $\{112\}\langle 110 \rangle$.

Figure 3 (continued)
c) deformation texture $\{111\}\langle 110 \rangle$.

Figure 3 (continued)
d) deformation texture $\{111\}\langle 112 \rangle$.

Figure 3 (continued)
e) deformation texture $\{11,11,8\}\langle 4,4,11 \rangle$.

With the $\{111\}\langle 112 \rangle$ component as starting texture we get a $\{111\}\langle 110 \rangle$ growth component, which has a rather high orientation density in experimental recrystallization textures. Furthermore the Goss component shows an enhanced growth rate, at least for small spread values ω_G (Figure 3d).

A shift of the maximum of this deformation texture of about 8° towards the orientation $\{11,11,8\}\langle 4,4,11 \rangle$ has a remarkable influence on the mean growth rate distribution. Now the Goss orientation is the maximum growth component for the whole range of the spread values ω_G (Figure 3e). Simultaneously the growth rate in the orientation $\{111\}\langle 110 \rangle$ is weakened considerably.

Obviously the results are strongly influenced by a variation of the spread value ω_G in the local growth law. This effect is elucidated by Figure 4: here the orientation densities in specific orientations are presented as a function of the spread ω_G in the local growth law with the deformation texture component (A–E) as parameter. This presentation allows to estimate the growth rate of nuclei with a certain orientation into the different deformation texture components. As we mentioned earlier, we must not compare the absolute intensities of the calculated results with experimental ones. Therefore we “normalized” the results in such a way, that the maximum of every calculated growth rate distribution $\bar{W}(g)^3$ corresponds to the value 1, the minimum to the value 0. That means that the normalization factor changes with the spread ω_G in the local growth law.

A growth component originating from a certain deformation texture component, which is very weak when a small spread ω_G is chosen, may become dominant when the spread

increases or vice versa. This effect is particularly strong for the growth rate of a $\{001\}\langle 110 \rangle$ nucleus into the deformation texture components $\{001\}\langle 110 \rangle$ and $\{112\}\langle 110 \rangle$ (Figure 4a).

Considering the recrystallization texture components $\{111\}\langle 110 \rangle$, $\{111\}\langle 112 \rangle$ and the Goss orientation $\{110\}\langle 001 \rangle$ (Figures 4c,d,f), each one has a maximum growth rate into one deformation texture component, but none is supported by all deformation texture components. Thus we expect that the composition of the deformation texture has a strong influence on the development of the recrystallization texture. A high volume fraction of the orientation $\{111\}\langle 112 \rangle$ in the deformation texture should support the development of the $\{111\}\langle 110 \rangle$ orientation in the recrystallization texture and vice versa (Figures 4c,d), whereas a shift of the deformation texture component $\{111\}\langle 112 \rangle$ to the $\{11,11,8\}\langle 4,4,11 \rangle$ orientation could explain the origin of the Goss component in the recrystallization texture.

Figure 4e shows that nuclei in the cube orientation have a very low growth rate into all deformation texture components.

A very interesting effect is the strong difference between the results from the two deformation textures $\{111\}\langle 112 \rangle$ and $\{11,11,8\}\langle 4,4,11 \rangle$ (D and E). Whereas the first one supports the growth of the $\{111\}\langle 110 \rangle$ component, the second leads to the formation

Figure 4 “Normalized” growth rate $\bar{W}(g)^3$ in the orientations a) $\{001\}\langle 110 \rangle$, b) $\{112\}\langle 110 \rangle$, c) $\{111\}\langle 110 \rangle$, d) $\{111\}\langle 112 \rangle$, e) $\{001\}\langle 100 \rangle$, f) $\{011\}\langle 100 \rangle$ as function of the spread value ω_G in the local growth law, obtained with the starting deformation textures A) $\{001\}\langle 110 \rangle$, B) $\{112\}\langle 110 \rangle$, C) $\{111\}\langle 110 \rangle$, D) $\{111\}\langle 112 \rangle$, and E) $\{11,11,8\}\langle 4,4,11 \rangle$. The maximum in the local growth law was chosen $\Delta g_0 = 27^\circ \langle 110 \rangle$, the nucleation distribution was assumed to be random. The value $f^R(g) = 1$ corresponds to the maximum, the value $f^R(g) = 0$ to the minimum of the calculated recrystallization texture, respectively.

of the Goss component. Since this shift of the maximum of the starting deformation texture of only 8° causes such a strong effect, the same calculations were carried out with some intermediate orientations $\{90^\circ, \Phi, 45^\circ\}$ with Φ values from 57° to 61° . The normalized orientation densities in the orientations $\{111\}\langle 110 \rangle$ and $\{110\}\langle 001 \rangle$ of the calculated recrystallization texture are shown in Figure 5. Even a small shift of the maximum in the starting deformation texture of about 2° has an evident influence on the growth rates of these two recrystallization texture components. Deformation texture components with a maximum intensity between $\{90^\circ, 55^\circ, 45^\circ\}$ and $\{90^\circ, 57^\circ, 45^\circ\}$ have a maximum growth rate in the orientation $\{111\}\langle 110 \rangle$, whereas the deformation texture components $\{90^\circ, 59^\circ, 45^\circ\}$ to $\{90^\circ, 63^\circ, 45^\circ\}$ cause a maximum growth rate in the Goss orientation $\{110\}\langle 001 \rangle$.

In contrast the growth rate in orientations with a large orientation difference from these specific growth components, e.g. in the orientations $\{001\}\langle 110 \rangle$, $\{111\}\langle 112 \rangle$ or $\{100\}\langle 001 \rangle$ (Figures 4a,d,e), is hardly influenced by this modification of the deformation texture.

3b. Simulations with a multi component deformation texture

Real rolling textures of α -iron generally have a broad spectrum of orientations. The exact composition of the deformation texture depends strongly on the material and the process parameters (v. Schlippenbach, Emren and Lücke 1986). A first rough approximation of such a rolling texture was made by composing the starting deformation texture of four ideal components: $\{112\}\langle 110 \rangle$, $\{111\}\langle 110 \rangle$ and $\{111\}\langle 112 \rangle$, with volume fractions of 30% each, and 10% of the $\{001\}\langle 110 \rangle$ component. Each component had a spread ω_d of 12° . In this way all deformation texture components had approximately the same maximum orientation density.

With this starting deformation texture and the same local growth law as in chapter 3a ($\Delta g_o = 27^\circ \langle 110 \rangle$) we obtain the results presented in Figure 6a. With the spread values $\omega_G = 8^\circ$ and 12° the maximum is found in the orientation $\{90^\circ, 60^\circ, 45^\circ\}$, close to the maximum of experimental recrystallization textures. With increasing spread a very strong component appears in the orientation range $\{25^\circ\text{--}50^\circ, 45^\circ, 0^\circ\}$. Table 1 and Figure 3 show that the deformation texture component $\{112\}\langle 110 \rangle$ has a major, the two

Figure 5 "Normalized" growth rate $\bar{W}(g)^3$ in the orientations a) $\{111\}\langle 110 \rangle$ and b) $\{110\}\langle 001 \rangle$ as function of the spread value ω_G in the local growth law, obtained with the starting deformation textures $\{90^\circ, \Phi, 45^\circ\}$, $55^\circ \leq \Phi \leq 63^\circ$.

Figure 6 Sections $\varphi_1 = 0^\circ, 45^\circ$ and 90° of the starting deformation texture and the mean growth rate $\bar{W}(g)^3$ calculated with a starting deformation texture consisting of 30% $\{112\}\langle 110 \rangle$, $\{111\}\langle 110 \rangle$, and $\{111\}\langle 112 \rangle$, respectively and 10% of the $\{001\}\langle 110 \rangle$ component.
a) $\Delta g_o = 27^\circ\langle 110 \rangle$. ω_G

Figure 6 (continued)
b) $\Delta g_o = 20^\circ\langle 110 \rangle$. ω_G

Figure 6 (continued)
c) $\Delta g_o = 35^\circ\langle 110 \rangle$. ω_G

components $\{111\}\langle 110 \rangle$ and $\{111\}\langle 112 \rangle$ have a minor growth component in this orientation range. This component becomes even stronger on costs of the γ fibre with increasing spread ω_G . In general the results gained with the "compromise" model (equation 2) show a rather complicated, non-linear dependence on the starting deformation texture as was already shown for fcc metals (Köhler *et al.*, 1992).

With the same starting deformation texture the influence of a change of Δg_o in the local growth law was investigated. In Figure 6 the results for different $\langle 110 \rangle$ orientation relationships are shown, the rotation angles are 20° , 27° and 35° .

The results obtained with $\Delta g_o = 20^\circ\langle 110 \rangle$ show a pronounced development of the γ -fibre $\{111\}$ (Figure 6b). Results obtained with $\Delta g_o = 35^\circ\langle 110 \rangle$ have no similarity with experimental textures at all (Figure 6c), the orientation density in the $\{111\}$ orientations is much too weak now.

Obviously the best results were produced with rotation angles between 20° and 27° . However, with both orientation relationships we get some essential deviations from experimental recrystallization textures: the local growth law with $\Delta g_o = 27^\circ\langle 110 \rangle$ and $\omega_G = 8^\circ$ to 12° leads to a maximum in the orientation $\{111\}\langle 112 \rangle$, which is in good agreement with experimental results. An increase of the spread value produces some further growth components which do not occur in experimental recrystallization textures. These are weakened when we change the maximum in the local growth law to the $20^\circ\langle 110 \rangle$ orientation relationship (considering the equivalent spread value ω_G). But simultaneously the maximum of the mean growth distribution is now in the orientation $\{111\}\langle 110 \rangle$ instead of the orientation $\{111\}\langle 112 \rangle$.

In growth selection experiments besides the major $27^\circ\langle 110 \rangle$ orientation relationship a $84^\circ\langle 110 \rangle$ relation was found between a certain fraction of the fast growing grains and the matrix crystals (Ibe, Lücke 1966). Thus calculations were carried out with local growth rate laws where both orientation relationships are considered. Figure 7 shows the results gained with varying fractions of the two Gaussian distribution (equation 5). The calculations represented in this Figure were carried out with $\omega_G = 16^\circ$, since with this spread value the best results were obtained. The mean growth rate distribution obtained with a pure $84^\circ\langle 110 \rangle$ orientation relationship in the local growth rate law has a strong maximum in the cube orientation, but a rather low intensity in the $\{111\}$ orientations.

Obviously an oriented growth mechanism according to a $84^\circ\langle 110 \rangle$ orientation relationship alone can not explain the texture formation during recrystallization at all. However, its introduction as minor component into the local growth law improves the results strongly: In comparison to the results obtained with a pure $27^\circ\langle 110 \rangle$ growth selection mechanism an addition of the $84^\circ\langle 110 \rangle$ orientation relationship leads to a remarkable density decrease in the orientations $\{\phi_1, 45^\circ, 0^\circ\}$. Instead the orientation density along the γ fibre is enhanced (Figure 7).

The best correspondence to experimental recrystallization textures with this particular starting deformation texture were obtained with a fraction of the $84^\circ\langle 110 \rangle$ orientation relationship of about 10 to 15% and spread values between 12° and 16° . A higher portion of the $84^\circ\langle 110 \rangle$ orientation relationship in the local growth law leads to an increase of the orientation density in the orientation range $\{001\}\langle 110 \rangle$ to $\{112\}\langle 110 \rangle$ on costs of the $\{111\}\langle 112 \rangle$ orientation. Thus the deviation between calculated and experimental results increases again.

Figure 7 Sections $\varphi_1 = 0^\circ, 45^\circ$ and 90° of the mean growth rate $\bar{W}(g)^3$ calculated with a starting deformation texture consisting of 30% $\{112\}\langle 110 \rangle$, $\{111\}\langle 110 \rangle$, and $\{111\}\langle 112 \rangle$, respectively and 10% of the $\{001\}\langle 110 \rangle$ component. The local growth law was combined of the two orientation relationships $\Delta g_{o,1} = 27^\circ\langle 110 \rangle$ and $\Delta g_{o,2} = 84^\circ\langle 110 \rangle$ with varying fractions ν_1 and ν_2 ($\nu_2 = 1 - \nu_1$). These calculations were carried out with a spread value $\omega_c = 16^\circ$.

3c. Simulations with a fibre type deformation texture DT1

In the modeled multi peak deformation texture the different components are clearly distinguishable. Since real deformation textures consist of a continuous spectrum of orientations (Figure 2), such a fibre type deformation texture was simulated by combining more Gaussian distributions to one deformation texture. The orientation distance between the maxima of the 17 Gaussian distributions superimposed in our model deformation texture DT1 was about 5° in the Euler Space.

The deformation texture DT1 was simulated as superposition of $\langle 110 \rangle$ orientations (α fibre, volume fraction 60%) and $\{111\}$ orientations ($\gamma 1$ fibre, volume fraction 40%). The α fibre has its skeleton line in the orientations $\{0^\circ, 0^\circ-45^\circ, 45^\circ\}$ with gaussian spreads varying from 18° in the orientation $\{0^\circ, 0^\circ, 45^\circ\}$ to 12° in the orientation $\{0^\circ, 45^\circ, 45^\circ\}$. The skeleton line of the $\gamma 1$ fibre was chosen $\{60^\circ-90^\circ, 55^\circ, 45^\circ\}$, the spread of each gaussian peak was 12° . The volume fractions of the different peaks in this fibre component were chosen in such a way, that the orientation density in the orientation $\{90^\circ, 55^\circ, 45^\circ\}$ is twice as high as in the orientation $\{60^\circ, 55^\circ, 45^\circ\}$. The resulting deformation texture has its maximum in the orientation $\{0^\circ, 40^\circ, 45^\circ\} = \{335\}\langle 110 \rangle$, where α and $\gamma 1$ fibre overlap, the intensity in the orientation $\{111\}\langle 112 \rangle$ has nearly the same value (Figure 8).

The calculations with this deformation texture and the same parameter combinations in the local growth law as in chapter 3b confirmed, that the same growth components were observed with different intensities now. The results with a maximum of the local

growth rate corresponding to a pure $27^\circ\langle 110 \rangle$ orientation relationship show a strong dependence on the spread ω_G (Figure 9a). With a spread value up to 12° we get a high intensity along the partial α fibre. Compared with the results obtained with the multi component deformation texture (Figure 6a) especially the orientation $\{100\}\langle 011 \rangle$ is much stronger now. This effect is caused by the high volume fraction of the α fibre components in the deformation texture DT1, which have a high growth rate in the orientation range $\{0^\circ, 0^\circ\text{--}30^\circ, 45^\circ\}$ (Figure 3, Table 1).

Apart from that the intensity of the growth component $\{111\}\langle 112 \rangle$ is rather weak now, what can be explained by a low volume fraction of the orientation $\{111\}\langle 110 \rangle$

Figure 8 Simulation of the deformation texture DT1:

- a) the $\langle 110 \rangle$ fibre components (α),
- b) the $\{111\}$ fibre components (γ_1),
- c) the ODF of the simulated deformation texture DT1.

Figure 9 Sections $\varphi_1 = 0^\circ, 45^\circ$ and 90° of the mean growth rate $\bar{W}(g)^3$ obtained with the starting deformation texture DT1 (figure 8c) and different local growth laws:

a) $\Delta g_{e,1} = 27^\circ\langle 110 \rangle$, ω_G .

b) $\Delta g_{e,1} = 27^\circ\langle 110 \rangle$, $v_1 = 0.85$, $\omega_{G,1} = \omega_G$
 $\Delta g_{e,2} = 84^\circ\langle 110 \rangle$, $v_2 = 0.15$, $\omega_{G,2} = \omega_G$.

in the simulated deformation texture. With high spread values ($\geq 16^\circ$) again a high growth rate in the orientation range $\{25^\circ\text{--}65^\circ, 45^\circ, 0^\circ\}$ occurs, which dominates the growth rate distribution.

Figure 9b shows the results obtained with a local growth rate combined of 85% of the $27^\circ\langle 110 \rangle$ and 15% of the $84^\circ\langle 110 \rangle$ orientation relationship. Again the addition of the orientation relationship $84^\circ\langle 110 \rangle$ reduces the growth rate in the orientations $\{\phi, 45^\circ, 0^\circ\}$, while simultaneously the intensities along the γ and the α fibre are increased. The best agreement with experimental results was obtained with a spread value of about 16° , but still there are some strong deviations.

Considering this particular, simulated deformation texture DT1, none of the parameter combinations leads to a sufficient correspondence with experimental recrystallization textures. Since similar results were obtained when experimental deformation textures were introduced as starting deformation texture (Köhler and Bunge 1994), we have to conclude that the texture formation can not be explained by an oriented growth mechanism alone, at least not with this model and the local growth laws used in our calculations.

Very often the texture formation was explained by oriented nucleation in matrix orientation (compare chapter 2). In our model calculations this mechanism can be considered by introducing a nuclei distribution, which simulates a high nucleation probability in the maxima of the deformation texture, into equation 1. However, an improvement of the results can not be expected in that way, because the strongest distinction between experimental results and simulated ones is the high orientation density along the α fibre. The orientations $\{001\}\langle 110 \rangle$ to $\{112\}\langle 110 \rangle$ would be further emphasized by a high nucleation probability in these orientations. Thus results obtained with the assumption of preferred nucleation in matrix orientation are not shown in detail.

4. THE MATHEMATICAL MODEL FOR INHOMOGENEOUS RECRYSTALLIZATION

The results presented in chapter 3 were obtained by calculations based on a model which presupposes a very homogenous recrystallization process. Numerous investigations of the recrystallization in α iron defeated this assumption. Kern and Bunge (1984) observed a very inhomogeneous microstructure in cold rolled α iron. They found that in the deformed state grains with different orientations showed a different etching behaviour. Very "smooth" regions without distinct structure included the orientations $\{001\}\langle 110 \rangle$ to $\{112\}\langle 110 \rangle$ and $\{001\}\langle 110 \rangle$ to $\{001\}\langle 100 \rangle$. Inside one region different orientations could be observed with a continuous orientation transition. Besides that, "rough" regions with traces of inhomogeneous deformation could be observed. These contained the $\{111\}$ orientations. Furthermore small "splintered" regions existed. The orientation of these regions could not be determined because of their very high lattice distortion. Influenced by this deformed microstructure the recrystallization behaviour of the sheet was also very inhomogeneous. Recrystallization started in the splintered regions with formation and growth of nuclei. A little later the same classical recrystallization process was observed in the rough regions. Up to that point the smooth regions showed only recovery. These regions were consumed by already recrystallized neighbouring grains in a later stage of the recrystallization process without any classical nucleation processes (Kern, Grewen, Bunge 1984).

Similar observations were made by Hu (1957) in cross rolled polycrystalline iron and by v. Schlippenbach and Lücke (1986) and Plutka and Hougardy (1991) in cold rolled deep drawing steels.

These observations were introduced into our recrystallization model as follows: according to equations 1 and 2 the recrystallization texture $f^R(g)$ resulting from the deformation texture $f^D(g)$ can be calculated by a “recrystallization operator” R , which is a function of the nucleation mechanism N , the stored energy P and the grain boundary mobility m :

$$f^R(g) = R(N, P, m) \otimes f^D(g) \quad (6)$$

In the extended model we assume the deformed structure to consist of different regions:

$$f^D(g) = \sum_i v_i \cdot f_i^D(g) \quad ; \quad \sum_i v_i = 1 \quad (7)$$

For each region the recrystallization texture can be calculated with its own recrystallization operator R . The final result is gained by an addition of the results obtained for the individual regions taking their volume fraction into account:

$$f^R(g) = \sum_i v_i \cdot R(N_i, P_i, m_i) \otimes f_i^D(g) \quad (8)$$

Since we have no knowledge about the texture of the splintered regions we consider only two regions with a different microstructure, a smooth and a rough one (if we assume that the splintered regions have a random orientation distribution, even orientation dependent processes will lead to a random recrystallization texture in these regions, anyway).

Figure 10 shall visualize the extended model of recrystallization texture formation. In the deformed state (Figure 10a) we assign the different regions considering their microstructure to the two texture parts “rough” (R) and “smooth” (S). At first, nucleation occurs only in the rough regions (Figure 10b), whereby we assume the nucleation sites to be statistically distributed in these grains. These nuclei undergo a growth selection process until the rough regions are consumed. At that time we have a very inhomogeneous microstructure: fully recrystallized regions exist between others which are still in the deformed (or recovered) state (Figure 10c). These deformed regions will be consumed by the already recrystallized grains after very long annealing times (Figure 10d). When we assume that this process undergoes a growth selection process as well, not all grains adjacent to the smooth regions are able to grow with the same growth rate into these grains, so that in the final state a rather inhomogeneous grain size distribution may occur.

As in the calculations presented in chapter 3 we do not take the orientation dependence of the stored energy into account:

$$P_1(g) = P_2(g) = 1 \quad (9)$$

In the rough regions we supposed a classical recrystallization process. The nuclei distribution can either be assumed to be random or we simulated a nuclei distribution function with an enhanced nucleation probability in the main components of the deformation texture according to equation 4.

The local growth law $m_1(\Delta g)$ was combined of the two different orientation relationships (equation 5).

Figure 10 Extended Model: the different stages of the recrystallization process:

- a) the deformed state consisting of "rough" regions R and "smooth" regions S,
- b) early recrystallization stage: beginning nucleation in the rough regions, no visible changes in the smooth regions,
- c) impingement of the growing grains in the rough regions, the smooth regions are still in the deformed (or recovered) state,
- d) very late recrystallization stage: consumption of the smooth regions by recrystallized grains of the rough regions.

With these parameter combinations we calculated the recrystallization texture of the rough regions according to equation 1 which then serve as nuclei distribution for the smooth regions:

$$f_2^N(g) = f_1^R(g) \quad (10)$$

The growth selection mechanism was assumed to be the same as in the rough regions:

$$m_2(\Delta g) = m_1(\Delta g) \quad (11)$$

The final results were obtained by superposing the results according to equation 8.

5. SIMULATIONS WITH THE EXTENDED MODEL

5a. Simulations with the deformation texture DT1

Considering the simulated deformation texture DT1 (Figure 8) the rough regions correspond to the $\{111\}$ fibre (γ_1), the smooth ones to the $\langle 110 \rangle$ fibre (α). At first the average growth rate functions $\bar{W}(g)$ were calculated with these two deformation textures, whereby the parameters in the local growth law were varied in the same limits as in chapter 3.

Figure 11 shows the mean growth rate distribution calculated with the α -fibre as starting deformation texture and a local growth rate law combined of 85% of the $27^\circ\langle 110 \rangle$ and 15% of the $84^\circ\langle 110 \rangle$ orientation relationship with a spread $\omega_G = 16^\circ$. The results for this starting deformation texture were rather insensitive against the variations of the local growth rate parameters. For all parameter combinations the maximum growth rate is found in the orientation $\{001\}\langle 110 \rangle$, only intensity and spread of this component depend on the local growth rate parameters.

In comparison with these results the mean growth rate distributions obtained with the γ_1 -fibre as deformation texture show a stronger dependence on the parameters in the local growth law. Figure 12a shows the results gained with a maximum of the local growth rate corresponding to a $27^\circ\langle 110 \rangle$ orientation relationship. For all spread values an enhanced growth rate in the orientations of the $\{111\}$ fibre can be observed, but with increasing spread values the orientation density in $\{\phi_1, 45^\circ, 0^\circ\}$ orientations increases.

The introduction of the $84^\circ\langle 110 \rangle$ orientation relationship into the local growth law had the effect, that the growth components in the orientations $\{\phi_1, 45^\circ, 0^\circ\}$, including the Goss orientation, were diminished essentially, whereas the γ -fibre was strengthened. As example the results gained with a fraction of 15% of the $84^\circ\langle 110 \rangle$ orientation relationship are presented in Figure 12b. In principle the results obtained with $\omega_G = 16^\circ$ are already very similar to experimental recrystallization textures. The growth rate function shows a good development of the $\{111\}$ fibre, yet the maximum is not in the orientation $\{111\}\langle 112 \rangle$.

The final recrystallization texture is gained by a superposition of the results obtained with the two texture parts. The results presented in Figure 13 were all obtained with the same local growth law, combined of 85% of the $27^\circ\langle 110 \rangle$ orientation relationship and 15% of the $84^\circ\langle 110 \rangle$ orientation relationship, with a spread value $\omega_G = 16^\circ$, respectively. With this local growth law generally the best results were obtained. Figure 13a shows once more the results obtained with the "homogeneous" model.

Figure 11 Sections $\phi_1 = 0^\circ, 45^\circ$ and 90° of the mean growth rate $\bar{W}(g)^3$ calculated with the α fibre (figure 8a), corresponding to the "smooth" regions of the deformation texture DT1, and a local growth law with $\Delta g_{\alpha,1} = 27^\circ\langle 110 \rangle$, $v_1 = 0.85$, $\Delta g_{\alpha,2} = 84^\circ\langle 110 \rangle$, $v_2 = 0.15$, $\omega_G = 16^\circ$.

As a first step we superimposed the results gained with the two partial textures (Figures 11 and 12b). In this way we assume that in both regions, the smooth and the rough one, the texture formation is determined by random nucleation and a selective growth process, but that none of the moving grain boundaries crosses the boundary between the two different microstructure types. The results gained with these assumptions and the starting deformation texture DT1 are shown in Figure 13b. The resulting ODF is dominated by the rotated cube component $\{001\}\langle 110 \rangle$, which is much too strong compared with experimental results. This is caused by the high growth rate of this component into the smooth part of our deformation texture. Thus we can say, that this variant of the model leads to an even more pronounced deviation from experimental results.

In order to adjust the model to the experimental observations we took the oriented nucleation mechanism in the smooth regions into account. For this purpose we introduced the results $\bar{W}(g)^3$ of the rough part (Figure 12b) as distribution of nuclei into the calculation of the recrystallization texture in the smooth part. Again the recrystallization texture is obtained by the superposition of the results of smooth and rough region (Figure 13c). Compared with the calculations based on the “homogeneous” model (Figure 13a) a strong improvement can be detected. Especially the high intensity of the rotated cube component $\{001\}\langle 110 \rangle$ is diminished, because the recrystallization process in the rough regions does not produce nuclei in this orientation which could grow into the smooth region. The $\{111\}$ fibre is well developed, with a maximum in the orientation $\{111\}\langle 110 \rangle$.

Figure 12 Sections $\varphi_1 = 0^\circ, 45^\circ$ and 90° of the mean growth rate $\bar{W}(g)^3$ calculated with the γ fibre (figure 8b), corresponding to the “rough” regions of the deformation texture DT1, and different growth laws:

a) $\Delta g_o = 27^\circ\langle 110 \rangle$, ω_G .

b) $\Delta g_{o,1} = 27^\circ\langle 110 \rangle$, $v_1 = 0.85$, $\omega_{G,1} = \omega_G$
 $\Delta g_{o,2} = 84^\circ\langle 110 \rangle$, $v_2 = 0.15$, $\omega_{G,2} = \omega_G$.

Figure 13 Sections $\varphi_1 = 0^\circ, 45^\circ$ and 90° of the recrystallization textures calculated according to different model variants with the starting deformation texture DT1. The parameters in the local growth law were: $\Delta g_{a,1} = 27^\circ\langle 110 \rangle$, $v_1 = 0.85$, $\Delta g_{a,2} = 84^\circ\langle 110 \rangle$, $v_2 = 0.15$, $\omega_G = 16^\circ$.

- a) homogeneous compromise model (compare figure 9b)
- b) extended model: the recrystallization takes place independently in the rough and the smooth regions, random nucleation in both regions.
- c) the grains recrystallized in the rough regions are introduced as nuclei distribution into the calculation of the recrystallization texture in the smooth regions. In the rough regions there is a random nucleation process.
- d) the grains recrystallized in the rough regions are introduced as nuclei distribution into the calculation of the recrystallization texture in the smooth regions. In the rough regions preferred nucleation in matrix orientation according to equation 12 is assumed.

A further strengthening of the $\{111\}$ fibre in the simulated recrystallization texture is gained by introducing an oriented nucleation process in matrix orientation in the rough regions, so that nuclei with $\{111\}$ orientations are more frequent than “random” nuclei:

$$f^N(g) = 0.9 + 0.1 \cdot f^D(g) \quad (12)$$

In this way the maximum intensity of the nuclei distribution is about $2.5R$, the minimum $0.9R$. So, oriented nucleation and oriented growth should have approximately the same influence. With this nuclei distribution and the extended model (equation 8) we get the results shown in Figure 13d. Compared with Figure 13c the intensity of the $\{111\}$ orientations is strengthened. Now the intensity of the orientation $\{111\}\langle 112 \rangle$ is the maximum of the simulated ODF because there we prescribed a very high nucleation probability.

Of course we can not expect a perfect coincidence between experimental results and simulated ones, because we make a lot of simplifying assumptions in the mathematical model (chapter 2 and 4). Considering the results gained with the modeled deformation texture DT1 and the different variants of the model we can conclude, that the extended model improves the results essentially. Figure 14 shows the orientation density of the

Figure 14 Orientation density of the experimental recrystallization texture (figure 2b) along the skeleton line of the $\{111\}$ -fibre, compared with the results calculated with the extended model (figure 13c and d) as function of the Euler-Angle φ_1 . The Euler-Angles Φ and φ_2 have the constant values $\Phi = 55^\circ$ and $\varphi_2 = 45^\circ$.

experimental recrystallization texture (Figure 2b) along the skeleton line of the $\{111\}$ -fibre, compared with the results calculated with the extended model (Figures 13c and d). Certainly the best correspondence to experimental recrystallization textures is obtained with the assumption of oriented nucleation in the rough regions. Oriented nucleation in matrix orientation can not explain the characteristic increase of the orientation $\{111\}\langle 112 \rangle$ during the recrystallization process, because this component usually is rather weak in the deformation textures. Furthermore, the occurrence of a strong Goss component in experimental recrystallization textures can not be interpreted by this mechanism, since this component is usually very weak in experimental rolling textures.

5b. Simulations with modified deformation textures

Since the exact composition of deformation textures in bcc metals depends strongly on the material and the process parameters, we investigated the influence of small variations of the deformation texture on the calculated results. We simulated other deformation textures with some slight deviations from DT1. All of them have the same α fibre (Figure 8a). The γ fibres of the four deformation textures were distinguished by the exact orientation of the skeleton line and the volume fractions of the gaussian peaks in the fibre (Figure 15a). The volume fractions of the two fibres are the same as in the deformation texture DT1, i.e. 60% (α) and 40% (γ), respectively. In this way we got three further deformation textures (Figure 15b):

- DT2: orientations and spread of the Gaussian distributions in the fibre γ_2 are the same as in γ_1 , but now the volume fraction of the orientation $\{60^\circ, 55^\circ, 45^\circ\}$ was twice as high as that of the $\{90^\circ, 55^\circ, 45^\circ\}$ component. As a consequence

Figure 15 Sections $\varphi_1 = 0^\circ, 45^\circ$ and 90° of the different $\{111\}$ fibre components and the simulated deformation textures combined of the α fibre (figure 8a) and the different $\{111\}$ fibre components, a) the $\{111\}$ fibre components γ_1 to γ_4 , b) the deformation textures DT1 to DT4.

the maximum in the resulting deformation texture is shifted to the orientation $\{0^\circ, 50^\circ, 45^\circ\} = \{556\}\langle 110 \rangle$. The orientation density in this orientation is much higher than in the orientation $\{111\}\langle 112 \rangle$. This is in good agreement with experimental deformation textures (Figure 2a).

- DT3: the skeleton line of the fibre γ_3 was chosen in such a way that it runs through the orientation $\{11,11,8\}\langle 4,4,11 \rangle$ instead of $\{111\}\langle 112 \rangle$, i.e. $\{60^\circ-90^\circ, 55^\circ-63^\circ, 45^\circ\}$, the spread of each Gaussian peak was 12° . Comparable to the fibre γ_1 the volume fraction of the different peaks was chosen in such a way, that the orientation density in the orientation $\{90^\circ, 63^\circ, 45^\circ\}$ was twice as high as in the orientation $\{60^\circ, 55^\circ, 45^\circ\}$. The resulting deformation texture had its maximum in the orientation $\{0^\circ, 40^\circ, 45^\circ\} = \{335\}\langle 110 \rangle$, as DT1. The intensity in the orientation $\{11,11,8\}\langle 4,4,11 \rangle$ had nearly the same value.
- DT4: orientations and spread of the Gaussian distributions in the fibre γ_4 are the same as in γ_3 , but now the intensity in the orientation $\{60^\circ, 55^\circ, 45^\circ\}$ was twice as high as in the orientation $\{90^\circ, 63^\circ, 45^\circ\}$. Like in DT2 the maximum in the resulting deformation texture is in the orientation $\{0^\circ, 50^\circ, 45^\circ\} = \{556\}\langle 110 \rangle$. The orientation density there is much higher than in the orientation $\{11,11,8\}\langle 4, 4,11 \rangle$.

With these deformation textures the same calculations as with DT1 were carried out. With the "homogeneous" model and the oriented growth mechanism the differences between the results of all four deformation textures were rather small. Therefore these results are not shown in detail.

The mean growth rate distributions obtained with the four different γ fibres show some characteristic distinctions. The results with a pure $27^\circ\langle 110 \rangle$ orientation relationship and a spread value of 16° are shown in Figure 16a. A high volume fraction of the

Figure 16 Sections $\phi_1 = 0^\circ, 45^\circ$ and 90° of the recrystallization texture obtained with the four different γ fibres γ_1 to γ_4 of the deformation textures DT1 to DT4 and different local growth laws:

a) $\Delta g_o = 27^\circ\langle 110 \rangle$, $\omega_g = 16^\circ$.

b) $\Delta g_{o,1} = 27^\circ\langle 110 \rangle$, $v_1 = 0.85$, $\omega_{g,1} = 16^\circ$
 $\Delta g_{o,2} = 84^\circ\langle 110 \rangle$, $v_2 = 0.15$, $\omega_{g,2} = 16^\circ$.

orientation $\{111\}\langle 110 \rangle$ in the deformation texture (γ_2 and γ_4) leads to an increase of the average growth rate $\bar{W}(g)$ in the orientation $\{111\}\langle 112 \rangle$. A shift of the orientation $\{111\}\langle 112 \rangle$ towards the orientation $\{11, 11, 8\}\langle 4, 4, 11 \rangle$ (as in γ_3 and γ_4) causes an increase of the growth rate in the Goss orientation $\{110\}\langle 001 \rangle$ at the expense of the growth rate along the $\{111\}$ fibre. This effect is particularly strong with the deformation texture γ_3 , because there the volume fraction of the orientation $\{11, 11, 8\}\langle 4, 4, 11 \rangle$ is especially high.

The results obtained with the four different γ -fibres and the local growth law considering both orientation relationships are shown in Figure 16b. As seen before, the introduction of the $84^\circ\langle 110 \rangle$ orientation relationship into the local growth law causes a decrease in the $\{\phi_1, 45^\circ, 0^\circ\}$ orientations, including the Goss component. Instead the $\{111\}$ fibre becomes stronger. Only the starting deformation texture γ_3 with a relative high volume fraction of the $\{11, 11, 8\}\langle 4, 4, 11 \rangle$ component still has an enhanced growth rate in the Goss orientation.

On account of these results we expect a remarkable influence of the composition of the deformation texture on the simulated recrystallization texture. Again the simulated results obtained with the rough regions (corresponding to the different γ fibres) and a combined growth law (Figure 16b) are introduced as nuclei distribution into the calculations of the recrystallization texture in the smooth region. Then the textures of both regions are added according to equation 8. The results presented in Figure 17 show a good development of the $\{111\}$ fibre for all four starting deformation textures. Obviously a high growth rate in the orientation $\{111\}\langle 112 \rangle$ as in DT2 and DT4 is connected with a strong deformation texture component in the orientation $\{111\}\langle 110 \rangle$, which is in good agreement with experimental observations. A shift of the skeleton

Figure 17 Sections $\varphi_1 = 0^\circ, 45^\circ$ and 90° of the recrystallization textures calculated according to extended recrystallization model obtained with the four different starting deformation textures DT1 to DT4 gained with the following parameters in the local growth law: $\Delta g_{o,1} = 27^\circ\langle 110 \rangle$, $v_1 = 0.85$, $\Delta g_{o,2} = 84^\circ\langle 110 \rangle$, $v_2 = 0.15$ and $\omega_G = 16^\circ$. The results from the rough regions, where random nucleation was assumed, were introduced as nuclei distributions of the smooth regions.

line of the γ fibre from the orientation $\{111\}\langle 112 \rangle$ toward $\{11,11,8\}\langle 4,4,11 \rangle$ as in DT3 and DT4 leads to a slight modification in the calculated results. Especially with the starting deformation texture DT3, which has a rather high volume fraction of the $\{11,11,8\}\langle 4,4,11 \rangle$ orientation, a broad spread to the Goss orientation can be observed. According to our investigations about the effect of the fraction of the $84^\circ\langle 110 \rangle$ orientation relationship in the local growth law (Figure 16) we can expect an even higher intensity of the Goss orientation with a lower portion of this orientation relationship in the growth law.

The same calculations with the assumption of oriented nucleation in the rough regions according to equation 12 produced the results in Figure 18. Now the $\{111\}$ fibres are further emphasized, because only nuclei in these orientations are available. The starting

Figure 18 Sections $\varphi_1 = 0^\circ, 45^\circ$ and 90° of the recrystallization textures calculated according to extended recrystallization model obtained with the four different starting deformation textures DT1 to DT4 gained with the following parameters in the local growth law: $\Delta g_{o,1} = 27^\circ\langle 110 \rangle$, $v_1 = 0.85$, $\Delta g_{o,2} = 84^\circ\langle 110 \rangle$, $v_2 = 0.15$ and $\omega_G = 16^\circ$. The results from the rough regions, where preferred nucleation in matrix orientation according to equation 12 was assumed were introduced as nuclei distributions of the smooth regions.

deformation textures DT2 and DT4 with a strong $\{111\}\langle 110 \rangle$ deformation component, cause a maximum recrystallization texture component in the same orientation. Against that, DT1 and DT3 cause a maximum $\{111\}\langle 112 \rangle$ recrystallization texture component, because the high nucleation probability in this orientation dominates the texture formation.

These results are contradictory to experimental observations (Figure 2), since usually a strong $\{111\}\langle 112 \rangle$ recrystallization texture emerges from a deformation texture with a high orientation intensity in the orientation $\{111\}\langle 110 \rangle$.

The development of the Goss component from the deformation texture DT3 was suppressed, because there are no nuclei with this orientation available.

6. DISCUSSION OF THE RESULTS

In this paper we present calculations of the primary recrystallization texture in bcc metals. Starting from simulated deformation textures we tried to reproduce the recrystallization texture formation by adjusting the mathematical model to experimental observations. The parameters in the different calculations were varied in sensible limits to optimize the results. Although no complete correspondence between experiment and simulation could be obtained, it is possible to explain some important features of the texture formation process by these model calculations. Obviously it was indispensable to take the inhomogeneity of the deformed microstructure and the orientation dependent recrystallization behaviour connected with this peculiarity into account.

The calculations of the mean linear growth rate were carried out on the basis of growth selection experiments by Ibe and Lücke (1966). In these investigations it was found, that grains with two particular misorientations to the matrix grew faster than other grains. Thus we considered at first local growth laws with a maximum at the misorientations 20° – $35^\circ\langle 110 \rangle$. The best agreement between simulation and experimental observations were obtained with a rotation angle between 20° and 27° . The results gained with a $35^\circ\langle 110 \rangle$ growth law had no similarity to experimental results at all. The introduction of the second observed orientation relationship $84^\circ\langle 110 \rangle$ in the local growth law improved the results essentially. With the extended model and a “combined” growth law the most important feature of the recrystallization texture development could be explained.

The determination of the grain size distributions of recrystallized grains with different orientation in a low carbon steel by Plutka and Hougardy (1991) showed that $\{111\}$ oriented grains were in general larger than $\langle 110 \rangle$ oriented grains. In a late recrystallization stage grains of the orientation $\{111\}\langle 112 \rangle$ gained a further size advantage over other $\{111\}$ oriented grains. The results obtained by our model calculations, based on the extended model and the oriented growth mechanism, agreed well with these observations.

All results shown in this paper were based on the “compromise” model, which assumes that the distance between nuclei is large compared with regions of equal orientation, so that every nucleus has to grow into all texture components. Another model assumes the distance of nuclei to be small compared with the grain size, so that each nucleus grows only into a single texture component (“superposition” model). Investigations of Inokuti and Doherty (1977) showed that in 40% cold rolled iron in the deformed grains orientation differences up to 30° were usual. Thus we can conclude that this effect is even stronger with higher deformation degrees. When we compare the size of regions with a constant orientation in the deformed state and the grain size after recrystallization

we can assume that the grains must have grown into regions with different orientations, at least in a later stage of recrystallization. Thus the superposition model can only be valid for the very early stage of the recrystallization process.

Considering the two basic mechanisms, oriented growth and oriented nucleation, it is difficult to decide which mechanism is the dominant one in the texture formation process. As mentioned above it is essential to consider the inhomogeneity of the whole process. Then we can explain the decrease of the $\langle 110 \rangle$ components by a lack of nuclei in these orientations (chapter 5). With oriented nucleation in matrix orientation alone the recrystallization texture corresponds to the ODF of the rough regions, that are the γ fibres in the simulated deformation textures. Of course we get strong $\{111\}$ fibres with this mechanism, but we can not explain the development of the $\{111\}\langle 112 \rangle$ recrystallization texture component from a high orientation density of the $\{111\}\langle 110 \rangle$ orientation in the deformation texture. Furthermore the occurrence of the Goss component is not interpretable. These two important features are reproduced much better with the oriented growth mechanism. Thus we can conclude that the mechanism of oriented nucleation in matrix orientation is not the essential one in the texture formation process. Probably we have a certain superposition of both mechanisms.

Investigations by Böttcher *et al.*, showed that the nucleation process leads to nuclei in matrix orientation as well as randomly oriented nuclei. It is difficult to judge whether there are different nucleation mechanisms taking place, since at this deformation degree the rolling texture shows usually a broad spread of orientations. However it is imaginable that regions with a very high lattice distortion show another nucleation behaviour than regions with a less distorted microstructure.

These observations were taken into account by combining random nucleation with oriented nucleation in matrix orientation (equation 4). With this equation we have the possibility to weight the influence of random and oriented nucleation, to a certain extend by varying the intensity of the maximum in the nuclei distribution function. In the calculations presented in chapter 4 we chose the "random part" in such a way, that oriented nucleation and oriented growth had approximately the same influence (equation 12). However, to optimize the results, we would have to use different distributions of nuclei for each parameter combination in the local growth law.

The development of the Goss component during recrystallization occurs especially in coarse grained materials at deformation degrees of about 50 to 70% (v. Schlippenbach, Lücke 1986). Single crystal experiments by Dunn (1953 and 1954) showed that this component may originate from a $\{111\}\langle 112 \rangle$ deformation texture component. Calculations with a $\{111\}\langle 112 \rangle$ deformation texture (chapter 3a) with the $27^\circ\langle 110 \rangle$ growth selection confirmed this observation. It was shown that the intensity of the Goss orientation in the simulated recrystallization texture depends strongly on the exact position of the maximum in the starting deformation texture. Unfortunately such small modifications of the deformation texture have never been investigated systematically in connection with the formation of the Goss component. The dependence of the intensity in the Goss orientation on the deformation degree may be caused by a broad spread of the texture components at smaller deformations. Furthermore the orientation density in the orientation $\{111\}\langle 112 \rangle$ is relatively strong at these deformation degrees, whereas the orientation $\{111\}\langle 110 \rangle$, which serves as matrix for the recrystallization texture component $\{111\}\langle 112 \rangle$, increases not before further rolling. Another point is, that when we take oriented nucleation in matrix orientation into account, the volume fraction of randomly oriented nuclei should be higher when the deformation texture is not yet as strong. Thus a connection between the exact composition of the deformation texture

and the strength of the two recrystallization texture components $\{111\}\langle 112 \rangle$ and $\{011\}\langle 100 \rangle$ can be established.

The calculations presented in this paper are obtained with simulated deformation textures as starting textures. Thus we must not expect a perfect correspondence between experimental and simulated results. The aim of this work was to show the systematic influence of the input parameters on the results by introducing well defined model distributions (chapter 2). In this way we can already interpret some important features of the texture development. This has to be confirmed by systematic experiments.

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