ON GENERALIZED NONLINEAR VARIATIONAL INEQUALITY PROBLEMS

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The following nonlinear variational inequality problem, for a nonempty closed convex subset K of a real Hilbert space H , is considered. [This generalizes an earlier work by the author accepted for publication to the *Applied Mathematics Letters*.]

Find x in H , w in $S(x)$, and z in $T(x)$ such that $f(x)$ is in K and

$$\langle f(x) - [w + f(z)], v - f(x) \rangle \geq 0$$

for all v in K , where $S: H \rightarrow P(H)$, $T: H \rightarrow C(H)$ and $f: H \rightarrow H$ are relaxed Lipschitz from H into the power set $P(H)$, Hausdorff-Lipschitz continuous from H into a family $C(H)$ of all nonempty compact subsets of H , and strongly monotone on H , respectively. The solvability of the problem is based on the reduction of the generalized nonlinear variational inequality to an equivalent nonlinear equation and then the generation of a modified iterative algorithm

$$f(x_{n+1}) = P_K[(1-t)f(x_n) + t(w_n + f(z_n))]$$

for a constant $t > 0$, where the sequences $\{x_n\}$, $\{w_n\}$, $\{z_n\}$ and $\{f(x_n)\}$ converge (under certain hypotheses) to x in H , w in $S(x)$, z in $T(x)$, and $f(x)$ in K , respectively. Here $P_K: H \rightarrow K$ is the projection of H onto K .