FRACTAL REVIEWS IN THE NATURAL AND APPLIED SCIENCES

Edited by Miroslav M. Novak

A BOOK REVIEW

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This is a collection of 37 articles on applications of fractals. A few of the applications are DNA sequences, stochastic sandpile automata, proton exchange in proteins, fractal geometry of quantum paths, microearthquake swarms, recognition of breast tumors, and a classification of writings based on fractal behavior. The common ground of mathematics in the articles was accessible with the aid of the elementary introduction by Hans Lauwerier [1]. Some of this is paraphrased in the next few paragraphs.

A classic mathematical approach has been to describe functions in terms of continuity, modulus of continuity, and further smoothness properties such as differentiability. By the end of the nineteenth century, it was clear to mathematicians that differentiability was not a property shared by all functions. For instance, Lebesgue gave an example of a function which was continuous but nowhere differentiable.

Constructs such as the Cantor set have been a part of the common background for mathematicians for the past few generations. In 1904, Koch gave a Cantor-like construction of a curve. For the Koch's set, one removes the middle third of sides of an equilateral triangle, so that a curve with four equal sized line segments results, with total length $4/3$ s as long as the original segment. For each of the four segments, replace the middle third with two equal sized segments. If the process is repeated indefinitely, the resulting curve has a unique tangent at no point. At each step, or if the total length is estimated at a corresponding length scale, length has a ratio $4/3$ of the previous step (use of scale twice as large). The fractal dimension of the curve is taken as $D = \log 4 / \log 3 = 1.26\dots$. Clearly, the limiting curve is of infinite length. For curves in the plane, the fractal dimension satisfies $1 \leq D \leq 2$, while for differentiable curves $D = 1$. For points on a straight line, imagine points covered with line-segments of length h with the smallest possible cover being $N(h)$. Clearly as h decreases, $N(h)$ increases. Then the capacity is defined as $D = \lim_{h \rightarrow 0} \log(N(h)) - \log(1/h)$. Then for the Cantor set, $D = \log 2 - \log 3 = .6309$. A similar construct allows fractal dimensions of surfaces to be between 2 and 3. The

Euclidean dimensions 1, 2, and 3 can be considered as special cases of the fractal dimension.

Fractals are self-similar when viewed at various scales. More complicated fractal constructs (multi-fractals) result if different transformations are used at different stages of the Koch constructions. For more complicated constructions, fractal dimension may or may not be uniquely defined.

The range of applications found in the proceedings illustrates that many scientists in a large variety of fields have found the concept of fractal useful in exploring the geometries they encounter. A recurrent theme is the extent to which a measure of fractal geometry can be useful in classification and identification of phenomena. Typically, one cannot take the limit as $h \rightarrow 0$, but are restricted to some minimal h , e.g., the size of a pixel. Moreover, the fractal $N(h)$ is not typically uniform as h approaches the smallest allowable value. A common approach is to determine D by a linear regression. The article by Beaver, Quirk, and Sattler (p. 63) discusses problems with such an approach and provides a readable introduction to the problem of applying fractals to image recognition. The article on classifying tumors of oral cancer from fractal dimension analysis by Obert, Bergmann, Linemann and Brust (p. 145) uses the spectra defined by $N(h)$ to effectively recognize tumors. Kurik and Lyukshutov (p. 163) analyze dramatic differences between diseased and healthy blood by a fractal analysis. Zwiggelaar and Bull (p. 204) relate the fractal spectra and power-law dependence of distributions to the power spectra returned by a fast Fourier transform, introducing also the effects of band-limiting in work on automatically distinguishing soil, weeds, and crops. A more theoretical discussion of the relation of bandlimiting to fractal analysis is found in the paper of Munteanu, C. Şuşeanu, C. Ioana, and E. Creţu (p. 259).

The papers mentioned above are among the easier to read and reflect the desire of the reviewer to get a good feel for application to practical image recognition. Readers who want a good feel for the virtues of fractals in other applications are likely to be well-served by this collection. This reviewer is not competent to judge the quality of the applications to many of the more specialized articles on quantum mechanics, etc., but in most instances I feel the application is appropriate and that the presentation will be accessible to readers more conversant with that particular field.

Overall, I am impressed by the quality of the work found here. I feel this book should be useful as a source book in upper-level undergraduate courses or graduate courses in mathematical modeling or approximation theory. It is also useful to those who study fractals in gaining insight not only into the success of their paradigm but also the modifications required to allow a practical description of nature. Finally, it should be accessible and useful to anyone whose scientific pursuits lead him to attempt mathematical description of shapes for which tangent lines do not exist.

References

- [1] Lauwerier, H., *Fractals, Endlessly Repeated Geometrical Figures*, Trans. by Sophia Gill-Hoffstädt, Princeton University Press, Princeton, NJ 1991.

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