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QUEUEING SYSTEMS WITH RENOVATION

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We study in this paper-single channel queueing systems with renovation. Such models are often encountered in computer science applications. Explicit solutions are discussed for the queues with Poisson input flow and for queues with general input and exponential service times.

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1. Introduction

Consider a single-channel queueing system with FIFO discipline and the following modification. After each customer is serviced, all the others leave the queue with probability q. Only a customer who has been serviced leaves the system with probability $p = 1 - q$. The probability q, which we call *renovation probability*, is essential to our analysis. This probability defines many important features of our model.

This model is a generalization of the classical single-server system. Below we present a short review of previous results concerning only the queues with general distribution of the input flow. The results related to such cases as Markovian input or deterministic input are well known and are described in many books and papers (see, e.g., monographs by Asmussen [1], Takagi [12], Klimov [9, 10] and Jaiswal [6]). The case of general input flow and exponentially distributed service times was considered in [6]. It is shown that if $N < \infty$, the Laplace transform of the stationary queue length distribution can be expressed through the roots of certain equations. A similar expression can be written down in the case $N = \infty$, but there is only one root to be found. More general results, such as ergodicity and heavy traffic limit theorems, were considered by Borovkov [2, 3].

Our aim is to generalize the results concerning the "classical" model to the case of the queueing systems $M/M^R/1/N$ and $GI/M^R/1/N$ with renovation.[1]

[1]We preserve the Kendall notation for a queueing system with renovation, equipping the second letter with superscript R.

These queueing systems are of significant interest for our applications. The first publications to study such models were papers by Kreinin [11] and by Towsley and Tripathi [13]. The motivation of these models is given below.

Kreinin [11] deals with a model of a so-called "prefetch instruction buffer". The prefetch buffer provides a pipeline mechanism in the most recent processors and microprocessors for systems with a single instruction flow. This architectural element improves the performance when the linear part of a program is running by allowing the simultaneous processing of independent parts of different instructions. If the recent instruction is "branch" (or "go to"), then the processor loses the content of the prefetch instruction buffer and renovates it.

Another example of a renovation mechanism is considered by Towsley and Tripathi in [13]. They studied the probabilistic models of buddy protocols for fault-tolerant systems. This model [13] leads to priority queues with renovation (in [13] the term "flushing" was used) which does not depend on the service process.

This paper is organized as follows. In Section 2 we discuss the ergodicity conditions for general queueing systems with renovation. In Section 3 we consider a system with renovation under the simplest assumptions on the interarrival and service times. Both cases, $N < \infty$ and $N = \infty$, are considered. Section 4 contains the results for the case of generally distributed interarrival times and Markovian service. Final remarks and some directions for further studies are discussed in Section 5.

2. Ergodicity Conditions

Consider a queueing system with renovation and general interarrival and service time distributions. Assume that the customers arrive from the outside at moments $t_1, t_2, \ldots$, and form a recurrent flow with interarrival times $I_n = t_{n+1} - t_n$, $n = 0, 1, \ldots$. Thus, the sequence $\{I_n\}$ forms a family of independent, identically distributed, random variables (r.v.'s) with a common distribution function $F(x)$. Assume

$$0 < EI = \int_0^\infty x dF(x) < \infty.$$

Let S_n be the service time of the nth customer. Assume that $\{S_n\}$ are independent and identically distributed r.v.'s with $ES_1 < \infty$.

Denote by ξ_t, the queue length at time t, $(t \geq 0)$. Assume that the initial queue length is finite with probability 1.

Proposition 1: *If $0 < p < 1$, then for each initial state ξ_0 of the queue satisfying the condition $Pr\{\xi_0 < \infty\} = 1$, the queueing system with renovation has a unique stationary distribution*

$$\pi_j = \lim_{t \to \infty} Pr\{\xi_t = j\}, \quad j = 0, 1, \ldots,$$

which does not depend on the initial state of the queue length process.

Proof: Denote by $\mathcal{A}_t = \{\inf_{0 \leq s < t} \xi_s = 0\}$ the event that the queue once became empty during $[0, t]$. The process ξ_t has a renovating event $\{\xi_t = 0\}$ (see Borovkov [3]). For all finite values of ξ_0, the probability of the complementary event

$$\mathfrak{B}_t = \bar{\mathcal{A}}_t = \{ \inf_{0 \le s \le t} \xi_s > 0\}$$

tends to zero

$$P_t = Pr\{\mathfrak{B}_t\} \to 0,$$

as $t \to \infty$. Indeed, let $N(t, t+T)$ be the number of customers served without renovation of the queue. Then for each $\epsilon > 0$ and for each integer M, there exists T sufficiently large, such that for every $t > 0$, $Pr\{N(t, t+T) > M\} < \epsilon$. Thus, the probability of the event $\mathcal{A}_t$ tends to 1 as $t \to \infty$. The existence of the renovating event provides the ergodicity of the queueing system. □

This results shows that the renovation probability is a critical parameter for the stability domain. Indeed, if $q > 0$, $EI_1 < \infty$, and $ES_1 < \infty$, the system is stable. In the case $q = 0$, the system is stable if and only if the inequality $EI_1 > ES_1$ is satisfied.

3. System with Poisson Input Flow

Consider a queueing system with renovation. We assume that the system has only one queue with FIFO discipline and one input flow of customers. The input flow is assumed to be Poisson. Denote by λ the input flow intensity. Service times are mutually independent and exponentially distributed random variables with parameter μ.

After a customer is serviced, all others can leave the queue with probability q. Only a customer who receives service leaves the system with probability $p = 1 - q$. Below we consider two classes of queues with renovation, Poisson input flow, and exponential service time distribution.

3.1 $M/M^R/N$ Queueing System

Denote by N the size of the waiting room. Let ξ_t be the number of customers in the system at moment t. Then $\xi_t \le N$. Denote

$$\pi_j(t) = Pr\{\xi_t = j\}, \quad j = 0, 1, \ldots, N.$$

The stationary distribution of the queue length is given by the following assertion.

Proposition 2: *If $0 < p < 1$ and $\mu > 0$, then for each initial state of the queue satisfying the condition*

$$\pi_j = \lim_{t \to \infty} Pr\{\xi_t = j\}, \quad (j = 0, 1, \ldots, N).$$

The generating function of this distribution satisfies the equation

$$\pi(z) = \frac{\mu p(1-z)\pi_0 - q\mu z - \lambda \pi_N (1-z) z^{N+1}}{Q(z)},$$

where $Q(z) = \lambda z^2 - (\lambda + \mu)z + p\mu$, and the probabilities π_0 and π_N are given by the formulas

$$\pi_0 = 1 - \frac{z_*^N - z_*^N}{z_*^{N+1} - z_*^{N+1}},$$

$$p_N = \frac{z^* - z_*}{z^{*N+1} - z_*^{N+1}},$$

with z^* and z_* being the roots of the equation $Q(z) = 0$.

Proof: The process ξ_t is a Markov chain. It is easy to verify that for $0 < p < 1$ and $\mu > 0$ this Markov chain is ergodic. The stationary distribution $\{\pi_j\}_{j=0}^{\infty}$ of ξ_t satisfies the equations

$$\begin{aligned} (\lambda + \mu)\pi_k &= \lambda p_{k-1} + \mu p \pi_{k+1}, \quad k = 1, 2, \ldots, N-1, \\ \lambda \pi_{N-1} &= \mu \pi_N, \\ \lambda \pi_0 &= \mu \pi_1 + \mu q \sum_{j=2}^{N} \pi_j, \end{aligned} \tag{1}$$

and the normalization condition

$$\sum_{j=0}^{N} \pi_j = 1.$$

Then, from (1) after standard calculations we obtain

$$\pi(z)Q(z) = \mu p(1-z)\pi_0 - q\mu z - \lambda \pi_N (1-z) z^{N+1}.$$

The equation $Q(z) = 0$ has two roots, z_* and z^*. Both roots are real and positive.[2] The generating function $\pi(z)$ is a polynomial. The stationary probabilities π_0 and π_N must satisfy the equations

$$\mu p \pi_0 (1 - z_*) - \lambda \pi_N z_*^{N+1}(1 - z_*) = \mu q z_*, \tag{2}$$

$$\mu p \pi_0 (1 - z^*) - \lambda \pi_N z^{*N+1}(1 - z^*) = \mu q z^*. \tag{3}$$

From (2) and (3), we obtain the relations for π_0 and π_N. Proposition 2 is proved. □

Corollary 1: *If the renovation probability $q = 0$, then the stationary distribution of the queue length is given by*

$$\pi_k = \pi_0 \rho^{k-1}, \quad k = 1, \ldots, N,$$

$$\pi_0 = \frac{1-\rho}{1-\rho^N},$$

where $\rho = \lambda/\mu$. If the renovation probability $q = 1$, then the stationary distribution of the queue length is given by

$$\pi_k = \frac{\mu}{\lambda + \mu}\left(\frac{\lambda}{\lambda + \mu}\right)^k, \quad k = 0, 1, \ldots, N-1,$$

$$\pi_N = \left(\frac{\mu}{\lambda + \mu}\right)^N.$$

Proof: The first part of the corollary is well known (see e.g., [1, 8]). The second

[2]It is not difficult to see that for $p > 0$ these roots satisfy the inequalities $z^* > 1$, $0 < z_* < 1$.

part follows directly from the equations (1) for the equilibrium distribution of the queue length process.

3.2 $M^X/M^R/1/\infty$ Queueing System

Let us study now the case $N = \infty$. We consider a more general situation of bulk input. Assume that the customers arrive in groups of random size, and interarrival times are mutually independent, exponentially distributed r.v.'s with parameter λ. This arrival flow is usually called a *compound Poisson process.* Denote

$$p_k = Pr\{l = k\}, \quad k = 1, 2, \ldots,$$

where l is the size of an arrival group of customers. We denote by $p(z)$ the generating function of this distribution. Thus,

$$p(z) = \sum_{k=0}^{\infty} p_k z^k,$$

where $|z| \leq 1$. The average size of the arriving groups is assumed to be finite. Then the function $p(z)$ is regular in the unit disc $D = \{z: |z| < 1\}$ and the average size of the arrival group is equal to $El = p'(1)$.

Proposition 3: *If $0 < p < 1$, $El < \infty$, and $\mu > 0$, then for each initial state, the queueing system has a unique stationary queue length distribution. The generating function of this distribution is given by the formula*

$$\pi(z) = \frac{\mu p(1-z)\pi_0 - q\mu z}{Q_p(z)},$$

where

$$Q_p(z) = \lambda z p(z) - (\lambda + \mu)z + p\mu, \quad \pi_0 = \frac{q z_*}{p(1 - z_*)},$$

and z_* is a unique root of the equation $Q_p(z) = 0$, that satisfies the inequality $0 < z_* < 1$.

Proof: Denote by q_m $(m = 1, 2, \ldots)$ the number of customers in the system at the arrival epoch τ_m of the mth customer. Using the standard technique it is not difficult to verify that $\{q_m\}$ is a positive supermartingale. On the other hand, the queue length ξ_t at the moment t, $\tau_m < t \leq \tau_{m+1}$ satisfies the inequality $0 \leq \xi_t \leq q_m$ for $m \geq 0$. The last inequality implies that the process ξ_t has a stationary distribution $\pi_n = \lim_{t\to\infty} Pr\{\xi_t = n\}$ that satisfies the equations

$$(\lambda + \mu)\pi_k = \lambda \sum_{n=1}^{\infty} \pi_{k-n} p_n + \mu p \pi_{k+1}, \quad k = 1, 2, \ldots, \tag{4}$$

$$\lambda \pi_0 = \mu \pi_1 + \mu q \sum_{j=2}^{N} \pi_j. \tag{5}$$

Then from (4) and (5) we have

$$\pi(z) = \mu \frac{p(1-z)\pi_0 - qz}{\lambda z p(z) - (\lambda + \mu)z + p\mu}. \tag{6}$$

The generating function $\pi(z)$ is regular in the unit disc D. By Rouche's theorem, the equation

$$\lambda z p(z) - (\lambda + \mu)z + p\mu = 0$$

has only one root, $z_* \in D$. Therefore, the numerator of the fraction in (6) should be equal to 0 at $z = z_*$. Hence,

$$\pi_0 = \frac{q z_*}{p(1 - z_*)}.$$

Therefore, the generating function of the stationary distribution satisfies the relation

$$\pi(z) = \mu q \frac{z - z_*}{z_* - 1} \frac{1}{Q_p(z)}. \qquad \square$$

Corollary 2: *If the input flow of customers is ordinary Poisson* ($p(z) = z$), *then*

$$\pi(z) = \frac{1 - z^*}{z - z^*},$$

where z^* *is the root of the equation* $\lambda z^2 - (\lambda + \mu)z + p\mu = 0$, *satisfying the inequality* $z^* > 1$.

4. General Arrival Process

In this section we consider a queueing system with renovation, general distribution of the interarrival times I_n, and exponential service time distribution. To describe the behavior of the system we will use an intensity function $\lambda(x)$, and parameter μ of the service time distribution. The intensity function satisfies the relations

$$\bar{F}(x) = \exp\left(- \int_0^x \lambda(t)dt \right), \quad \lambda(x) \geq 0,$$

where $1 - F(x) = \bar{F}(x)$. The behavior of the queue length process is described by semi-Markov process.[3]

4.1 $GI/M^R/1/N$ Queue with Renovation and Lost Customers

Consider a queueing system with finite waiting room and renovation. Let the process $\zeta_t = (k_t, x_t, t > 0)$, $k_t \in \mathbb{Z}_+$, $x_t \in \mathbb{R}^+$, where k_t is the number of customers in the system ($k_t \leq N$), and x_t is the elapsed time since the last arrival at moment t. The process (k_t, x_t) is a Markov process.

Denote

$$P_n(t, x) = Pr\{k_t = n, x_t \leq x\}, \quad n = 0, 1, \ldots, N.$$

It is known (see, e.g., [6]) that there exist the densities of the probability distribution

$$p_n(t, x) = \frac{\partial P_n(t, x)}{\partial x},$$

[3]Examples of the general theory of semi-Markov processes and their application in queueing theory may be found in the books by Gnedenko and Kovalenko [5] and by Jaiswal [6]; see also the book by Çinlar [4].

and their stationary limits

$$p_n(x) = \lim_{t\to\infty} \frac{\partial P_n(t,x)}{\partial x}, \quad n = 0,1,\ldots,N, \quad x \geq 0. \tag{7}$$

The densities $p_n(x)$ do not depend on the initial state of the process ξ_t. If the stationary densities $p_n(x)$ are known, the stationary probabilities can be easily found in the form:

$$\pi_k = \int_0^\infty p_k(x)dx, \quad k = 0,1,\ldots,N. \tag{8}$$

There are two different ways to control the system when the queue buffer is saturated ($k_t = N$). The first is to lose the arriving customer, while the second is to block the input flow. We consider the case of the system with lost customers. To formulate the desired assertion we need the following notations:

Let

$$\Gamma_n = \int_0^\infty e^{-\mu x} x^n \overline{F}(x)dx, \quad \gamma_n = \int_0^\infty e^{-\mu x} x^n dF(x), \quad n = 0,1,\ldots,$$

and

$$R_m = \sum_{j=0}^{m} \frac{\mu^j}{j!}\Gamma_j.$$

Proposition 4: *The stationary densities of the queue length process in the system with lost customers are given by*

$$p_k(x) = e^{-\mu x}\overline{F}(x)\sum_{i=0}^{N-k} C_{i,k}x^i, \quad k = 1,\ldots,N, \tag{9}$$

where the coefficients $C_{i,k}$ satisfy the relations

$$C_{n+1,k} = \frac{\mu p}{n+1}C_{n,k+1}, \quad n = 0,\ldots,N-k-1,$$

$$C_{0,k} = \sum_{j=0}^{N-k+1} \gamma_j C_{j,k-1} \quad k = 2,\ldots,N-1, \tag{10}$$

$$C_{0,N} = \frac{\gamma_0}{1-\gamma_0}C_{0,N-1} + \frac{\gamma_1}{1-\gamma_0}C_{1,N-1},$$

and

$$p_0(x) = \overline{F}(x)\sum_{m=0}^{N-1} C_m \mu^{-m-1} m! \left(1 - exp(-\mu x)\sum_{k=0}^{m}\frac{(\mu x)^k}{k!}\right), \tag{11}$$

where the coefficients C_m satisfy the relations

$$C_k = \mu C_{1,k} + \mu q \sum_{l=2}^{N-k} C_{l,k}, \quad k = 1,\ldots,N-2,$$

$$C_{n-1} = \mu C_{1,N-1}, \tag{12}$$

and the normalization condition

$$\sum_{k=1}^{N} \sum_{m=0}^{N-k} C_{m,k}\Gamma_m + \sum_{m=0}^{N-1} C_m m!\mu^{-m-1}(EI - R_m) = 1. \tag{13}$$

Relations (8)-(13) *define the ergodic distribution of system* $GI/M^R/1/N$.

Proof: Denote by

$$\dot{p}_k(x) = \frac{d}{dx}p_k(x), k = 0, 1, \ldots, N,$$

the derivatives of the stationary density functions. Then the equations for the stationary densities can be written in the form

$$\begin{aligned} \dot{p}_k(x) &= -(\lambda(x) + \mu)p_k(x) + \mu p p_{k+1}(x), \quad k = 1, \ldots, N-1, \\ \dot{p}_N(x) &= -(\lambda(x) + u)p_N(x), \\ \dot{p}_0(x) &= -\lambda(x)p_0(x) + \mu p_1(x) + \mu q \sum_{j=2}^{N} p_j(x). \end{aligned} \tag{14}$$

The boundary conditions for the functions $p_k(x)$ are

$$\begin{aligned} p_0(0) &= 0, \\ p_k(0) &= \int_0^\infty p_{k-1}(x)\lambda(x)dx, \quad k = 1, \ldots, N-1, \\ p_N(0) &= \int_0^\infty (p_{N-1}(x) + p_N(x))\lambda(x)dx. \end{aligned} \tag{15}$$

Substituting (9) into (14) we obtain that, for all $x > 0$,

$$\sum_{i=0}^{N-k} iC_{i,k}x^{i-1} = \mu p \sum_{i=0}^{N-k-1} C_{i,k+1}x^i.$$

Hence, the relation

$$(i+1)C_{i+1,k} = \mu p C_{i,k+1}$$

should be satisfied for $i = 0, \ldots, N-k-1$, that proves te first equality in (10).

To prove the second equality, we consider the values of $p_k(x)$ for $x = 0$. Then the second equality in (10) follows from the first equality in (15) and (9). From (14) it follows that $p_N(x)$ has the form

$$p_N(x) = C_{0,N}\bar{F}(x)\exp(-\mu x), C_{0,N} > 0. \tag{16}$$

Then, from (15) and (16) we deduce that

$$C_{0,N} = \frac{\gamma_0}{1-\gamma_0}C_{0,N-1} + \frac{\gamma_1}{1-\gamma_0}C_{1,N-1},$$

as was to be proved.

From (9) and (11), we find

$$\mu q \sum_{j=2}^{N} p_j(x) + \mu p_1(x) = \exp(-\mu x)\bar{F}(x) \sum_{m=0}^{N-1} C_m x^m, \tag{17}$$

where the coefficients C_m satisfy (12). Then, substituting (17) into the differential equation for $p_0(x)$ we obtain (11). Using the normalization condition for the stationary probabilities π_k, $k = 0, 1, \ldots, N$ we derive (13). □

4.2 $GI/M^R/1/\infty$ Queueing System

The final form of the stationary distribution is simpler in the case of infinite queueing length. To formulate the assertion we will use the notation $\lambda = 1/(EI)$ for the intensity of the input flow.

Proposition 5: *Let the average service time and the probability of renovation satisfy the inequalities $\mu < \infty$, $q > 0$. Then, the stationary densities $p_k(x)$ are given by*

$$p_k(x) = c\alpha^k \bar{F}(x) e^{-\mu(1-p\alpha)x}, \; k = 0, 1, \ldots,$$

$$p_0(x) = c\frac{\alpha}{1-\alpha}\bar{F}(x)(1 - e^{-\beta x}),$$

where

$$c = \lambda\frac{1-\alpha}{\alpha}, \quad \beta = \mu(1 - \alpha p), \tag{18}$$

and α is a unique root of the equation

$$\alpha = \int_0^\infty e^{-\mu(1-\alpha p)x} dF(x), \quad \alpha \in (0, 1). \tag{19}$$

Proof: The stationary densities satisfy the equations

$$\frac{dp_0(x)}{dx} = -\lambda(x)p_0(x) + \mu p_1(x) + \mu q \sum_{j=2}^{\infty} p_j(x), \tag{20}$$

$$\frac{dp_k(x)}{dx} = -(\lambda(x) + \mu)p_k(x) + \mu p p_{k+1}(x), \quad k = 1, 2, \ldots, \tag{21}$$

with the boundary conditions

$$p_k(0) = \int_0^\infty \lambda(x) p_{k-1}(x) dx, \quad k = 1, 2, \ldots, \tag{22}$$

$$p_0(0) = 0. \tag{23}$$

We will seek the solution of (20) and (21) in the form

$$p_k(x) = c\alpha^k \bar{F}(x) e^{-ax}, \quad x \geq 0, \quad k = 1, 2, \ldots. \tag{24}$$

Substituting (24) into (21) after simple transformations, we obtain that $\mu - a = \mu p \alpha$, which is equivalent to $a = \beta$. Then we obtain

$$\sum_{k=2}^{\infty} p_k(x) = c\frac{\alpha^2}{1-\alpha}\bar{F}(x)e^{-\beta x}.$$

Using this formula we can rewrite (20) as follows:

$$\frac{d_0(x)}{dx} = -\lambda(x)p_0(x) + A\overline{F}(x)e^{-\beta x},$$

where

$$A = c\mu\alpha\frac{1-\alpha p}{1-\alpha}.$$

Solving this ordinary differential equation we find

$$p_0(x) = \frac{A}{\beta}(1-e^{-\beta x})\overline{F}(x). \tag{25}$$

It can easily be seen that boundary condition (23) is met.

Using expression (24) we can rewrite boundary condition (22) as

$$c\alpha^k = \int_0^\infty c\alpha^{k-1}\lambda(x)\overline{F}(x)e^{-\beta x}dx.$$

Therefore, α should satisfy the equation

$$\alpha = \int_0^\infty e^{-\beta x}dF(x),$$

which is equivalent to (19). Standard arguments concerning the convexity of the Laplace transform lead to the conclusion that equation (19) has only one root in the interval $(0,1)$.

To find c, we express the stationary probabilities π_k through this unknown constant and then, use the normalization condition $\sum_{j=0}^{\infty}\pi_k = 1$. Namely, for $k \geq 1$ we have

$$\pi_k = \int_0^\infty p_k(x)dx = c\alpha^k\int_0^\infty e^{-\beta x}\overline{F}(x)dx.$$

Integrating by parts the latter integral, we obtain

$$\int_0^\infty e^{-\beta x}\overline{F}(x)dx = \frac{c\alpha^k(1-\widetilde{f}(\beta))}{\beta}.$$

Therefore,

$$\pi_k = c\alpha^k\frac{1-\widetilde{f}(\beta)}{\beta} \quad k = 1,2,\ldots.$$

On the other hand, (25) leads to

$$\pi_0 = \frac{c\alpha}{1-\alpha}\left(\frac{1}{\lambda} - \frac{1-\widetilde{f}(\beta)}{\beta}\right).$$

From this formula and the normalization condition, we obtain the constant c. Thus, the relation (18) is proved. □

5. Final Remarks

The purpose of this paper is to present the results for queueing system with renovation in Markov case, and in the case of the general recurrent arrival flow. Using the same technique, one can study the case of Poisson arrival and generally distributed service times. The calculations should be performed in this case using the method of embedded Markov chains. In Section 4.1 we studied the $GI/M^R/1/N$ queueing system with renovation and lost customers. In the same manner one can study the queueing system with blocking input when the queue is saturated.

More complicated problems arise when the interarrival and service times have general distributions. Almost nothing is known for such systems. The problem of calculating stationary characteristics has remained open. In this case, the approach based on heavy traffic limit theorems should be fruitful. The approximation for the waiting time for the customers in heavy traffic has the form of a diffusion process in the line modified to include jumps to origin at random moments. Such processes appeared in the paper by Kella and Whitt [7], devoted to the diffusion approximations for queues with server vacations.

Another direction for generalizing the model discussed in this paper is a multiphase queueing system with Poisson arrival flows and exponentially distributed service times. This queue leads to non-homogeneous random walks that can be studied in several important cases. This problem will be considered in future publications.

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