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ABOUT THE EQUATIONS OF MOTION OF A MAGNETIC PARTICLE IN A MAGNETIC SEPARATOR

YU.S. MOSTIKA, V.I. KARMAZIN*,
V.YU. SHUTOV and L.Z. GREBENYUK

*The National Mining Academy of the Ukraine,
19 Karl Marx Prospect, 320027 Dnepropetrovsk, The Ukraine*

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Equations of motion of magnetic particles in the flow medium near a cylindrical ferromagnetic collector have been developed. A vector equation, expressing the balance of the inertia force and the vector sum of three forces: the magnetic, the gravitational and the hydrodynamic drag force of a particle was taken as the initial equation. Reduced equations obtained from the initial one by rejection of terms corresponding to the inertia or the gravitational force were also considered. Examples of numerical evaluation in which the motion trajectories calculated with the use of the initial equation were compared with the reduced equations of motion are given.

Keywords: Magnetic separation; Gravitational force; Inertial force; Hydrodynamic drag; Magnetic force; Particle trajectory

INTRODUCTION

In the number of papers [1–4] dealing with the theory of magnetic separation, in addition to the initial differential equation of movement of a magnetic particle also simplified equations, obtained from the initial one by rejecting the terms conforming the inertia force were considered.

It is assumed that such a replacement is admissible for small sizes of particles or with a specific combination of sizes of particles, their density and viscosity of the medium flow, for example, with the.use the Parker's

* Corresponding author.

criterion [2]. It is usually considered that if with some conditions the influence of the inertia force on the movement of a particle is negligibly small, then for these conditions it is possible to neglect the gravitational force. It is of interest to carry out numerical evaluation for two specific cases.

THEORETICAL MODEL

Let us write the equation of motion of a magnetic particle in the flow medium near the cylindrical collector:

$$m_{\mathrm{p}} \frac{\mathrm{d}\vec{V}_{\mathrm{p}}}{\mathrm{d}t} = \vec{F}_{\mathrm{m}} + \vec{F}_{\mathrm{d}} + \vec{F}_{\mathrm{g}}, \tag{1}$$

where m_{p} is the mass of a particle; $\vec{V}_{\mathrm{p}}$ is the velocity vector of a particle and $\vec{F}_{\mathrm{m}}$, $\vec{F}_{\mathrm{d}}$, $\vec{F}_{\mathrm{g}}$ are the vectors of magnetic, hydrodynamic and gravitational forces acting on a particle.

The gravitational force is an algebraic sum of the weight and Archimedes forces. The magnetic force is caused by joint influence of the external magnetic field H_0 and the magnetization of the cylinder that is placed in this field.

Equation (1) can be expressed as its projection onto the coordinate directions r, θ of the polar coordinate system, the center of which coincides with the center of cylinder cross section:

$$m_{\mathrm{p}} w_r = F_{\mathrm{m},r} + F_{\mathrm{d},r} + F_{\mathrm{g},r}; \tag{2}$$

$$m_{\mathrm{p}} w_\theta = F_{\mathrm{m},\theta} + F_{\mathrm{d},\theta} + F_{\mathrm{g},\theta}, \tag{3}$$

where w_r, w_θ are the projections of particle acceleration vector onto coordinate directions r, θ.

The second indices r or θ of the quantities $F_{\mathrm{m},r}, \ldots, F_{\mathrm{g},\theta}$ denote the components of corresponding vectors with directions r, θ.

Sometimes reduced equations of motion [1,2] are used in which the inertial force is neglected. Such equations can be obtained by substituting the expressions for the components of the forces into the following system of equations:

$$F_{\mathrm{m},r} + F_{\mathrm{d},r} + F_{\mathrm{g},r} = 0; \tag{4}$$

$$F_{\mathrm{m},\theta} + F_{\mathrm{d},\theta} + F_{\mathrm{g},\theta} = 0. \tag{5}$$

Usually, although not always, for the separation conditions that allow the use of Eqs. (4) and (5), it is possible to accept one additional simplification: to neglect the gravitational force in comparison with magnetic and hydrodynamic ones. Then the movement of a particle is described approximately by equations equivalent to the next system of equations

$$F_{\mathrm{m},r} + F_{\mathrm{d},r} = 0; \tag{6}$$

$$F_{\mathrm{m},\theta} + F_{\mathrm{d},\theta} = 0. \tag{7}$$

Calculations show that the criterion:

$$\frac{2\rho_{\mathrm{p}} r_{\mathrm{p}}^2}{9\eta} \ll 1,$$

that was proposed in [2], does not always guarantee the validity of the replacement of Eqs. (2) and (3) by Eqs. (4) and (5) or by Eqs. (6) and (7). A more reliable admissibility criterion of such a substitution is the condition that the inertial force F_{in} is much smaller than the magnetic force F_{m}, hydrodynamic drag F_{d} force or than the sum of one of them with the gravitational force F_{g}:

$$Z_{\mathrm{m}} \ll 1, \tag{8}$$

$$Z_{\mathrm{d}} \ll 1, \tag{9}$$

where

$$Z_{\mathrm{m}} = \frac{F_{\mathrm{in}}}{|\vec{F}_{\mathrm{m}} + \vec{F}_{\mathrm{g}}|}; \quad Z_{\mathrm{d}} = \frac{F_{\mathrm{in}}}{|\vec{F}_{\mathrm{d}} + \vec{F}_{\mathrm{g}}|}; \quad F_{\mathrm{in}} = m_{\mathrm{p}} \left| \frac{\mathrm{d}\vec{V}_{\mathrm{p}}}{\mathrm{d}t} \right|.$$

Let us express the components of the forces on the right-hand side of Eqs. (2) and (3). The magnetic force, acting on a particle, is determined by the formula [3,5]:

$$\vec{F}_{\mathrm{m}} = \mu_0(\kappa_{\mathrm{p}} - \kappa_{\mathrm{f}}) V_{\mathrm{p}} H \text{ grad } H, \tag{10}$$

where κ_{p} and κ_{f} are the volume magnetic susceptibilities of a particle and the medium, respectively, V_{p} is the volume of a particle and $H = H(r, \theta)$ is the magnetic field strength in the vicinity of a ferromagnetic collector.

We shall determine the function $H = H(r, \theta)$ assuming a single cylinder with its axis perpendicular to the vector of the external magnetic field using formulas [3,6]. Then from Eq. (10) we obtain following expressions for the $F_{m,r}$, $F_{m,\theta}$ components of magnetic force vector $\vec{F}_m$:

$$F_{m,r} = m_p Q_m \left(\frac{A}{\tilde{r}^2} + \cos 2\theta \right); \tag{11}$$

$$F_{m,\theta} = m_p Q_m \sin 2\theta, \tag{12}$$

where

$$Q_m = -\mu_0 \frac{\kappa_p - \kappa_f}{\rho_p} \frac{MH_0}{r_w} \frac{1}{\tilde{r}^3}, \tag{13}$$

where r_w is the radius of a cylindrical collector; $\tilde{r} = r/r_w$; ρ_p is the density of the particle material and $M = 2AH_0$ is the magnetization of the collector;

$$A = \frac{\mu_w - \mu_f}{\mu_w + \mu_f}, \tag{14}$$

μ_w and μ_f are the magnetic permeabilities of the cylinder and the medium ($\mu_f \approx \mu_0$), respectively.

The vector of the hydrodynamic drag force can be expressed as:

$$\vec{F}_d = -\tfrac{1}{2} C_d \rho_f |\vec{V}_{p,f}| \vec{V}_{p,f} S_m, \tag{15}$$

where C_d is the coefficient of the hydrodynamic drag $\vec{V}_{p,f} = \vec{V}_p - \vec{V}_f$; $\vec{V}_p$, $\vec{V}_f$ are the vectors of velocity of a particle and medium flow; ρ_f is the medium density and S_m is the cross sectional area of a particle.

The hydrodynamic drag coefficient C_d depends on the shape of a particle and on the Reynolds number

$$\mathrm{Re} = \frac{\rho_f |\vec{V}_{p,f}| d_p}{\eta},$$

where η is the dynamic viscosity of the carrying medium and d_p is the diameter of a particle.

For a spherical particle a number of different formulas, approximating the experimental dependence $C_d(\mathrm{Re})$ are known. A formula for

$C_d(\mathrm{Re})$ offered in [7] is used in this work. Let us assume that the shape of a particle is spherical, then $S_m = \pi d_p^2/4$.

In this case we obtain from Eq. (15) the following expression of projections of the hydrodynamic drag vector onto the coordinate directions r, θ:

$$F_{d,r} = -m_p \frac{N_d}{\tau}(V_{p,r} - V_{f,r}); \tag{16}$$

$$F_{d,\theta} = -m_p \frac{N_d}{\tau}(V_{p,\theta} - V_{f,\theta}), \tag{17}$$

where $N_d = C_d/C_{d,0}$; $C_{d,0}$ and C_d are the hydrodynamic drag coefficients based the Stokes formula and on a more general formula corresponding to the given Reynolds number Re;

$$\tau = \frac{\rho_p d_p^2}{18\eta}, \tag{18}$$

$V_{p,r}$, $V_{f,r}$ are the radial components of $\vec{V}_p$ and $\vec{V}_f$ vectors and $V_{p,\theta}$ and $V_{f,\theta}$ are the azimuthal components of these vectors.

The $F_{g,r}$ and $F_{g,\theta}$ projections of the $\vec{F}_g$ vector of the gravitational force are determined by formulas:

$$F_{g,r} = Q_g \cos(\theta - \beta); \tag{19}$$

$$F_{g,\theta} = -Q_g \sin(\theta - \beta), \tag{20}$$

where $Q_g = m_p g(1 - (\rho_f/\rho_p))$; g is the acceleration of gravity and β is the angle between x axis and the vector $\vec{g}$.

The x axis is parallel to the $\vec{H}_0$ vector and the positive direction of the axis x is opposite to the $\vec{V}_0$ vector of the medium flow velocity in the "longitudinal configuration" ($\vec{V}_0 \parallel \vec{H}_0$).

Let us express the w_r and w_θ components of the vector of particle acceleration with the time derivative of coordinates r, θ of the polar system:

$$w_r = \frac{d^2 r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2; \tag{21}$$

$$w_\theta = r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}. \tag{22}$$

Let us substitute Eqs. (21) and (22) into the left-hand sides of Eqs. (2) and (3) and the Eqs. (11)–(13), (16), (17), (19) and (20) into the right-handsides of the Eqs. (2) and (3) using a notation:

$$V_{\mathrm{m}} = \mu_0(\kappa_{\mathrm{p}} - \kappa_{\mathrm{f}})\frac{d_{\mathrm{p}}^2}{18\eta}\frac{MH_0}{r_{\mathrm{w}}}, \tag{23}$$

as introduced in [1]. Taking into account Eq. (18), we obtain:

$$\begin{aligned}\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} - r\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2 &= \frac{1}{\tau}\left[-\frac{V_{\mathrm{m}}}{\tilde{r}^3}\left[\frac{A}{\tilde{r}^2} + \cos 2\theta\right] - N_{\mathrm{d}}(V_{\mathrm{p},r} - V_{\mathrm{f},r})\right]\\ &\quad + g\left(1 - \frac{\rho_{\mathrm{f}}}{\rho_{\mathrm{p}}}\right)\cos(\theta - \beta);\end{aligned} \tag{24}$$

$$\begin{aligned}r\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + 2\frac{\mathrm{d}r}{\mathrm{d}t}\frac{\mathrm{d}\theta}{\mathrm{d}t} &= \frac{1}{\tau}\left[-\frac{V_{\mathrm{m}}}{\tilde{r}^3}\sin 2\theta - N_{\mathrm{d}}(V_{\mathrm{p},\theta} - V_{\mathrm{f},\theta})\right]\\ &\quad - g\left[1 - \frac{\rho_{\mathrm{f}}}{\rho_{\mathrm{p}}}\right]\sin(\theta - \beta).\end{aligned} \tag{25}$$

Components of the medium flow velocity vector $V_{\mathrm{f},r}$, $V_{\mathrm{f},\theta}$ in the right-hand sides of Eqs. (24) and (25) are considered to be known functions of coordinates r, θ. These functions have different forms for different models of flow around the cylindrical collector by fluid, e.g. the potential, viscous laminar flow and the potential flow with viscous boundary layer on the surface of the cylinder.

Taking into account the analysis of these models carried out in [3], we shall consider the model of the potential flow around the cylinder, with the vector of the unperturbered flow $\vec{V}_0$, normal to the axis of cylinder.

We shall consider additional limitations: (1) the diameter of particles and the medium flow velocity are quite small, so that when determining the hydrodynamic drag of a particle it is possible to use the Stokes formula; (2) the mass of a particle or the difference of densities of the particles and the liquid are quite small, so that the gravitational force (the algebraic sum of the gravity and Archimedes forces) can be neglected; (3) the magnetization of a cylindrical ferromagnetic element reached the value of saturation ($M = M_{\mathrm{s}}$). Under these assumptions Eqs. (24) and (25) then yield equations given in [1]. Analogical equations given in [3] are applicable to a more general case. In contrast

Eqs. (24) and (25) can be used for any arbitrary flow regime around the cylinder, not only for the Stokes condition.

DISCUSSION OF RESULTS

Let us consider examples of trajectories of particle motion calculated by numerical solution of the Eq. (24) and (25) equivalent to Eq. (1). At the same time we shall consider trajectories, which were calculated using the same initial data, from the reduced equations obtained from Eqs. (24) and (25) assuming that the left-hand sides are equal to zero (neglecting the inertia force).

The influence of the gravitational force on the capture cross-section of particles can be significant or negligibly small, depending on the diameter of the particle d_p, the medium flow velocity V_0, strength of the external magnetic field H_0, densities of a particle and the medium ρ_p and ρ_f respectively, the volume magnetic susceptibilities of particles and the medium κ_p and κ_f, the angle β between the vectors $\vec{F}_g$ and $\vec{H}_0$, and on the angle α between the vectors $\vec{V}_0$ and $\vec{H}_0$.

With $\alpha = 0$ and $\beta = \pi/2$ the influence of the gravitational force becomes apparent in the displacement of the central line of the particle capture cross-section zone with respect to the axis x, which is the axis of symmetry of the magnetic field and the slurry flow. Calculations show that for a given configuration the inclusion of the F_g force in the equations of motion leads to insignificant alterations of the capture zone. It is explained by that fact that increments of the capture cross-sections for $Y > 0$ and for $Y < 0$ have opposite signs and differ insignificantly in absolute value.

Fig. 1 shows the limiting trajectories of a particle with the following values of parameters: $d_p = 100\ \mu\text{m}$, $V_0 = 0.1$ m/s, $B_0 = 0.5$ T ($H_0 = 3.98 \times 10^5$ A/m), $d_w = 1$ mm; $\alpha = 0$ (the longitudinal configuration); $\chi_p = 1.5 \times 10^{-6}\ \text{m}^3/\text{kg}$ ($\chi_p = \kappa_p/\rho_p$); $\rho_p = 4.7 \times 10^3\ \text{kg/m}^3$; $\rho_f = 10^3\ \text{kg/m}^3$.

Trajectories 1a and 1b are calculated by neglecting the gravitational force ($F_g = 0$); while trajectories 2a, 2b – with the gravitational force included. The vector $\vec{F}_g$ is perpendicular to the vectors $\vec{H}_0$ and $\vec{V}_0$. The inertial force was taken into account in all four cases. For $F_g = 0$ (curves 1a and 1b) width of the capture zone is $\tilde{h} = 2\tilde{y}_c = 5.54$ at the distance $\tilde{x} = 3\tilde{r}_*$ from the axis of the cylinder; for $F_g \neq 0$ (curves 2a and 2b)

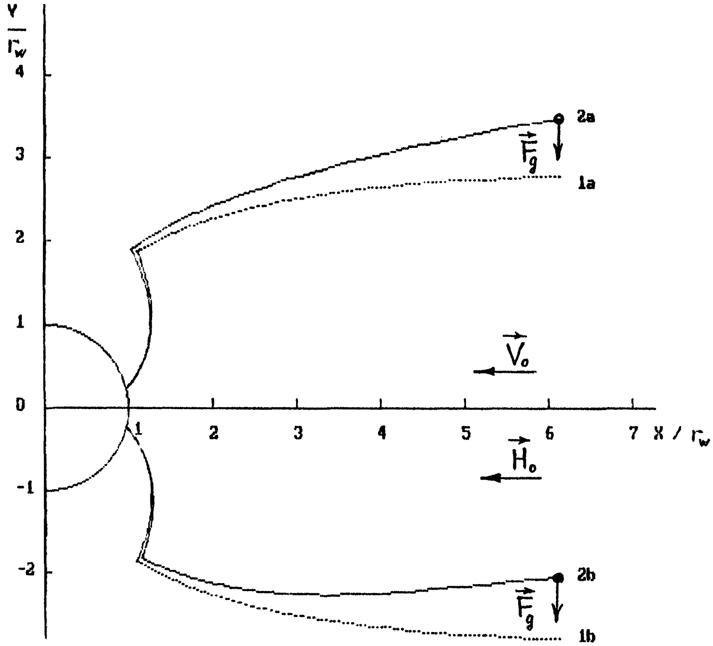

FIGURE 1 The influence of gravitational force F_g on the particle trajectories and the capture cross section when the flow velocity vector V_0 is normal to the vector F_g; $d_p = 100\,\mu m$; $d_w = 1$ mm ($d_w = 2r_w$); $V_0 = 0.1$ m/s; $B_0 = 0.5$ T; 1a, 2a – upper limits of the capture cross-section; 1b, 2b – lower limits; 1a, 1b – $F_g = 0$; 2a, 2b – $F_g \neq 0$.

$\tilde{h} = (\tilde{y}_c)_{2a} + |(\tilde{y}_c)_{2b}| = 5.47$, i.e. 1.3% less than for $F_g = 0$. Here $\tilde{h} = h/r_w$; $\tilde{y}_c = y_c/r_w$; r_w is the radius of the cylinder; indices 2a and 2b designate the capture cross-section y_c, corresponding to the trajectories 2a and 2b in Fig. 1. For comparison we shall note that if the inertia force is neglected ($F_{in} = 0$; $F_g = 0$), then the capture cross-section will be 4% higher than in the case when the inertia force is included ($F_{in} \neq 0$; $F_g = 0$).

With the decreasing diameter of particle d_p and also with the increasing velocity V_0 the difference between the results obtained with $F_g = 0$ and $F_g \neq 0$ will be reduced.

In the case when vectors $\vec{F}_g$, $\vec{H}_0$ and $\vec{V}_0$ are parallel, the influence of the gravitational force on the capture section has a different character. In particular, there could be a case when the above mentioned influence will be significant and, at the same time, the influence of the inertial force will be negligibly small. Our calculations show that for configuration

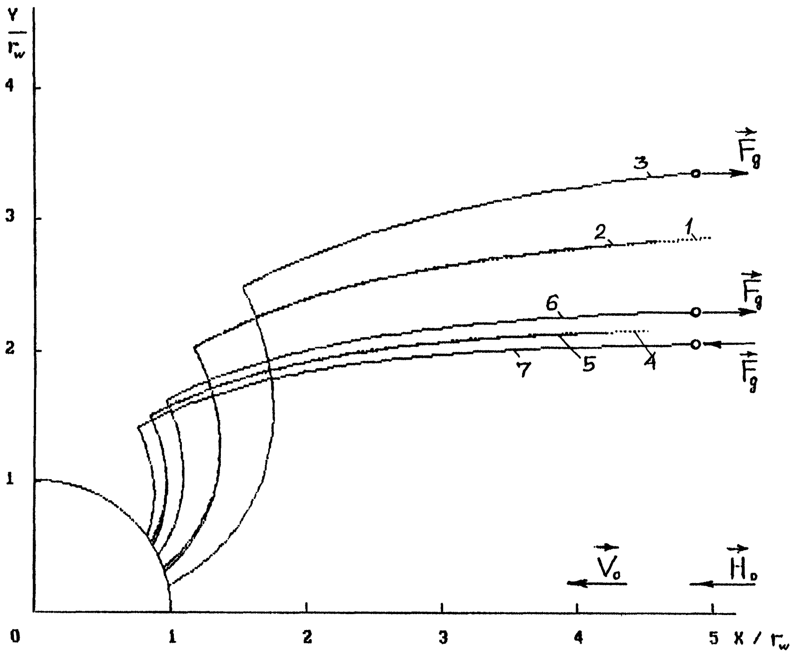

FIGURE 2 The influence of the gravitational force F_g and the intertial force on the particle trajectories and the capture cross-section when the flow velocity vector $\vec{V}_0$ is parallel to the vector $\vec{F}_g$; $d_p = 50\,\mu m$, $d_w = 1$ mm ($d_w = 2r_w$); $B_0 = 0.25$ T; 1, 2, 3, – $V_0 =$ 0.01 m/s; 1 – without consideration of the inertial force; 2, 3 – with consideration of the inertial force; 1, 2 – $F_g = 0$; 3 – $F_g \neq 0$, vertical ascending flow; 4, 5, 6, 7 – $V_0 = 0.02$ m/s; 4 – without consideration of the inertial force; 5, 6, 7 – with consideration of the inertial force; 4, 5 – $F_g = 0$; 6, 7 – $F_g \neq 0$; 6 – vertical ascending flow; 7 – vertical descending flow.

$\vec{H}_0 \,||\, \vec{V}_0$, $\vec{F}_g \,||\, \vec{V}_0$ (the vertical slurry flow) with the value of V_0 approaching the value of "soaring" velocity of a particle in the rising flow the difference of the capture sections, calculated with and without inclusion of the gravitational force, becomes more significant. This situation is illustrated in Fig. 2 where the trajectories for $V_0 = 0.01$ m/s and 0.02 m/s, are compared, assuming that other parameters in both cases are the same: $d_p = 50\,\mu m$, $B_0 = 0.25$ T; the values d_w, ρ_p, ρ_f, χ_p are equal to the values in the example considered above.

With $V_0 = 0.02$ m/s the trajectories of particles, estimated with and without the inclusion of the inertia force with $F_g = 0$ (curves 4 and 5 respectively), practically coincide. The small difference between them is visible in the vicinity of the cylinder. When the gravitational force is included the trajectories and the capture section are appreciably changed. For the vertical ascending flow (curve 6) the capture section is increased by 6.5%, for the descending one (curve 7) is decreased by 5.1%. Let us note that the capture cross-section is changed with the change of the distance from the initial point to the axis of the cylinder; here this distance is fixed and assumed equal to $x_0 = 3r_*$, where r_* is the radial coordinate of the equilibrium point on the trajectory of a particle.

The analogous calculation for variant $V_0 = 0.01$ m/s gives the following results: the trajectories estimated with and without inclusion of the inertial force with $F_g = 0$ (curves 1 and 2 in Fig. 2) practically coincide; with $F_g \neq 0$ (the gravitational force is included) for the vertical ascending flow (curve 3) the capture cross-section increases by 19.6%.

CONCLUSION

The effect of the gravitational and inertial forces, for various configurations, on the particle capture cross-section has been investigated. It transpires from the results that if, for a certain combination of parameters d_p, V_0, H_0 and others, one of the above forces can be neglected, it does not necessarily indicate the other force can also be neglected. It was also observed that with a decreasing size of a particle, the influence of both forces tends to decrease.

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