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# THE ASSESSMENT OF THE APPLICABILITY OF A MODEL OF NONINERTIAL MOTION OF A PARTICLE IN A MAGNETIC SEPARATOR

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Using two models, namely the initial differential equation of motion and the reduced equation obtained from the initial one by neglecting the inertial force comparative calculations of the capture cross section of paramagnetic particles by a cylindrical ferromagnetic collector (a wire) were carried out. For various combinations of the particle diameter, the wire diameter, medium flow velocity, strength of the magnetic field, the error in the capture cross section caused by neglecting the inertial force in the equations of motion was determined. Approximate formulae for the determination of this error and of the ratio of the inertial force to the magnetic force as a function of the main separation parameters were proposed.

*Keywords:* Magnetic separation; Inertial force; Hydrodynamic drag; Magnetic force; Particle trajectory; Reynolds number

## INTRODUCTION

When designing magnetic filters or separators it is expedient to carry out preliminary determination of the efficiency of extraction of material depending on its properties (magnetic susceptibility, density and the diameter of particles) and technological parameters (strength of the magnetic field, velocity of the slurry movement). For the working space

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filled with ferromagnetic elements (rods, wire mesh, steel wool) this efficiency can be approximately determined on the basis of calculation of the capture cross section of particles of the extracted material by a single wire. This problem is well described in [1–4] and in other publications.

## THEORETICAL MODELS

In order to determine the capture cross section a theoretical model can be, in some cases used [1,2], which we shall call for short a model of "noninertial" motion of particles. It is a system of differential equations of motion, where the terms corresponding to the inertial force are rejected. As a criterion of validity of such a model a condition that quantity $\tau$ is much smaller than unity was proposed in [2]:

$$\tau = \frac{2}{9}\frac{\rho_p}{\eta} r_p^2; \quad \tau \ll 1.$$

(It has to be noted that $\tau$ is much smaller than 1 s, since $\tau$ has a dimension of time.)

Here $\rho_p$ and $r_p$ are density and radius of a particle, respectively and $\eta$ is the dynamic viscosity of the carrying medium.

In this work, in order to estimate the influence of the inertial force, two methods are used: the first one – determination of the ratio of the inertial force to the magnetic and hydrodynamic drag forces, the second one – determination of the capture cross sections calculated with and without the inertial force. In each theoretical model, "inertial" and "noninertial", the capture cross section was estimated for given combinations of the determining parameters on the basis of the limiting trajectory calculations. Differential equations of motion proposed in work [6] were used. By a numerical solution of the initial equations and of the corresponding reduced equations assuming that

$$m_p \frac{d\vec{V}_p}{dt} = 0,$$

a parametric study of errors of calculation of the capture cross section of paramagnetic particles caused by neglecting the inertial force in the equations of motion was carried out. Here $m_p$ is the mass of a particle,

$\vec{V}_p$ is the particle velocity. The results thus obtained allow to estimate the validity of the simplified model of the "noninertial" movement of particles for a given combination of parameters: the diameter of a particle $d_p$, the wire diameter $d_w$, the medium flow velocity $V_0$, and the strength of the magnetic field $H_0$.

Taking into account the estimates of the influence of the gravitational force $F_g$ carried out in [6] we shall consider separately the influence of the inertial force on the capture cross section of particles assuming $F_g = 0$. In this case the criterion of validity of the "noninertial" equations of motion can be expressed by the following inequalities: $Z_m \ll 1$ or $Z_d \ll 1$, where $Z_m = F_{in}/F_m$; $Z_d = F_{in}/F_d$; $F_{in} = m_p|d\vec{V}_p/dt|$ is the inertial force; $F_m$ is the magnetic force; $F_d$ is the hydrodynamic drag. It must be noted that for the calculation of $F_d$ the dependence of the hydrodynamic drag factor on Reynolds number as given in [5] was used.

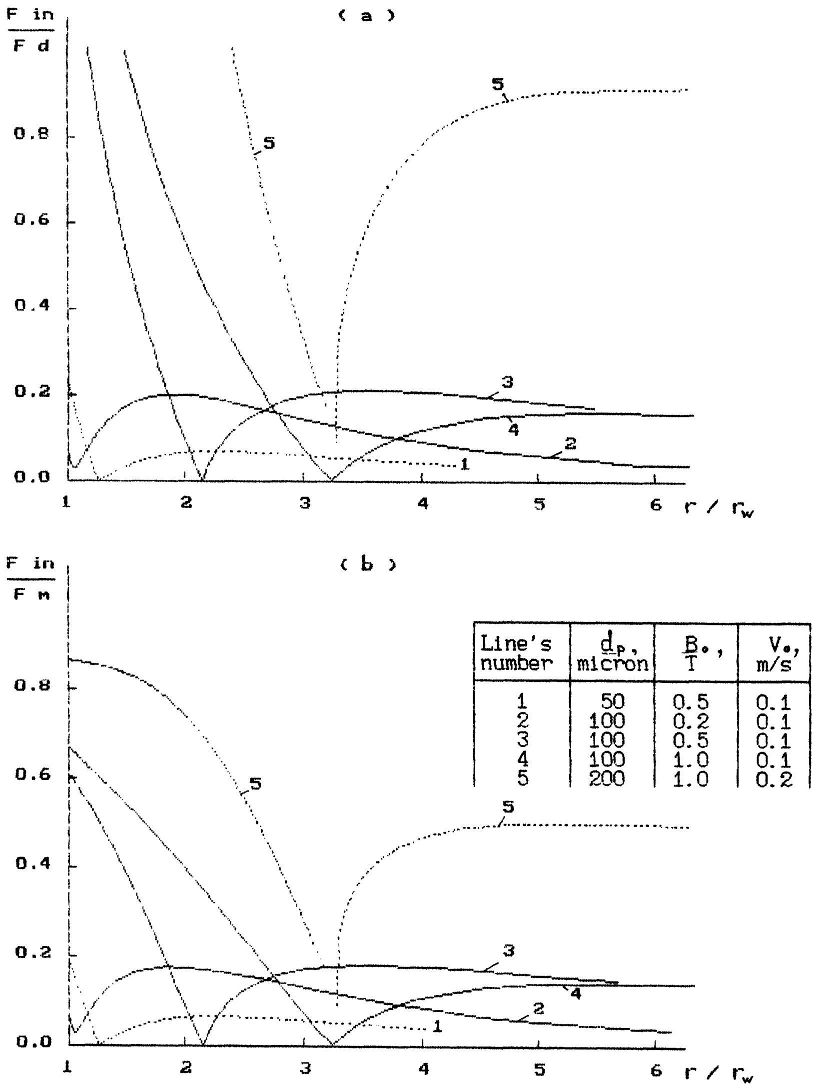
## DISCUSSION

The results of calculation of distribution of quantity $Z_d$ (the ratio of the inertial force to the hydrodynamic drag) along the trajectory of particles, in the form $(F_{in}/F_d)\ \{r/r_w\}$, where $r$ is the radial coordinate of a given point of trajectory ($r_w = d_w/2$), are shown in Fig. 1(a). The variables were the diameter of a particle $d_p$, the magnetic induction $B_0$ of external magnetic field and the medium flow velocity $V_0$. Other determining parameters in all variants were equal and had following values:

density of particles $\rho_p = 4.7 \times 10^3$ kg/m$^3$;
specific magnetic susceptibility of particles $\chi_p = 1.5 \times 10^{-6}$ m$^3$/kg;
diameter of the cylindrical ferromagnetic collector $d_w = 1$ mm.

The longitudinal configuration [3], i.e. $\vec{V}_0 \parallel \vec{H}_0$ was considered. The medium fluid was assumed to be water at room temperature.

Magnetic permeability of the cylinder material (a steel containing 0.2% of carbon), which is used to estimate the magnetization of the cylinder, was determined using the reference tables and approximations as a function of $H_0$. Fig. 1(b) shows the dependencies $(F_{in}/F_m)\ \{r/r_w\}$; the curve numbers correspond to the same combinations as in Fig. 1(a). These dependencies, in the first approximation, are similar to the




| Line's number | $d_p$, micron | $B_0$, T | $V_0$, m/s |
|---|---|---|---|
| 1 | 50 | 0.5 | 0.1 |
| 2 | 100 | 0.2 | 0.1 |
| 3 | 100 | 0.5 | 0.1 |
| 4 | 100 | 1.0 | 0.1 |
| 5 | 200 | 1.0 | 0.2 |

FIGURE 1 The ratio of the inertial force $F_{in}$ to the hydrodynamic drag $F_d$ (a) and to the magnetic force $F_m$ (b) as functions of the relative radial coordinate $r/r_w$ of the limiting trajectory.

dependencies $(F_{in}/F_d)$ $\{r/r_w\}$. However the value $Z_m$ (provided $F_g = 0$ which is assumed above) at any point of the trajectory is less than unity, whereas the value $Z_d$ can exceed unity. This usually takes place for $r < r*$ and for larger values $d_p$ and $V_0$ and as well for $r > r*$, where $r*$ is the radial coordinate of the equilibrium point for a given trajectory.

It can be seen from Fig. 1(a) and (b) that the dependencies $Z_d(\tilde{r})$ and $Z_m(\tilde{r})$, where $\tilde{r} = r/r_w$, have two characteristic sections, located on the left and right-hand sides of the $Z_d$ and $Z_m$ zero value points. Each of these points corresponds to a special point of the system of differential equations of motion of a particle (the equilibrium point). For some combinations of values of $d_p$, $V_0$, $H_0$ and other parameters the points of zero value of $Z_d$ or $Z_m$ is absent; in these cases equations of motion do not have special points in the interval $\tilde{r} > 1$.

It is obvious that the capture cross section is determined by the parameters of the particle motion in the section of the trajectory located upstream from the equilibrium point, i.e. in the section $r > r*$. The values of $F_m$, $F_d$, $F_{in}$ change along the trajectory, therefore let us consider, for the estimation of the dependence of ratios $Z_d = F_{in}/F_d$ and $Z_m = F_{in}/F_m$ on parameters $d_p$, $d_w$, $V_0$, $B_0$ the average-integral values $Z'_d$, $Z'_m$ of these ratios:

$$Z'_d = \frac{1}{L_1}\int_0^{L_1} Z_d \, dL; \quad Z'_m = \frac{1}{L_1}\int_0^{L_1} Z_m \, dL;$$

where $L_1$ is the length of a section of the limiting trajectory from the point $r = r_1$ to the point $r = r*$; d$L$ is an element of length of the trajectory.

In preliminary calculations the $r_1$ value was varied, after that it was chosen to be equal to $r_1 = 3r*$. The $Z'_d$ and $Z'_m$ dependencies on induction $B_0$ of the external magnetic field for various values of $d_p$, $d_w$, and $V_0$ are shown in Fig. 2. Values of these parameters corresponding to the variants of curves 1 to 9 are shown in the Table in Fig. 3. The reduction of values $Z'_d$ and $Z'_m$ with increasing $B_0$, keeping other parameters fixed, is caused by the increase of the distance $r*$ from the axis of the cylinder to the equilibrium point on the limiting trajectory. Thus the section of the limiting trajectory $r* < r < r_1$ is displaced in the area of relatively small gradients of the magnetic field and relatively weak perturbation of the flow. This results in the reduction of

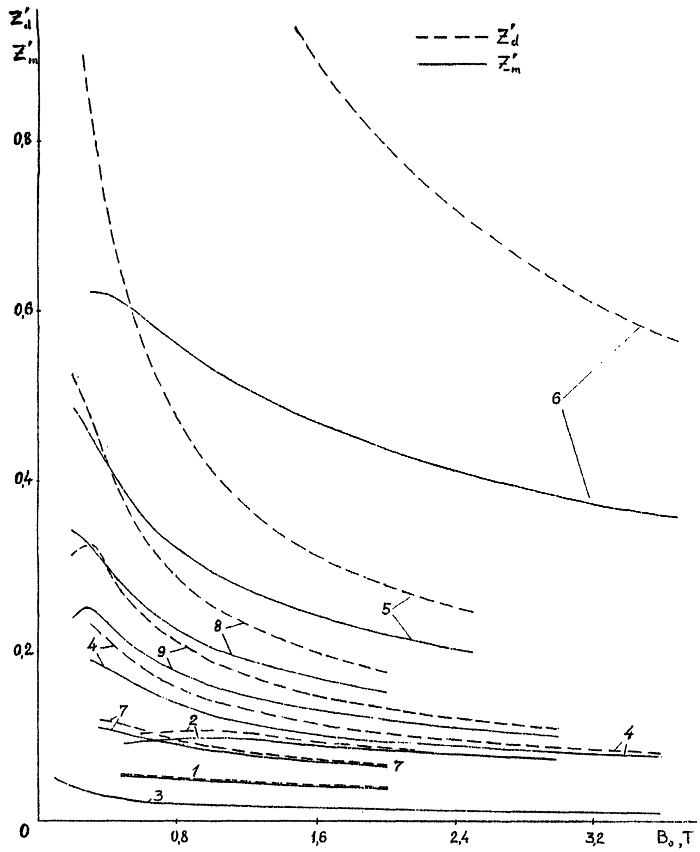



FIGURE 2 The average-integral values of the ratio of the inertial force to the hydrodynamic drag ($Z'_{\mathrm{d}}$) and to the magnetic force ($Z'_{\mathrm{m}}$) as functions of the background magnetic induction $B$.

the acceleration of a particle in a given section. The difference between dependencies $Z'_{\mathrm{d}}(B_0)$ and $Z'_{\mathrm{m}}(B_0)$ increases with the increase of the diameter of a particle $d_{\mathrm{p}}$ and the velocity $V_0$ and with the reduction of the wire diameter $d_{\mathrm{w}}$, when the above parameters are fixed. Parameters $d_{\mathrm{p}}$ and $V_0$ are related to the Reynolds number $\mathrm{Re}_0$ by:

$$\mathrm{Re}_0 = \frac{\rho_{\mathrm{f}} V_0 d_{\mathrm{p}}}{\eta} \tag{1}$$

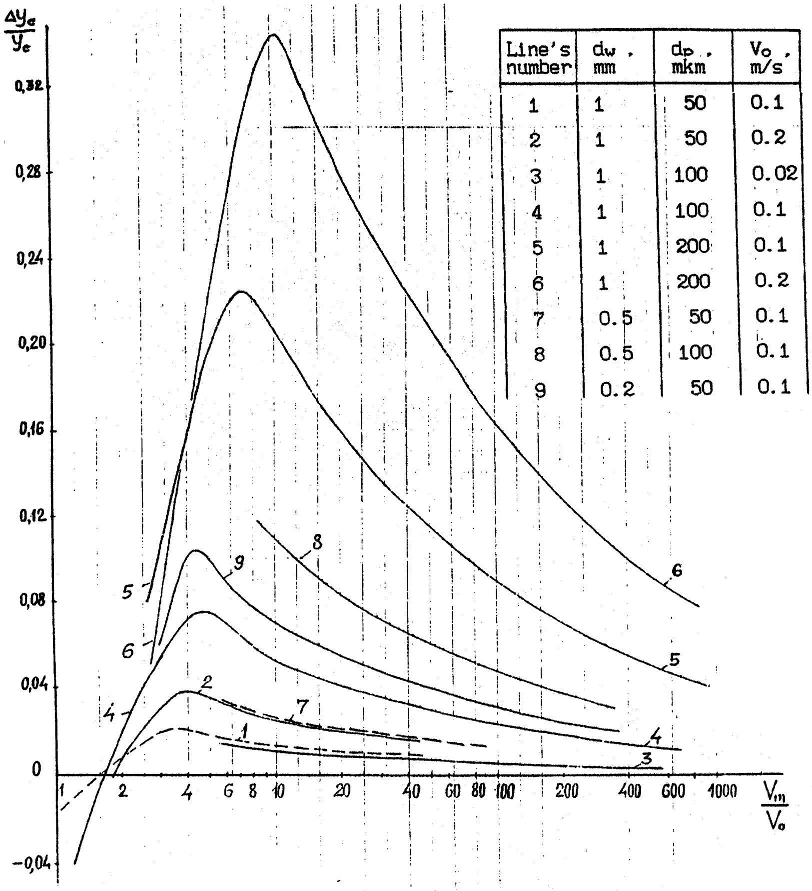



FIGURE 3 The relative errors $\Delta Y_c/Y_c$ of the determination of the capture cross section caused by neglecting the inertial force in the equations of particle motion as functions of the relative magnetic velocity $V_m/V_0$; $F_g = 0$; longitudinal configuration.

and the field strength $H_0$ is related to the "magnetic" velocity $V_m$ according to [1,3]:

$$V_m = \mu_0(\kappa_p - \kappa_f)\frac{d_p^2 M H_0}{18\eta r_w}, \tag{2}$$

$Z'_m$ can then be expressed by following approximate formula:

$$Z'_m = \frac{0.58\mathrm{Re}_0^{\alpha} d_1^{1.2}}{1 + 0.14(V_m/V_0)^{0.5}}, \tag{3}$$

where $\alpha = 1 - 1.2d_1$; and $d_1 = d_p/d_w$. In (1) to (3) $\rho_f$ is the density of the flow medium; $M$ is the magnetization; $M = 2AH_0$;

$$A = \frac{\mu_w - \mu_f}{\mu_w + \mu_f};$$

$\mu_w$ and $\mu_f$ are the magnetic permeabilities of the cylinder and the medium, respectively, $\kappa_p$ and $\kappa_f$ are the volume magnetic susceptibilities of a particle and the medium, respectively. Equation (3) approximates the averaged (in the section $r* < r < 3r*$ of the limiting trajectory) values of the ratio of the inertial force to the magnetic force for the values $d_p$, $d_w$, and $V_0$ corresponding to the variants 1 to 9 (see Fig. 3).

The range of the $H_0$ values of the magnetic field strength where Eq. (3) is applicable is determined on the basis of inequality $V_m/V_0 > (V_m/V_0)_{min}$, with $V_m$ on the left-hand side given by Eq. (2). The right hand side of this inequality is given by the following approximation:

$$\left(\frac{V_m}{V_0}\right)_{min} \approx \mathrm{Re}_0.$$

The dependencies of the relative error $\Delta Y_c/Y_c$ of the capture cross section on the ratio $V_m/V_0$ caused by neglecting the inertial force in the movement equations is shown in Fig. 3. Here $\Delta Y_c = Y_{c,0} - Y_c$ where $Y_c$, $Y_{c,0}$ are the capture cross sections at distance $x = 3r*$ from the axis of the cylinder calculated with and without the inertial force; in both cases the gravitational force was neglected. The curves 1, 2, 7, 9 are obtained for the particles with diameter $d_p = 50$ μm; 3, 4, 8 – 100 μm; 5, 6 – 200 μm; the value of the medium flow velocity $V_0$ for these curves: 3 – $V_0 = 0.02$ m/s; 1, 4, 5, 7, 8, 9 – 0.1 m/s; 2 and 6 – 0.2 m/s; $d_w = 1$ mm for the variants 1–6; $d_w = 0.5$ for the variants 7, 8; $d_w = 0.2$ for the variant 9. For all variants $\rho_p = 4.7 \times 10^3\,\mathrm{kg/m^3}$; $\rho_f = 10^3\,\mathrm{kg/m^3}$; $\chi_p = 1.5 \times 10^{-6}\,\mathrm{m^3/kg}$.

The maximum point of curve $(\Delta Y_c/Y_c)(V_m/V_0)$ corresponds to such a combination of parameters when the equilibrium point of the limiting trajectory is close to the surface of the cylinder. When $V_m/V_0$ increases in the section $V_m/V_0 > (V_m/V_0)_m$ where $(V_m/V_0)_m$ is the value of $V_m/V_0$ at the maximum point the value $\Delta Y_c/Y_c$ decreases at the expense of the growth of the function $Y_c(V_m/V_0)$. Function $\Delta Y_c(V_m/V_0)$ increases slowly, asymptotically approaching the constant value for high values of $V_m/V_0$. The value of $(V_m/V_0)_m$ can be calculated from the approximate formula:

$$\left(\frac{V_\mathrm{m}}{V_0}\right)_\mathrm{m} \approx 1 + \mathrm{Re}_0^{0.6}.$$

As can be seen from Fig. 3 the value $\Delta Y_\mathrm{c}/Y_\mathrm{c}$ can be negative for small $V_\mathrm{m}/V_0$. In this case the capture cross section calculated without the inclusion of the inertial force is smaller than with the inertia force: $Y_{\mathrm{c},0} < Y_\mathrm{c}$.

However, for the majority of the considered conditions $Y_{\mathrm{c},0}$ is greater than $Y_\mathrm{c}$. From Fig. 3 it is also seen that the value $\Delta Y_\mathrm{c}/Y_\mathrm{c}$ increases with the increase of the diameter of particles $d_\mathrm{p}$ and the medium flow velocity $V_0$. As these two parameters are included in the Reynolds number Re, determined by Eq. (1), it is possible to express the value of $\Delta Y_\mathrm{c}/Y_\mathrm{c}$ as the function of Re, $V_\mathrm{m}/V_0$ and some other variables. Of practical interest are the relative errors $\Delta Y_\mathrm{c}/Y_\mathrm{c}$ in the range $(V_\mathrm{m}/V_0) > (V_\mathrm{m}/V_0)_\mathrm{m}$, corresponding to the section of the declining curve $(\Delta Y_\mathrm{c}/Y_\mathrm{c})(V_\mathrm{m}/V_0)$. In this range the results of calculation shown in Fig. 3 can be approximated by following formula:

$$\frac{\Delta Y_\mathrm{c}}{Y_\mathrm{c}} = \frac{0.27\mathrm{Re}_0^{\beta} d_1^{1.25}}{0.14 + (V_\mathrm{m}/V_0)^{0.35}}; \quad \frac{V_\mathrm{m}}{V_0} > \left(\frac{V_\mathrm{m}}{V_0}\right)_\mathrm{m}, \tag{4}$$

where

$$\beta = 1 - 0.8d_1;\ d_1 = d_\mathrm{p}/d_\mathrm{w}. \tag{5}$$

On the other hand, for each combination of parameters $d_\mathrm{p}$, $d_\mathrm{w}$, $V_0$, $H_0$ the relative error $\Delta Y_\mathrm{c}/Y_\mathrm{c}$ can be associated with the averaged value of the ratio of the inertial force to the magnetic force with the formula:

$$\frac{\Delta Y_\mathrm{c}}{Y_\mathrm{c}} = kZ'_\mathrm{m}, \quad \text{with } \frac{V_\mathrm{m}}{V_0} > \left(\frac{V_\mathrm{m}}{V_0}\right)_\mathrm{m}. \tag{6}$$

When determining the values $Z'_\mathrm{m}$ and $\Delta Y_\mathrm{c}/Y_\mathrm{c}$ under approximate formulae (3) and (4) respectively, then the factor $k$ in Eq. (6) has value in the range 0.18–0.41 for the values $d_\mathrm{p}$, $d_\mathrm{w}$, $V_0$ corresponding to the variants 1–9 with $V_\mathrm{m}/V_0 > (V_\mathrm{m}/V_0)_\mathrm{m}$.

Equation (4) shows that if the parameter $V_\mathrm{m}/V_0$ is fixed, then the relative error $\Delta Y_\mathrm{c}/Y$ increases with the increasing diameter $d_\mathrm{p}$ of a particle through two parameters: Reynolds number $\mathrm{Re}_0$ and the ratio $d_1 = d_p/d_w$. If the diameter of a particle is small in comparison with the diameter of the wire (then under Eq. (5) $\beta = 1$) then with $V_m/V_0 = \text{const}$ taking into account Eq. (1) we obtain that the value $\Delta Y_c/Y_c$ is proportional to the factor $V_0 d_p^{2.25}/d_w^{1.25}$. If the ratio $V_m/V_0$ is not fixed or we cannot neglect the value $d_p/d_w$ in comparison with unity then the dependence $\Delta Y_c/Y_c$ on $d_p$ and on $d_w$ becomes more complex, since these parameters also belong to the expression for the "magnetic" velocity (2) (taking into account $r_w = d_w/2$).

The results of calculations show that, for example, for the case $d_w =$ 1 mm it is possible to neglect the inertial force in the equations of motion when $d_p = 50\,\mu\text{m}$, if the slurry velocity $V_0$ does not exceed 0.1 m/s; and for $d_p = 100\,\mu\text{m}$ – when $V_0 < 0.02$ m/s. For an arbitrary combination of parameters the relative error can be estimated using Eq. (4), where the Reynolds number $\mathrm{Re}_0$ is determined by Eq. (1) and the "magnetic" velocity $V_m$ by Eq. (2).

## CONCLUSIONS

1. Trajectories of paramagnetic particles moving in the medium flow near a cylindrical ferromagnetic collector (a wire) have been calculated. Two various theoretical models were used: a system of the initial differential equations of motion where the inertial force was taken into account and the system of the reduced equations where the inertial force was neglected.
2. The values of the ratio of the inertial force to the magnetic force $Z_m = F_{in}/F_m$ and to the hydrodynamic drag force $Z_d = F_{in}/F_d$ along the trajectories of particles have been calculated. The average values of these ratios $Z'_m$ and $Z'_d$ depending on the diameter of particles $d_p$, a wire diameter $d_w$, the medium flow velocity $V_0$ and the magnetic field strength $H_0$ were also determined. The values $Z'_m$ and $Z'_d$ were calculated as the average-integral values of the ratios $Z_m$ and $Z_d$ along the section of a trajectory determining the capture of particles ($r* < r < 3r*$).
3. The relative errors $\Delta\tilde{Y}_c = \Delta Y_c/Y_c$ of determination of the capture cross section of particles caused by neglecting in the equations of motion of the inertial force, depending on parameters $d_p$, $d_w$, $V_0$, $H_0$, have been calculated.

4. Using the given results of calculation the approximate formulas for the values $Z'_m$ and $\Delta\tilde{Y}_c$ were obtained. The estimate of the proportionality factor $k$ in the approximate formula: $\Delta\tilde{Y}_c = kZ'_m$, relating the calculation error $\Delta\tilde{Y}_c$ of the capture cross section to the average-integral value of the ratio $Z'_m$ (the inertial to the magnetic force) for a given section of the trajectory was also determined.

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