TOPICS IN RANDOM POLYNOMIALS

by Kambiz Farahmand

A BOOK REVIEW

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(Received July, 1999; Revised December, 1999)

This book conveys a comprehensive account of the important results of random polynomials. The content is accessible to senior undergraduates in mathematics and graduates from disciplines with a significant mathematics element. On a first reading a great deal can be gleaned from the text without concentrating on the detail of the proofs. The book is very readable and the chapter introductions prove particularly interesting. The concise nature of the text, combined with the almost exhaustive bibliography, will make it popular with both experts and novices alike.

Polynomials represent an important step in the mathematical development of the school going student. It is therefore not surprising that the study of polynomials, with random coefficients, attracts a substantial level of interest from mathematical researchers. It is through the study of polynomials that the young student first perfects his/her graph drawing skills. The graph of the real polynomial represents the ultimate in efficient characterization. Without the option of graphing random polynomials, it is important to invent other means of characterizing their behavior.

In this book, the traditional methods for describing the behavior of random polynomials, such as, the number and location of zeros and turning points are well covered. However, the main strength of this text is that Farahmand introduces innovative devices for capturing the behavior of random polynomials. This is the first time that these methods appear in book form. The author determines the number of "sharp crossings," "maxima below a level" and the "exceedance measure" to compensate for the absence of a graphical image. Farahmand communicates the historically interesting aspects of random polynomials in a tractable manner. The reader learns that for the algebraic polynomial, the majority of zeros are concentrated around $+1$ and 1 . The surprising fact that, for all known symmetric coefficient distributions, the number of real zeros for each algebraic polynomial is of the same order is emphasized. The small number of real zeros for the algebraic and hyperbolic polynomials compared with the trigonometric and orthogonal polynomials is also stressed. Interesting results for non-zero means and dependent coefficients are also extensively covered. Results for random polynomials with normal coefficients are presented in separate

chapters for algebraic, trigonometric, orthogonal, and hyperbolic polynomials.

The self-contained nature of these chapters make it possible for the reader to obtain a deep understanding of a particular random polynomial without reference to the other result chapters. The reader can proceed to a particular result chapter, after studying the results for relevant stochastic and Gaussian processes in chapter 2. However, random polynomials become most fascinating when results for one polynomial are compared to those for another. In fact, the consistency of the presentation makes it easier for the reader to track and compare the evolution of the discipline within each of these polynomials. In the final two chapters, random polynomials with other coefficient distributions and complex cases are dealt with. Cauchy distributed coefficients for the algebraic polynomial is examined. The reader is taken through the mathematics required to move from the characteristic function for the Cauchy distribution to obtain an integral representation for the number of zeros. The mathematics employed is not trivial, and the solution is evaluated further using numerical integration methods. Results for random coefficients that belong to the domain of attraction of the normal law are also given for both the algebraic and trigonometric polynomials.

The last chapter concerns algebraic and trigonometric polynomials with complex coefficients. The real and imaginary parts of the complex coefficients examined are both normally distributed with zero means. Farahmand outlines why real roots have probability zero, and he obtains the number of "almost zeros." Important results for complex roots are also contained here. The author traces the study of random polynomials, in its present form, back to the year 1938. It is evident from the text that the study is only in its infancy. The author identifies many areas, in which potential developments in the theory could originate.

Although this text concentrates primarily on large degree polynomials, with normally distributed coefficients, opportunities for further research are identified for other distributions. The possibility of deriving a similarly comprehensive set of results for other distributions is an obvious possible area of further study. Within the sphere of normal coefficients, the absence of good quality results for non-identically distributed coefficients is clear. The author refers also to the little use that has been made of the "exceedance measure" in studying random polynomials. It is apparent from the results that the study of orthogonal and hyperbolic polynomials has yet to progress to the same extent as the study of algebraic and trigonometric polynomials. Many of these gaps in the theory will undoubtedly be addressed in the years ahead, and equally undoubtedly the results obtained will not always be as expected. The modern discipline of random polynomials, encompasses a broad set of mathematical skills including probability theory, asymptotic methods, complex analysis, and simulation methods. This expansion in the topic will have the effect of broadening the interest in random polynomials further.

This text provides an easy entry into the topic for someone previously not familiar with the discipline. With areas of possible further study flagged in the book, and a good balance between rigor and coverage, this text is ideal vehicle for gaining a grasp of the subject. The text may also reinvigorate those with knowledge of random polynomials to search for results, or technique, that shed further light on their behavior.

Topics in random polynomials, by **Kambiz Farahmand**

Publisher: Pitman Research Notes in Mathematics Series, No. 393, Longman

Publication Year: 1998

ISBN 0-582-35622-9

Price: \$58.00