Identification of Disjoint Bi-decompositions in Boolean Functions through Walsh Spectrum

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A new algorithm for the identification of disjoint bi-decomposition in Boolean functions from its Walsh spectrum is proposed. The type of bi-decomposition and its existence is derived from the knowledge of a subset of Walsh spectrum for a Boolean function. All three types of bi-decomposition are considered including OR, AND and EXOR type. A filtering procedure that uses just few Walsh spectral coefficients (SC) is applied to quickly eliminate the functions that are not bi-decomposable and hence the algorithm is very efficient. The type of bi-decomposition and affirmation/negation of variables in its logic sub-functions are directly identified by manipulation on the reduced cubical representation of Boolean functions and their corresponding Walsh spectra. The presented algorithm has been implemented in C and tested on the standard benchmark functions. The number of Boolean functions having various disjoint bi-decompositions has also been enumerated.

Keywords: Boolean functions; Functional decomposition; Disjoint bi-decomposition; Walsh spectrum; Walsh transform

INTRODUCTION

Functional decomposition of Boolean functions allows decomposition of original logical system into sub-circuits that can be realized by some basic blocks from the library of various programmable logic devices. There are different types of functional decomposition and various methods for their identification [1–4,9–15]. The earlier classical methods used decomposition charts and the algebra of Boolean derivatives [1–4], while the recent methods are more powerful techniques that manipulate on reduced representations of Boolean functions such as decision diagrams or cubes [10–13]. Spectral methods based on Walsh transform of Boolean functions have also been used for identification of disjoint decomposition [9,15] as well as to identify parallel and serial linear decomposition (extraction of linear pre- and post-filters) [8,9]. It should be noticed that the identification of a linear decomposition is only possible through the Walsh spectral domain.

Due to recent advances in programmable logic devices that have two-input logic elements in the outputs, an interest in bi-decomposition has developed. The methods

that are used for this type of decomposition apply either decision diagrams or spectral representation of Boolean functions using the Reed–Muller transform [10,14]. For the first time, spectral conditions in the Walsh domain of the existence for bi-decomposition of Boolean functions were reported in Ref. [7]. In this paper, all types of bi-decomposition are identified with exact determination of logic sub-functions with examples for each case. In majority of cases only a subset of the whole Walsh spectrum is necessary to either reject or confirm the existence of various types of bi-decomposition. The new algorithm has been implemented in C and tested on the standard benchmark functions. Finally, the number of Boolean functions having various types of considered bi-decompositions have been given.

PRELIMINARIES

Cubical Representation of a function

An n -bit string is a vertex of an object called a 0-cube. An n -variable Boolean function is represented as an

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FIGURE 1 Disjoint bi-decomposition.

n -dimensional space (n -hypercube) in which each vertex represents a minterm. A collection of 2^i , $i \in \{0, 1, \dots, n\}$ adjacent minterms is called an i -cube [5]. A cube can be represented by an n -string of symbols 0, 1 and $-$, where 0 corresponds to the negation of the variable, 1 to its affirmation value and $-$ to the variable that is missing in the cube.

Hadamard–Walsh Transform

Let $f(x)$ denote a completely specified Boolean function over a set of n input variables $X = (x_1, x_2, x_3, \dots, x_n)$, $x_u \in \{0, 1\}$, $1 \leq u \leq n$. The truth vector and the encoded 2^n

output values of f may be treated as column vectors $f \rightarrow$ and $F \rightarrow$ where the original elements of $f \rightarrow$ confined to $\{0, 1\}$ are mapped to an S -coding of $F \rightarrow$ which is defined as [logic “0” $\rightarrow 1$] and [logic “1” $\rightarrow -1$], respectively. Then the Walsh–Hadamard spectrum $S \rightarrow$ of $f(x)$ is a column vector given by, $S \rightarrow = H^n F \rightarrow$ and $F \rightarrow = (1/2^n) H^n S \rightarrow$ where $F \rightarrow = [f_0, f_1, f_2, \dots, f_{2^n-1}]^T$ and $S \rightarrow = [s_0, s_1, s_2, s_{12}, \dots, s_{123\dots n}]^T$, s_1 are the spectral coefficients (SC) in Hadamard ordering and I is the index of the coefficient of any order o , ranging from one to n . The DC coefficient or s_0 is related to the number of minterms for the function that have value 1 denoted by a and its value is $s_0 = 2^n - 2a$. H^n is the Hadamard–Walsh matrix or order n [8].

Disjoint Bi-decomposition

Let $X = (X_1, X_2)$ be a partition of the variables representing a function $f(X)$ such that $X_1 \cap X_2 = \emptyset$ and X_1 is a group of m variables and X_2 is a group of $n - m$ variables. $f(X_1, X_2) = h(g_1(X_1), g_2(X_2))$ is said to be disjoint EXOR bi-decomposable if $f(X)$ can be realized as shown in Fig. 1 where h is a two input EXOR gate and g_1 and g_2 are either AND, OR or EXOR gate. If h is either AND or OR gate with g_1 and g_2 defined as above, then the

FIGURE 2 Flow chart to identify functions with bi-decomposition.

decomposition is represented as disjoint AND or disjoint OR bi-decomposition, respectively [10–13].

DISJOINT BI-DECOMPOSITION

The flow chart in Fig. 2 shows the procedure to select a given function to test for the decomposition based on the DC coefficient and the first order Walsh SCs. In certain cases, where the first order SCs are all zero, the higher order coefficients need to be considered. If the preliminary conditions are satisfied, the algorithm discussed in the next section is applied on the selected function to identify the type of the decomposition. By this filtering procedure, the algorithm is very efficient and is effective in identifying the type and also the exact function from its cubical representation.

ALGORITHM TO IDENTIFY TYPE OF DECOMPOSITION

In the algorithm that is given below all the possible cases as outlined in the flow chart in Fig. 2 are discussed. The following three letters abbreviation scheme illustrated by an example is used throughout the algorithm for the type of the decomposition. The symbol AAE means that g_1 and g_2 are AND gates and h the EXOR gate so that the output $f = a \oplus b$ (Fig. 1). Here A, E, O represent the AND, EXOR and OR gates, respectively, while symbols $\wedge$, $\oplus$ and $\vee$ represent the logical product, EXOR operation and logical sum, respectively.

Case 1: (AAE)

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if  $((s_0 = 0) \text{ and } (\text{one } s_i \neq 0) \text{ and } (\text{all the other } s_i = 0))$ 
   $\{j = 2; \text{ while } (j \leq n) \{ \text{Calculate all SCs for the order } j;$ 
  if  $((i \in I) \text{ and } (s_1 \neq 0) \text{ and } (|s_1| + |s_i| = 2^n))$ 
     $j++;$  else break;
  if  $(j = n + 1)$ 
    if  $(s_i < 0) \{f = \bar{x}_i \oplus \text{product of the remaining variables}$ 
    else if  $(s_i > 0)$ 
       $\{f = x_i \oplus \text{product of the remaining variables}\}$ 

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Note: The second order coefficients identify if any variables in the product term are in negation.

Example 1 Consider the function $f(x_4, x_3, x_2, x_1) = \Sigma(2, 3, 6, 7, 10, 11, 13, 14)$. $s_0 = 0$; Calculating the first order coefficients, $s_1 = s_3 = s_4 = 0$; $s_2 = 12$. The values of the first order coefficients indicate that there is only one SC of first order that is not zero and all the other first order SCs are zero. As the values of the first order SCs satisfy the condition, further calculating the other order SCs, $s_{12} = s_{23} = s_{24} = s_{1234} = 4$; $s_{123} = s_{124} = s_{234} = -4$. Hence any $|s_1| + |s_i| = 2^n = 16$. As the values have satisfied the conditions and as $s_2 = 12$, the function can be represented as: $x_2 \oplus x_1 x_3 x_4$.

Case 2: (EEE) & (AEE)

Case 2a: (EEE)

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if  $((s_0 = 0) \text{ and } (s_i = 0))$ 
  if  $(n\text{-th order SC} = 2^n) \{f = \oplus \text{ of all the variables}\}$ 
  else if  $(n\text{-th order SC} = -2^n)$ 
     $\{f = \oplus \text{ of all the variables out of which odd number are}$ 
     $\text{in negation}\}$ 

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Example 2 Consider the function $f(x_4, x_3, x_2, x_1) = \Sigma(1, 2, 4, 7, 8, 11, 13, 14)$. As $s_0 = 0$ and all the first order SCs are zero, and $s_{1234} = 16$, the function is represented as: $x_1 \oplus x_2 \oplus x_3 \oplus x_4$. As s_{1234} is positive, there are no variables in negation in the representation of the function. Also this function can be represented as the EXOR sum of even number of negated variables.

Case 2b: (AEE)

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if  $((s_0 = 0) \text{ and } (s_i = 0))$ 
   $\{j = 2; \text{ while } (j \leq n) \{ \text{Calculate all the SCs corre-}$ 
   $\text{sponding to } j;$ 
  if  $((\text{any one } s_i \neq 0) \text{ and } (\text{all the other SC for that } j\text{-th}$ 
   $\text{order} = 0))$ 
     $\{I = X; \text{ break; else } j++;\}$   $k = j;$ 
  if  $(X < n) \{ \text{Calculate all the SCs corresponding to}$ 
   $j + 1;\}$ 
  if  $((X \in I) \text{ and } (s_1 \neq 0)) \{j++; \text{ else break;}\}$ 
  if  $(j = n + 1)$ 
     $\{f = (\oplus \text{ of all the variables of the index } X) \oplus (\wedge \text{ of}$ 
     $\text{the remaining variables})\}$ 

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Example 3 A four variable function represented as, $f(x_4, x_3, x_2, x_1) = \Sigma(3, 4, 5, 6, 8, 9, 10, 15)$ is considered. The DC coefficient and all the first order SCs are zero. Hence proceeding further to calculate the second order coefficients, $s_{34} = 8$ and all the other SCs of second order are zero. Similarly, $s_{134} = s_{234} = 8$ and $s_{1234} = -8$. All the other SCs are zero. As this function satisfies the condition for Case 2b, the function is represented as $x_1 x_2 \oplus (x_3 \oplus x_4)$. Also as 34 is the index of the second order SC that is not zero, the variables x_3 and x_4 are represented as the EXOR function.

Case 3: (AAA) & (OOO)

```

if  $((s_0 \neq 0) \text{ and } (s_i = \pm 2) \text{ and } (2^n = |s_0| + 2))$ 
   $\{j = 2; \text{ while } (j \leq n) \{ \text{Calculate all the SCs of order } j;$ 
  if  $(|s_j| \neq 2) \{ \text{break;}\}$ 
  else } j++;
  if  $(j = n + 1) \text{ and } (s_0 < 0)$ 
     $\{f = \wedge \text{ of all the variables}\}$  (AAA)
  if  $((j = n + 1) \text{ and } (s_0 > 0))$ 
     $\{f = \vee \text{ of all the variables}\}$  (OOO)

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Note: x_i is in negation for all $s_i = -2$.

Example 4 Considering a four variable function, $f(x_4, x_3, x_2, x_1) = \Sigma(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)$, the DC coefficient is -14 and all the other SC are 2. Neglecting the sign of the DC coefficient, $|s_0| + |\text{any } s_i| = 16$. Also as the DC coefficient is negative, the function is represented as $x_1 \vee x_2 \vee x_3 \vee x_4$.

Case 4: (AAO) & (OOA)

Case 4a:

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if  $((s_0 \neq 0) \text{ and } (s_i = \pm X) \text{ and } (2^n - |s_0| = (|X| + 1)^* 2) \text{ and } (n \text{ is even}))$ 
   $\{j = 2; \text{while } (j \leq n) \{ \text{Calculate all the SCs};$ 
  if  $((s_i \neq |X|) \text{ and } (s_i \neq \pm 2)) \{ \text{break}; \}$ 
  if  $((j = n/2) \text{ and } (s_i = |X|)) \{ I = Y; j++; \}$ 
  if  $(j = n + 1) \text{ and } (s_0 > 0)$ 
     $\{ f = (\wedge \text{ all the variables of } Y) \vee (\wedge \text{ the remaining variables}) \}$  (AAO)
  if  $((j = n + 1) \text{ and } (s_0 < 0))$ 
     $\{ f = (\vee \text{ all the variables of } Y) \wedge (\vee \text{ the remaining variables}) \}$  (OOA)

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Note: x_i is in negation for all i where $s_i < 0$.

Case 4b:

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if  $((s_0 \neq 0) \text{ and } (s_i = \pm X \text{ or } \pm Y) \text{ and } (2^n - |s_0| = |X| + |Y| + 2))$ 
   $\{j = 2; \text{while } (j \leq n) \{ \text{Calculate all the SCs};$ 
  if  $((s_i \neq |X|) \text{ and } (s_i \neq |Y|) \text{ and } (s_i \neq \pm 2)) \{ \text{break};$ 
  else  $j++; \}$ 
  if  $((j = n + 1) \text{ and } (s_0 > 0))$ 
     $\{ f = (\wedge \text{ all the variables whose } s_i = \pm X) \vee (\wedge \text{ all the variables whose } s_i = \pm Y) \}$  (AAO)
  if  $((j = n + 1) \text{ and } (s_0 < 0))$ 
     $\{ f = (\vee \text{ of all the variables whose } s_i = \pm X) \wedge (\vee \text{ of all the variables whose } s_i = \pm Y) \}$  (OOA)

```

Note: Any x_i is in negation if $s_i < 0$.

Example 5 A four variable function $f(x_4, x_3, x_2, x_1) = \Sigma(3, 7, 12, 13, 14, 15)$ is considered. The DC coefficient is 2 and all the first order coefficients have the value 6. Further calculating the second order coefficients, $s_{12} = s_{34} = -6$ and all the other second order SCs are 2. All the third and fourth order SC are -2 and 2, respectively. Hence the function is represented as: $x_1 x_2 \vee x_3 x_4$. This is an AAO decomposition from Case 4a since the DC spectral coefficient is greater than zero.

Case 5: (AOA) & (OAO)

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if  $((s_0 \neq 0) \text{ and } (s_i = \pm X \text{ or } \pm 2) \text{ and } (2^n = |s_0| + |X|))$ 

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   $\{j = 2; \text{while } (j \leq n) \{ \text{Calculate all the SCs};$ 
  if  $((s_i \neq |X|) \text{ and } (s_i \neq \pm 2)) \{ \text{break}; \text{else } j++; \}$ 
  if  $(j = n + 1) \text{ and } (s_0 > 0)$ 
     $\{ f = (\wedge \text{ of all the variables whose } s_i = \pm X) \wedge (\vee \text{ of all the remaining variables}) \}$  (AOA)
  if  $((j = n + 1) \text{ and } (s_0 < 0))$ 
     $\{ f = (\vee \text{ of all the variables whose } s_i = \pm X) \vee (\wedge \text{ of all the remaining variables}) \}$  (OAO)

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Note: Any x_i is in negation if $s_i < 0$.

Example 6 Consider a four variable function given by: $f(x_4, x_3, x_2, x_1) = \Sigma(1, 2, 3, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15)$. The DC coefficient is -10 and the first order SCs with the index 1 and 2 are 6 and those with the index 3 and 4 are 2. Hence calculating the second order SCs $s_{12} = 6$ and $s_{34} = -2$ and all the other SCs are 2. $s_{123} = s_{124} = 2$, $s_{134} = s_{234} = s_{1234} = -2$. Hence the function can be represented as $x_1 \vee x_2 \vee x_3 x_4$.

Case 6: (AAE) & (OAE)

Case 6a:

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if  $((s_0 \neq 0) \text{ and } (s_i = \pm X) \text{ and } (2^n - |s_0| = (|X| + 2)^* 2) \text{ and } (n \text{ is even}))$ 
   $\{j = 2; \text{while } (j \leq n) \{ \text{Calculate all the SCs of order } j;$ 
  if  $((s_i \neq |X|) \text{ and } (s_i \neq 4)) \{ \text{break}; \}$ 
  if  $((j = n/2) \text{ and } (s_i = |X|)) \{ I = Y; j++; \}$ 
  if  $(j = n + 1) \text{ and } (s_0 > 0)$ 
     $\{ f = (\wedge \text{ of all the variables of } Y) \oplus (\wedge \text{ of the remaining variables}) \}$  (AAE)
  if  $((j = n + 1) \text{ and } (s_0 < 0))$ 
     $\{ f = (\vee \text{ of all the variables of } Y) \oplus (\wedge \text{ of the remaining variables}) \}$  (OAE)

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Note: (i) x_i is in negation for all $s_i < 0$. (ii) The AAE can also be represented as an OOE decomposition but the only change is that, that all the variables are complemented.

Case 6b:

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if  $((s_0 \neq 0) \text{ and } (s_i = \pm X \text{ or } \pm Y) \text{ and } (2^n - |s_0| = |X| + |Y| + 4))$ 
   $\{j = 2; \text{while } (j \leq n) \{ \text{Calculate all the SCs of order } j;$ 
  if  $((s_i \neq |X|) \text{ and } (s_i \neq |Y|) \text{ and } (s_i \neq \pm 4) \text{ and } (j \leq n/2 + 1) \text{ and } (n \text{ is even})) \{ \text{break}; \}$ 
  if  $((s_i \neq \pm 4) \text{ and } (j > n/2 + 1) \text{ and } (n \text{ is even})) \{ \text{break}; \}$ 
  if  $((s_i \neq |X|) \text{ and } (s_i \neq |Y|) \text{ and } (s_i \neq \pm 4) \text{ and } (j \leq n + 1/2) \text{ and } (n \text{ is odd})) \{ \text{break}; \}$ 
  if  $((s_i \neq \pm 4) \text{ and } (j > (n + 1)/2) \text{ and } (n \text{ is odd})) \{ \text{break}; \}$ 
  if  $(j = n + 1) \text{ and } (s_0 > 0)$ 
     $\{ f = (\wedge \text{ of all the variables whose } s_i = \pm X) \oplus (\wedge \text{ of all the variables whose } s_i = \pm Y) \}$  (AAE)
  if  $(j = n + 1) \text{ and } (s_0 < 0)$ 

```

TABLE I The possible number of decompositions of a specific type for Boolean functions

Type of decomposition	# of such decompositions possible	Type of decomposition	# of such decompositions possible
OOA, AAO	$\sum_{i=1}^n C_i^n 2^n$	AOO	$\sum_{i=2}^{n-2} C_i^n 2^n$
AEA, OEA, OEO, AEO	$\sum_{i=1}^n C_i^n 2^{i+1}$	EEA, EEO	$4 \sum_{i=2}^{n-2} C_i^n$
AOA	$\sum_{i=1}^n C_i^n 2^n$	AEE	$\sum_{i=2}^{n-2} C_i^n 2^{i+1}$
AAE	$n 2^n$	AAA, OOO	2^n
EEE	2		

$\{f = (\vee \text{ of all the variables whose } s_i = \pm X) \oplus (\wedge \text{ of all the variables whose } s_i = \pm Y)\}$ **(OAE)**

Note: Any x_i is in negation if $s_i < 0$.

Example 7 Consider a four variable function given by: $f(x_4, x_3, x_2, x_1) = \Sigma(3, 7, 11, 12, 13, 14)$. The DC coefficient is 4 and as all the other SCs are 4 or -4, which follows all the conditions as outlined for Case 6a, the function can be represented as $x_1 x_2 \oplus x_3 x_4$.

Case 7: (AEA) & (OEO)

if $((s_0 > 0) \text{ and } (s_i = \pm X \text{ or } 0) \text{ and } (2^n = |s_0| + |X|))$
 $\{j = 2; \text{ while } (j \leq n) \{ \text{Calculate all the SCs of order } j;$
if $((s_l \neq |X|) \text{ and } (s_l \neq 0)) \{ \text{break}; \}$
else if $((\text{only one } s_l = -|X| \text{ and all other } s_l \text{ are zero})$
and $(n \text{ is odd})) \{k = 1; \}$
else if $((n \text{ is even}) \text{ and } (s_l = -X, \text{ where } i \text{ for which } s_i = \pm X \notin I)) \{k = 1; \}$
 $j++;$
if $(j = n + 1) \text{ and } (s_0 > 0)$
 $\{f = (\wedge \text{ of all the variables whose } s_i = \pm X) \wedge (\oplus \text{ of all the other variables})\}$ **(AEA)**
if $(j = n + 1) \text{ and } (s_0 < 0)$
 $\{f = (\vee \text{ of all the variables whose } s_i = \pm X) \vee (\oplus \text{ of all the other variables})\}$ **(OEO)**
if $(k = 1) \{ \text{odd number of } x_i, \text{ whose } s_i = 0 \text{ are in negation} \} \}$

Note: Any x_i is in negation if $s_i < 0$.

Example 8 The four variable function is represented as: $f(x_4, x_3, x_2, x_1) = \Sigma(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15)$. The DC coefficient is -12 and further calculating the first

order coefficients, $s_1 = s_2 = 4; s_3 = s_4 = 0$; The second order coefficients are: $s_{12} = s_{34} = 4$ and all the other SCs are zero. Similarly, the other coefficients that are non zero of third and fourth order are s_{134}, s_{234} and s_{1234} . As the conditions outlined for Case 7 are satisfied, the function can be represented as $(x_1 \vee x_2) \vee (x_3 \oplus x_4)$.

Case 8: (OEA) & (AEO)

if $((s_0 \neq 0) \text{ and } (s_i = \pm X \text{ or } 0) \text{ and } (s_0 = |A|))$
 $\{j = 2; Y = 2^n - |X|; \text{ while } (j \leq n) \{ \text{Calculate all the SCs};$
if $((s_l \neq |X|) \text{ and } (s_l = 0) \text{ and } (s_l \neq |Y|)) \{ \text{break}; \}$
else if $((\text{only one } s_l = -|Y|) \text{ and } (\text{all other } s_l \text{ are zero or } \pm X))$
 $k = 1; j++;$
if $((j = n) \text{ and } (s_0 + Y = 2^n) \text{ and } (s_0 > 0))$
 $\{f = (\vee \text{ of all the variables whose } s_i = \pm X) \wedge (\oplus \text{ of all the other variables})\}$ **(OEA)**
if $((j = n) \text{ and } (s_0 + Y = 2^n) \text{ and } (s_0 < 0))$
 $\{f = (\wedge \text{ of all the variables whose } s_i = \pm X) \vee (\oplus \text{ of all the other variables})\}$ **(AEO)**
if $(k = 1) \{ \text{odd number of } x_i, \text{ whose } s_i = 0 \text{ are in negation} \} \}$

Note: Any x_i is in negation if $s_i < 0$.

Example 9 The four variable function is represented as: $f(x_4, x_3, x_2, x_1) = \Sigma(5, 6, 7, 9, 10, 11)$. The DC coefficient is 4 and the first order coefficients, $s_1 = s_2 = 4; s_3 = s_4 = 0$; The non zero second order coefficients are: $s_{12} = 4$ and $s_{34} = 12$ and all the other SC are zero. The third and fourth order non zero SCs are s_{134}, s_{234} , and s_{1234} and their value are -4. As the conditions outlined for Case 8 are

TABLE II Experimental results

Standard benchmark function	No. of inputs	No. of outputs	Run time (s)	No. of functions with bi-decomposition
Xor5	5	1	0.03	1
Cml38a	6	8	0.03	3
Ex5	8	63	0.35	15
Exps	8	38	0.21	4
Luc	8	27	0.04	1
Rd84	8	4	0.04	2
Br1	12	8	0.21	1
Br2	12	8	0.18	2

satisfied, the function is represented as $(x_1 \vee x_2) \wedge (x_3 \oplus x_4)$

Case 9: (EEA) & (EEO)

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if ( $s_0 = \pm 2^n/2$ )
   $\{j = 1; k = 0; neg = 0; \text{while } (j \leq n) \{ \text{Calculate all the}$ 
     $\text{SCs of } j\text{-th order;}$ 
    if ( $s_I = 2^n/2$ )
       $k++; X = I;$ 
    if ( $s_I = -2^n/2$ )  $\{k++; Y = I; neg++; \}$ 
    if ( $(s_I \neq \pm 2^n/2) \text{ and } (s_I \neq 0)$ )  $\{ \text{break; } j++; \}$ 
    if ( $(j = n) \text{ and } (k = 3) \text{ and } (s_0 = 2^n/2) \text{ and } (neg = 0)$ )
       $\{f = (\oplus \text{ of all the variables in } X) \wedge (\oplus \text{ of the}$ 
         $\text{remaining variables}) \quad \text{(EEA)}$ 
      if ( $(j = n) \text{ and } (k = 3) \text{ and } (s_0 = -2^n/2) \text{ and } (neg = 0)$ )
         $\{f = (\oplus \text{ of all the variables in } X) \vee (\oplus \text{ of the}$ 
           $\text{remaining variables}) \}$  (EEO)

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Note: Odd number of variables in g_2 are in negation if $neg = 1$, odd number of variables in both the groups g_1 and g_2 are in negation if $neg = 2$.

Example 10 The four variable function is represented as: $f(x_4, x_3, x_2, x_1) = \Sigma(5, 6, 9, 10)$. The DC coefficient is 8 and all the first order coefficients are zero. The non zero second order coefficients are: $s_{12} = s_{34} = 8$ and all the other SC are zero. The fourth order non zero SC is -8 . As the conditions outlined for Case 9 are satisfied, the function is represented as $(x_1 \oplus x_2) \wedge (x_3 \oplus x_4)$.

ENUMERATION OF BI-DECOMPOSITIONS

For any given n -variable Boolean function the number of decompositions of a particular type is listed in the Table I.

EXPERIMENTAL RESULTS

The algorithm described in the “Algorithm to identify type of decomposition” section was implemented in C and run on the HP 9000 workstation. The standard benchmark logic functions have been considered for testing. The results are shown in Table II. Columns in Table II from left to right represent the benchmark function, the number of input variables for that function, the run time in seconds, and the number of outputs of the same benchmark function that has any one of the bi-decompositions discussed here.

CONCLUSION

Spectral conditions in the Walsh domain for the identification of existence and type of the bi-decomposition for Boolean functions have been presented. The new algorithm is efficient in finding the type of bi-decomposition and its exact logical sub-functions. By using a filtering procedure, the algorithm can eliminate the lack of existence of the bi-decomposition without the necessity of calculating and operating on the whole Walsh spectrum. This work extended the Walsh spectral theory of standard types of decomposition known earlier [6,8,9,15] to the bi-decomposition. Consideration of various types of bi-decomposition is important not only from theoretical but also from practical point of view as there are many recent programmable logic devices that can be efficiently used for direct implementation of types of bi-decompositions discussed in this article.

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