SYNERGISTIC MODELING OF CALL CENTER OPERATIONS

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We synergistically apply queueing theory, integer programming, and stochastic simulation to determine an optimal staffing policy for a repair call handling center. A stationary Markovian queueing model is employed to determine minimal staffing levels for a sequence of time intervals with varying call volumes and mean handling times. These staffing requirements populate an integer program model for determining the mix of call agent shifts that will achieve service quality standards at minimum cost. Since the analytical modeling requires simplifying assumptions, expected performance of the optimal staffing policy is evaluated using stochastic simulation. Computational efficiency of the simulation is improved dramatically by employing the queueing model to generate an analytic control variate.

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1. Introduction

Many commercial enterprises and public agencies operate centralized call centers to provide effective and responsive service for patrons. For example, communication service providers operate call centers to ensure timely restoration of service following equipment failures within the communications network or at the customer premises. The call center is staffed by repair service agents who are trained to effectively interact with the customer, diagnose the problem, and dispatch appropriate repair resources. Typically, the goal is to completely restore service within a few hours.

The operating cost of a repair call center is dominated by personnel expense, so the economic efficiency of the system is determined almost entirely by the quality of the agent scheduling process. The scheduling problem is characterized by a highly variable demand pattern and a requirement to schedule agents in shifts that are constrained by labor rules. Fortunately, the weekly demand profile is quite predictable and seasonally consistent. The fundamental challenge is to schedule agent shifts such that resulting agent availability will enable consistent achievement of a specified service level at minimum cost.

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The importance of the call center scheduling problem is indicated by a large body of relevant literature (see Gans et al. [8]). Reported application areas include retail sales (Andrews and Parsons [1]), transportation (Linder [16]), public services (Harris et al. [11]), and the telecommunications industry (Buffa et al. [5], Church [7]). Solution approaches have incorporated diverse operations research methods such as mathematical programming (Burns and Carter [6], Segal [21]), analytical queueing models (Sze [23]), simulation (Paul and Stevens [18]), and various heuristic procedures (Baker [2], Henderson and Berry [12]). Brigandi et al. [4] document deployment of a call center modeling system that delivered $750 million in increased profits for a diverse set of client enterprises in a single year. The system relied on simulation as the primary modeling tool, but used a queueing model to calculate agent requirements and network flow programming to determine optimal schedules. In this paper, we synergistically apply queueing theory, integer programming, and simulation to derive and evaluate an optimal staffing policy for repair call handling. We employ a Markovian queueing model to obtain a conservative minimum staffing level for each service interval. These staffing requirements populate constraints in an integer programming formulation that selects an optimal mix of agent shifts. The analytical solution is evaluated using a simulation model that relaxes the required assumptions of exponential handling times and stationary conditions. The simulation application is unique in that we employ the analytical queueing model to generate a novel type of control variate, resulting in substantial improvement in computational efficiency.

2. Analytical modeling and optimization

The demand profile for a repair call handling center can accurately be predicted from historical data and appropriate forecasting techniques. Figure 2.1 displays the expected call volume for each of the 336 30-minute intervals within a single week, along with the average handling time for calls arriving within each interval. Variability in realized call volume within an interval can be treated as random, so the customer arrival process can be modeled as a nonstationary Poisson process with an expected number of arrivals n_i for each interval i. The schedule must ensure that sufficient agents are assigned on each interval to satisfy an overall quality of service requirement. For scheduling purposes, quality of service is narrowly defined as the probability that a random customer will not wait more than a specified time for agent contact.

In any given week, repair service agents must be scheduled in shift groupings called "tours" that span multiple intervals. Each agent is assigned to one tour from a specified set T, and any number of agents can be assigned to any tour $j \in T$. Each tour is uniquely characterized by its combination of start time, workday schedule, and shift type (standard or split). The standard shift consists of 8.5 consecutive hours (4 hours of work, 30 minutes off, and another 4 hours of work), whereas the split shift consists of 12 consecutive hours (4 hours of work, 4 hours off, and another 4 hours of work). There are 48 different tour start times, which are spaced at 30-minute intervals starting at midnight. Finally, there are seven different workday schedules. Each schedule has five days of work and two consecutive days off. Since we require that a shift must have the same start time for every workday, our basic tour set consists of $2 \times 48 \times 7 = 672$ different tours. This set may be

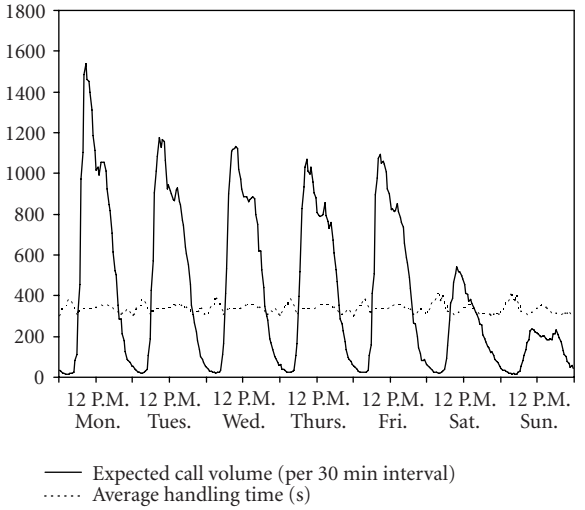

Figure 2.1. Expected call volume and handling time for a typical week.

modified or augmented to facilitate evaluation of alternative scheduling approaches (e.g., inclusion of overtime, nonconsecutive days off, or short tours for part-time agents).

The split tours (denoted $S \subset T$) are included in the tour set because standard shifts alone tend to specify an inefficient set of basis functions for accommodating the call volume profile. Not suprisingly, agents generally prefer standard tours, so our model includes the capability to limit the portion of agents p who are assigned to split tours. For example, setting $p = 0.20$ will ensure that a typical agent will be assigned to no more than one split tour every five weeks.

The tour scheme makes it impractical to exactly match staffing levels to the expected call volume on every interval. We require that a random customer waits no longer than a specified time t (e.g., 20 s) with probability α (e.g., 0.80). Service quality below α may be acceptable on some intervals provided the weekly average exceeds α. However, since reasonable uniformity in service quality is desirable, we specify an associated minimum standard $\phi \le \alpha$ that must be achieved on *every* interval. The corresponding staffing requirement s_i for a particular interval i can be approximated by assuming exponentially distributed handling times (with mean of h_i s) and stationary conditions (with call arrival rate of n_i per interval or $n_i/1800$ per second). Letting W_i represent customer waiting time, we initialize the staffing level at $s_i = \lceil n_i h_i/1800 \rceil$ and then increment s_i until $P(W_i \le t) \ge \phi$. Computation of service quality follows from well-known analytical results for multichannel Markovian queues (e.g., see Gross and Harris [10, pages 69–72]):

$$\begin{aligned} P(W_i \le t) &= F(t, n_i, h_i, s_i) \\ &= 1 - \left\{ \frac{(n_i h_i/1800)^{s_i} \exp\left[(n_i/1800 - s_i/h_i)t\right]}{(s_i - 1)!(s_i - n_i h_i/1800)} \right\} \\ &\quad \times \left\{ \sum_{k=0}^{s_i - 1} \frac{(n_i h_i/1800)^k}{k!} + \frac{(n_i h_i/1800)^{s_i}}{(s_i - 1)!(s_i - n_i h_i/1800)} \right\}^{-1}. \end{aligned} \tag{2.1}$$

The assumption of independent stationary operation on each interval may result in understatement of a particular staffing requirement (Green et al. [9]), but this effect is overwhelmed by the assumption of exponential handling times. A more realistic handling time distribution approximates a symmetric triangular form with mean h_i and variance $h_i^2/24$ (much lower than the exponential variance h_i^2), so the analytically derived estimate of minimum s_i is predominately conservative. The aggregate conservatism of our analytical results is verified in subsequent simulation-based performance modeling.

The operational objective is to assign repair service agents to tours such that a staffing level of at least s_i is achieved on each interval, while minimizing the total number of agent-hours assigned for the week. The ancillary restriction on the frequency of split tours must also be enforced. To satisfy these requirements, we formulate a simple integer program (Wolsey [24]). We define the decision variable x_j as the number of agents assigned to tour j. Letting I_j be the set of intervals covered by tour j (with cardinality $|I_j|$), we write the formulation.

$$\begin{aligned}
&\text{Minimize}\\
&\sum_{j\in T} |I_j|\, x_j\\
&\text{subject to}\\
&\sum_{j\in T: i\in I_j} x_j \geq s_i, \quad i = 1,\dots,336,\\
&\sum_{j\in S} x_j \leq p \sum_{j\in T} x_j,\\
&x_j = 0,1,\dots, \quad j \in T.
\end{aligned} \tag{2.2}$$

By employing commercial optimization software (CPLEX), we can solve the model in a few seconds on a personal computer. Staffing levels resulting from the optimal tour assignment are derived as

$$s_i' = \sum_{j\in T: i\in I_j} x_j, \quad i = 1,\dots,336. \tag{2.3}$$

This solution follows from a particular value of ϕ. To ensure that aggregate service quality meets the specified standard, we iteratively adjust the global parameter ϕ, recalculate each s_i, and resolve the integer program until

$$A(t,n,h,s') = \frac{\sum_{i=1}^{336} n_i F(t,n_i,h_i,s_i')}{\sum_{i=1}^{336} n_i} \approx \alpha, \tag{2.4}$$

where n, h, and s' are the respective vector representations of n_i, h_i, and s_i', $i = 1,\dots,336$.

Table 2.1 displays the expected call volume, average handling times, minimum staffing requirements, optimal staffing levels, and quality of service estimates for the first 24 scheduling intervals shown in Figure 2.1 (Monday A.M.). All staffing results are based on $t = 20$ seconds, $\phi = 0.50$, and our basic tour set with $p = 0.20$; the corresponding aggregate

Table 2.1. Interval parameters and analytical results for Monday A.M.

Interval i	Call volume n_i	Handling time h_i	Minimum staff s_i	Optimal staff s'_i	Service quality $F(20, n_i, h_i, s'_i)$
1 (0 : 00)	33.1	299.8	7	10	0.953
2 (0 : 30)	30.3	309.5	7	10	0.966
3 (1 : 00)	20.7	336.0	5	11	0.998
4 (1 : 30)	20.7	349.3	6	10	0.994
5 (2 : 00)	16.6	348.0	5	10	0.999
6 (2 : 30)	15.2	347.0	4	10	0.999
7 (3 : 00)	15.2	385.0	5	10	0.999
8 (3 : 30)	17.9	379.1	5	17	1.000
9 (4 : 00)	17.9	358.9	5	12	1.000
10 (4 : 30)	25.5	341.7	6	17	1.000
11 (5 : 00)	80.0	320.4	16	34	1.000
12 (5 : 30)	111.0	303.0	21	62	1.000
13 (6 : 00)	317.9	322.8	60	99	1.000
14 (6 : 30)	456.5	324.1	86	141	1.000
15 (7 : 00)	971.0	334.2	185	204	0.987
16 (7 : 30)	1100.6	334.7	210	233	0.994
17 (8 : 00)	1482.0	334.6	281	281	0.532
18 (8 : 30)	1537.2	333.0	290	290	0.535
19 (9 : 00)	1463.4	335.5	278	279	0.582
20 (9 : 30)	1450.3	335.7	276	277	0.597
21 (10 : 00)	1402.0	335.7	267	271	0.745
22 (10 : 30)	1308.9	334.9	249	250	0.606
23 (11 : 00)	1185.5	336.2	227	228	0.625
24 (11 : 30)	1114.4	338.5	215	222	0.858

service quality is $A(t, n, h, s') = 0.798$. The optimal solution assigns 306 agents to 59 standard tours (with 28 unique start times), and 76 agents to 12 split tours (with 8 unique start times). All seven off-day schedules are employed, with the largest group of agents having Saturday and Sunday off (224), and only three agents having Monday and Tuesday off. Overall efficiency of the tour set in covering staffing requirements can be computed as

$$\frac{\sum_{i=1}^{336} s_i}{\sum_{i=1}^{336} s'_i} = \frac{27365}{30560} = 0.895. \tag{2.5}$$

A full-week comparison of the minimum staffing requirements s and optimal staffing levels s' is presented in Figure 2.2.

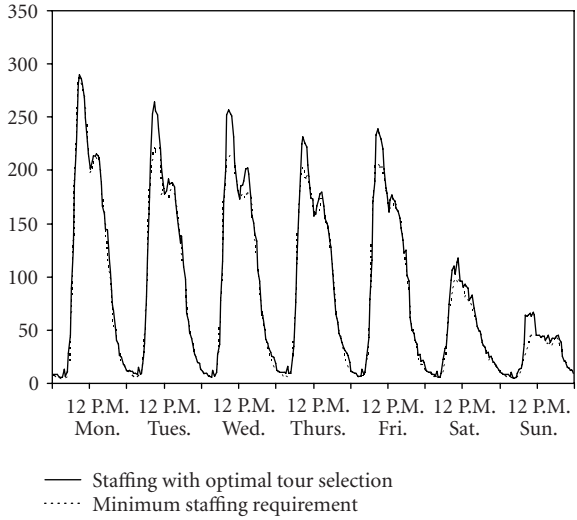

Figure 2.2. Minimum staffing requirements and optimal tour-based staffing.

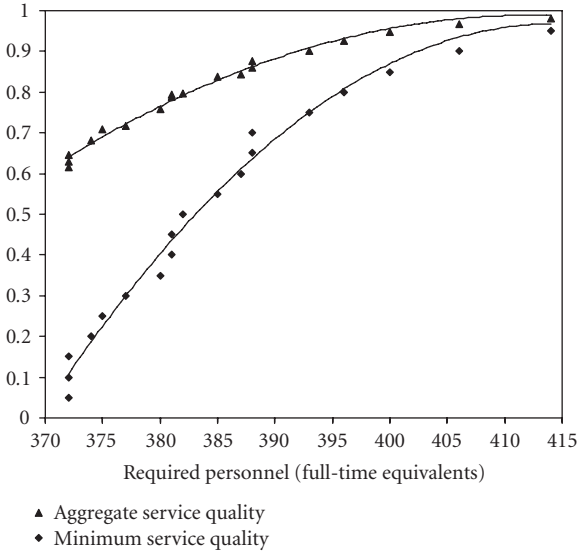

Figure 2.3. Relationship between service quality and required personnel.

Figure 2.3 illustrates the relationship between minimum service quality ϕ, aggregate service quality $A(t,n,h,s')$, and the total personnel requirement. The personnel requirement is expressed in 40-hour "full-time equivalents" ($\sum_{i=1}^{336} s_i'/80$). Diminishing returns for increased staffing are apparent, though it is noteworthy that aggregate service quality could be increased from 0.80 to 0.90 with only a 3% increase in total personnel. Note that if staffing levels could be idealized on every interval ($s' = s$), the plots for minimum

and aggregate service quality would be identical. The difference between the two plots is therefore an informative indicator of tour set efficiency.

Other relationships can be conveniently studied using the analytical modeling. For example, sensitivity analysis of the split tour restriction reveals that not many split tours are needed to achieve almost all of the realizable benefit. The solution associated with $p = 0.20$ yields a personnel requirement of 382, which represents only a slight increase over the 378 personnel required to achieve the same aggregate service quality when split tours are completely unrestricted ($p = 1$). The corresponding personnel requirement when no split tours are permitted ($p = 0$) is 444. Similar insights on other scheduling issues could be obtained by including additional constraints in the integer program (e.g., overtime restrictions).

3. Analytically controlled simulation

The analytical models described in the previous section require some simplifying assumptions, so we are compelled to employ stochastic simulation to more accurately predict performance of the optimal staffing policy. Unlike the analytical model, the simulation captures the nonstationary arrival process and generates call handling times from appropriate triangular distributions. The service quality response is produced by explicitly tracking the portion of arriving customers who begin service within t seconds after their arrival. The simulation must be applied on a repetitive basis since each week offers a new call volume profile and the available tour set may be arbitrarily modified. For every case considered, we must replicate the simulation to generate statistical confidence limits on the mean response. We are therefore interested in maximizing computational efficiency through an appropriate variance reduction technique. Fortunately, we can leverage our existing analytical model by employing a *control variate* approach to variance reduction (Lavenberg and Welch [14]).

Our objective is to obtain a precise estimate of $\mu_Y = E[Y]$, where the random variable Y is the simulation response for aggregate service quality. To reduce variance in the estimate of μ_Y, we will exploit another random variable, Z, which is correlated with Y and has known expectation μ_Z. A new *controlled* response

$$Y(b) = Y - b(Z - \mu_Z) \tag{3.1}$$

can thus be constructed for each simulation replication, where the constant b is called the *control coefficient*. The variance of $Y(b)$ is given by

$$\operatorname{Var}[Y(b)] = \operatorname{Var}[Y] + b^2 \operatorname{Var}[Z] - 2b \operatorname{Cov}[Y,Z], \tag{3.2}$$

and the value of b that minimizes $\operatorname{Var}[Y(b)]$, which can be found by differentiating (3.2), is

$$\beta = \frac{\operatorname{Cov}[Y,Z]}{\operatorname{Var}[Z]}. \tag{3.3}$$

The true value of $\mathrm{Cov}[Y,Z]$ is unknown but, for r replications, β can be estimated by

$$\hat{\beta} = \frac{\sum_{j=1}^{r}(Y_j - \bar{Y})(Z_j - \bar{Z})}{\sum_{j=1}^{r}(Z_j - \bar{Z})^2}, \tag{3.4}$$

where $\bar{Y}$ and $\bar{Z}$ are the respective means of the r observations of Y and Z. A controlled estimate of μ_Y can then be obtained as

$$\bar{Y}(\hat{\beta}) = \frac{1}{r}\sum_{j=1}^{r}[Y_j - \hat{\beta}(Z_j - \mu_Z)]. \tag{3.5}$$

The approach can be extended to exploit multiple random variables that are correlated with Y and have known expectation.

Traditional control variate methods are classified as either *internal* or *external*. The internal method is poorly suited for our application since we have a very large number of equally valid control variate candidates (336 call volumes and 336 mean handling times) (Lavenberg and Welch [14]). The external method is also inappropriate because the assumption of stationary operation makes it impractical to create an efficient auxiliary simulation that corresponds exactly with our analytical model (a methodological requirement). Instead, we employ the *analytic* control variate (ACV) concept first proposed by Nelson [17]. This approach can be considered a hybrid of both internal and external methods since an external analytical model is used to produce a single control variate that is a function of multiple internal random variables.

To illustrate how our ACV is generated, consider a particular simulation replication with response Y. Let $N = (N_1, N_2, \ldots, N_{336})$ be the vector of generated call volumes with $E[N] = (n_1, n_2, \ldots, n_{336})$. Similarly, let $H = (H_1, H_2, \ldots, H_{336})$ be the vector of realized *mean* handling times with $E[H] = (h_1, h_2, \ldots, h_{336})$. The analytical result $Z = A(t, N, H, s')$ should be highly correlated with Y, and is therefore well suited for employment in (3.5). We note that if $N_i H_i/1800 \geq s_i'$ for some i, we must let $F(t, N_i, H_i, s_i') = 0$ in the computation of Z.

We now require a value for μ_Z. If the analytical result Z was a linear function of N and H, then the linearity of the expectation operator would ensure that $\mu_Z = E[A(t,N,H,s')] = A(t, E[N], E[H], s') = A(t,n,h,s')$. Unfortunately, the analytical model is nonlinear, so computation of μ_Z in this manner will produce a biased result (Sharon and Nelson [22]). Analytical determination of μ_Z is impractical, but Irish et al. [13] have shown that it can be accurately estimated through an empirical approach. Since the joint distribution of N and H is known, we can generate a very large number of static random samples from this distribution (denoted $\tilde{N}^k$, $\tilde{H}^k$, $k = 1, \ldots, m$), and then estimate μ_Z as

$$\hat{\mu}_Z = \frac{1}{m}\sum_{k=1}^{m} A(t, \tilde{N}^k, \tilde{H}^k, s') \tag{3.6}$$

since $\hat{\mu}_Z \to \mu_Z$ as $m \to \infty$.

Repair call arrivals occur according to a memoryless process on each interval i, so each call volume $\tilde{N}_i^k$ can be independently generated from a Poisson distribution with mean n_i (Ross [20, page 76]). The sampling of a mean handling time is more complex since the distribution of $\tilde{H}_i^k$ is dependent on the associated realization of $\tilde{N}_i^k$. We generate each

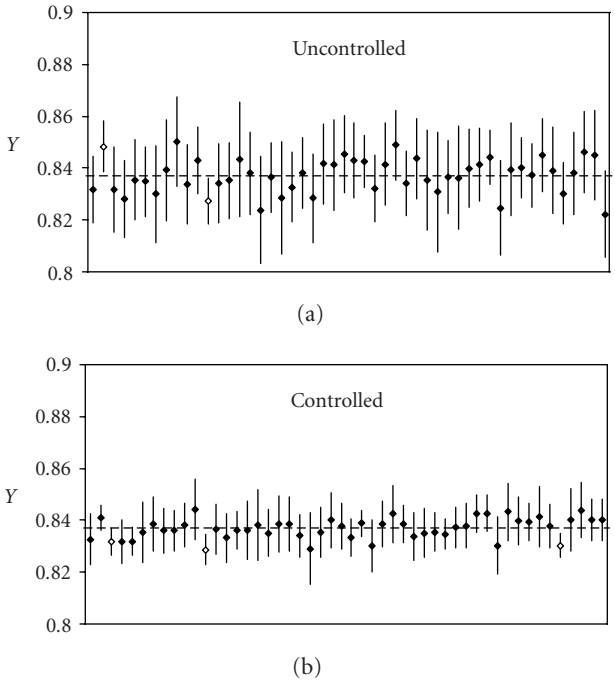

Figure 3.1. ACV performance for 50 batches of 10 replications each.

mean handling time as

$$\widetilde{H}_i^k = \begin{cases} \left(\widetilde{N}_i^k\right)^{-1} \sum_{j=1}^{\widetilde{N}_i^k} \mathrm{Tri}\left[h_i, \dfrac{h_i^2}{24}\right] & \text{if } \widetilde{N}_i^k \le 5, \\ \mathrm{Norm}\left[h_i, \dfrac{h_i^2}{24\widetilde{N}_i^k}\right] & \text{otherwise,} \end{cases} \tag{3.7}$$

where the functions $\mathrm{Tri}[\mu, \nu]$ and $\mathrm{Norm}[\mu, \nu]$ represent random samplings from symmetric triangular and normal distributions respectively (each with mean μ and variance ν). The employment of a normal approximation for $\widetilde{N}_i^k > 5$ in (3.7) balances statistical precision with computational efficiency by making appropriate use of the central limit theorem. Using this approach, we can obtain an accurate estimate of μ_Z (based on $m = 10^4$ samples) in about 245 seconds on a personal computer. In comparison, a single replication of the simulation consumes more than 297 seconds of run time on the same computer. We generate the required Poisson random variables using the procedure cited by Ross [19, pages 194-195], which is particularly efficient for large mean values. The triangular samples are produced using a standard inverse-transform method (Law and Kelton [15, page 469]), and the normal samples are obtained using the efficient numerical procedure offered by Beasley and Springer [3].

Figure 3.1 illustrates the efficacy of the ACV method by comparing uncontrolled and controlled 95% confidence intervals for fifty batches of ten replications each. The ACV

Table 3.1. Average efficiency results for different batch sizes.

Batch size	Number of batches	Correlation coefficient	Uncontrolled CI half-width	Controlled CI half-width	Percent reduction in CI half-width
5	100	0.930	0.0259	0.0138	46.7
10	50	0.912	0.0154	0.0089	42.0
20	25	0.910	0.0102	0.0061	40.6
50	10	0.906	0.0062	0.0038	39.2
100	5	0.905	0.0043	0.0027	38.9
500	1	0.909	0.0019	0.0012	38.6

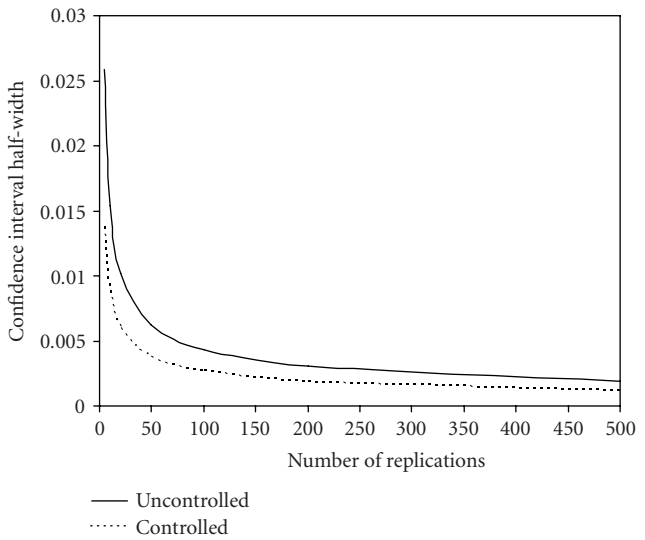

Figure 3.2. Graphical comparison of confidence interval half-widths.

method reduces the confidence interval half-width by an average of more than 40%. No evidence of appreciable bias in the controlled response is apparent. Using the uncontrolled mean response for all 500 simulation runs as a surrogate for truth (represented by the dotted line at $Y = 0.837$), we find that realized uncontrolled coverage (48/50 = 96%) and realized controlled coverage (47/50 = 94%) both approximate the nominal expectation.

Table 3.1 presents a summary of average efficiency results when the 500 runs are partitioned into different batch sizes. The estimated control coefficient maintains a value near 0.91, and substantial variance reduction is consistently achieved. The graphical depiction in Figure 3.2 highlights the realizable computational savings. Note that the same statistical precision achieved by 500 uncontrolled replications can be achieved by about 150 replications when the ACV method is applied (a 70% reduction). The computational

overhead required to execute the ACV method is minimal, and the only accommodation required within the simulation itself is the collection of generated values of N and H for each replication.

Applying the ACV method, a single batch of ten simulation runs yields a 95% confidence interval of 0.83 ± 0.01 on aggregate service quality. This result offers strong assurance that the staffing policy recommended by the analytical modeling is conservative and will provide service quality well in excess of 0.80. The computational cost of attaining this assurance (including the cost of estimating μ_Z) is about 54 minutes of run time. The computational burden is light enough that iterative application of the analytical and simulation models might be considered to develop a very comprehensive understanding of the relationship between service quality and personnel requirements.

4. Concluding remarks

We have synergistically applied a Markovian queueing model, integer program, and stochastic simulation to derive and evaluate staffing policies for a repair call handling center. The objective is to minimize total personnel requirements while strictly enforcing service quality standards. Due to the transient nature of the call center operation, only stochastic simulation can produce an accurate estimate of aggregate service quality. However, the large size of the candidate tour set precludes enumerative simulation of alternative policies. We therefore employ analytical modeling to obtain an approximate optimal solution, and then evaluate the performance of this solution through detailed simulation. While this approach is commonly adopted in operations research practice, we carry the synergy further by exploiting our analytical queueing model to substantially improve the computational efficiency of the simulation study. Computational savings of about 70% are realized when we apply an ACV method with empirical estimation of the control variate expectation. Our implementation demonstrates that this variance reduction technique can be applied to a range of problems beyond the queueing network examples reported in the current literature. Arguably, any simulation model is a candidate for the ACV method if we can formulate an approximate analytical model that captures key stochastic inputs, and if we can properly model dependencies between these inputs in estimating the control variate expectation.

We have implemented our analytical approach in call center operations at Qwest Communications. Specifically, we have produced monthly schedules for customer support of "designed services" (complex business accounts). In this application, we employ many weeks of historical data to obtain the shape of the call volume profile, and then scale this profile to reflect recent trends in total volume. We have also applied our methods to monitor and evaluate contracted call handling for a separate Qwest operation that delivers integrated video, data, and voice service via telephone lines in select geographic markets. Our current activities focus on supporting managerial efforts to efficiently maintain service quality standards. The telecommunications industry is extremely competitive, so high service quality and operational efficiency are both essential to future corporate success. In our experience, we observe a 10–15% reduction in personnel requirements from those produced by previous scheduling methods. Beyond operational utility, our methods have useful applications in strategic planning. For example, during labor negotiations, we have

applied our models to reveal the economic consequences of changes in contract restrictions on overtime or split tours. We can also provide analytical support for human resources planning that considers seasonal or macroeconomic fluctuation in demand. More generally, our synergistic approach to call center modeling could be applied to myriad application areas such as travel reservations, public services, and retail sales.

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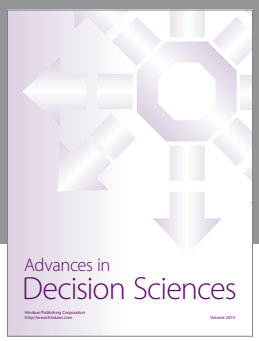

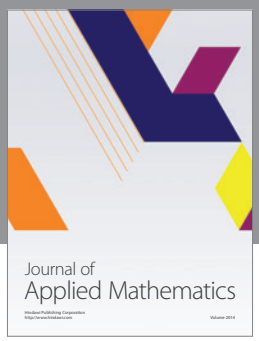

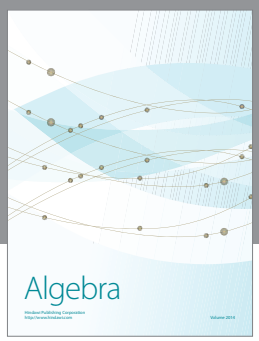

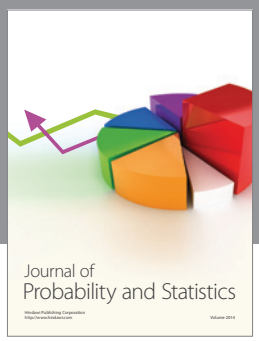

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