Scholarly Research Exchange

Volume 2008 • Article ID 105235 • doi:10.3814/2008/105235

Research Article

Advances in the Measurement of the Lense-Thirring Effect with Planetary Motions in the Field of the Sun

Lorenzo Iorio

Istituto Nazionale di Fisica Nucleare (INFN), Sezione di Pisa, 56127 Pisa, Italy

Correspondence should be addressed to Lorenzo Iorio, lorenzo.iorio@libero.it

Received 4 September 2008; Revised 22 September 2008; Accepted 11 October 2008

By processing more than 400 000 planetary observations of various types with the dynamical models of the EPM2006 ephemerides, E.V. Pitjeva recently estimated a correction to the canonical Newtonian-Einsteinian Venus' perihelion precession of -0.0004 ± 0.0001 arcseconds per century. The prediction of general relativity for the Lense-Thirring precession of the perihelion of Venus is -0.0003 arcseconds per century. It turns out that neither other mismodelled/unmodelled standard Newtonian/Einsteinian effects nor exotic ones, postulated to, for example, explain the Pioneer anomaly, may have caused the estimated extra-precession of the Venus orbit which, thus, can be reasonably attributed to the gravitomagnetic field of the Sun, not modelled in the routines of the EPM2006 ephemerides. However, it must be noted that the quoted error is the formal, statistical one; the realistic uncertainty might be larger. Future improvements of the inner planets' ephemerides, with the inclusion of the Messenger and Venus-Express tracking data, should further improve the accuracy and the consistency of such a test of general relativity which would also benefit from the independent estimation of the extra-precessions of the perihelia (and the nodes) by other teams of astronomers.

Copyright © 2008 Lorenzo Iorio. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

In the weak field and slow motion approximation, the Einstein field equations of general relativity get linearized resembling the Maxwellian equations of electromagnetism. As a consequence, a gravitomagnetic field arises [1, 2]; it is induced by the off-diagonal components g_{0i}, $i = 1, 2, 3$ of the space-time metric tensor related to the mass-energy currents of the source of the gravitational field. It affects orbiting test particles, precessing gyroscopes, moving clocks and atoms, and propagating electromagnetic waves [3, 4]. The most famous gravitomagnetic effects are, perhaps, the precession of the axis of a gyroscope [5, 6], whose detection in the gravitational field of the rotating Earth is the goal of the space-based GP-B experiment [7] (see http://einstein.stanford.edu/), and the Lense-Thirring precessions [8] of the orbit of a test particle for which some disputed satellite-based tests in the gravitational fields of the spinning Earth [9–14] and Mars [15–17] have been reported. (According to an interesting historical analysis recently performed in [18], it would be more correct to speak about an Einstein-Thirring-Lense effect.)

We focus on the detection of the solar gravitomagnetic field through the Lense-Thirring planetary precessions of the longitudes of perihelia $\varpi = \omega + \cos i\Omega$,

$$\frac{d\varpi}{dt} = -\frac{4GS\cos i}{c^2 a^3 \left(1 - e^2\right)^{3/2}}, \tag{1}$$

where G is the Newtonian gravitational constant, S is the proper angular momentum of the Sun, c is the speed of light in vacuum, a and e are the semimajor axis and the eccentricity, respectively, of the planet's orbit. (here ω is the argument of pericentre, reckoned from the line of the nodes, i is the inclination of the orbital plane to the equator of the central rotating mass and Ω is the longitude of the ascending node). It may be interesting to know that in [19] it was proposed to measure the solar gravitomagnetic field through the Schiff effect with a drag-free gyroscope orbiting the Sun in a polar orbit.

The impact of the Sun's rotation on the Mercury's longitude of perihelion was calculated for the first time with general relativity by de Sitter [20] who, by assuming a homogenous and uniformly rotating Sun, found a secular rate of -0.01 arcseconds per century ($''\,\mathrm{cy}^{-1}$ in the following). This value is also quoted in [21, page 111]. Cugusi and Proverbio [22] yield $-0.02\,''\,\mathrm{cy}^{-1}$ for the argument of perihelion of Mercury. Instead, recent determinations of the Sun's proper angular momentum $S_{\odot} = (190.0 \pm 1.5) \times$

Table 1: Lense-Thirring precessions, in $'' \mathrm{cy}^{-1}$, of the longitudes of the perihelion ϖ of the inner planets of the solar system induced by the gravitomagnetic field of the Sun. The value $S_\odot = (190.0 \pm 1.5) \times 10^{39}\ \mathrm{kg\,m^2\,s^{-1}}$ has been assumed for its angular momentum.

Mercury	Venus	Earth	Mars
−0.0020	−0.0003	−0.0001	−0.00003

$10^{39}\ \mathrm{kg\,m^2\,s^{-1}}$ from helioseismology [23, 24], accurate to 0.8%, yield a precessional effect one order of magnitude smaller. The predicted gravitomagnetic precessions of the four inner planets, according to the recent value of the Sun's angular momentum, are reported in Table 1; they are of the order 10^{-3}–$10^{-5}\ '' \mathrm{cy}^{-1}$. Due to their extreme smallness, it has been believed for a long time, until recently, that the planetary Lense-Thirring effect would have been undetectable; see, for example, [21, page 23]. A preliminary analysis showing that recent advances in the ephemerides field are making the situation more favorable was carried out in [25]. Pitjeva [26] processed more than 317 000 planetary observations of various kinds collected from 1917 to 2003 with the dynamical force models of the EPM2004 ephemerides [27]. This produced a global solution in which she also estimated, among many other parameters, corrections $\Delta\dot{\varpi}$ to the canonical Newton-Einstein perihelion precessions for all the inner planets. Since the gravitomagnetic force was not modelled at all, contrary to the static, Schwarzschild-like component of the general relativistic force of order $\mathcal{O}(c^{-2})$, such corrections to the usual perihelia evolutions account, in principle, for the Lense-Thirring effect as well, in addition to the mismodelled parts of the standard Newtonian/Einsteinian precessions. Thus, the estimated corrections for the perihelion rates of Mercury, the Earth, and Mars have been used in [28] to perform a first test. The errors $\delta(\Delta\dot{\varpi})$ released in [26] were slightly larger than the gravitomagnetic precessions whose predicted values, however, were found compatible with the estimated corrections. Venus was not used because of the poor dataset used in the estimation of its extra-precession whose value, indeed, turned out to be too large due to a physically plausible effect amounting to $+0.53 \pm 0.30\ '' \mathrm{cy}^{-1}$. The Lense-Thirring prediction for the Venus perihelion precession was incompatible with such a result at about $2-\sigma$ level.

Now, the situation for the second planet of the solar system has remarkably improved allowing for a more stringent test of the Lense-Thirring effect. Indeed, Pitjeva [29, 30], in the effort of continuously improving the planetary ephemerides, recently processed more than 400 000 data points (1913–2006) with the EPM2006 ephemerides which encompass better dynamical models with the exception, again, of the gravitomagnetic force itself. Also in this case, she estimated, among more than 230 parameters, the corrections to the usual perihelion precessions for some planets [29]. In the case of Venus, the inclusion of the radiometric data of Magellan [30] as well allowed her to obtain

$$\Delta\dot{\varpi}_{\mathrm{Venus}} = -0.0004 \pm 0.0001\ '' \mathrm{cy}^{-1}, \qquad (2)$$

in which the quoted uncertainty is the formal, statistical one (personal communication by Pitjeva to the author, June 2008). By looking at Table 1, it turns out that such an extra-precession can be well accommodated by the general relativistic prediction for the Lense-Thirring rate of the Venus' perihelion whose existence would, thus, be confirmed at 25%. It may be objected that the gravitomagnetic force should have been explicitly modelled, and an ad-hoc parameter accounting for it should have been inserted in the set of parameters to be estimated. Certainly, it may be an alternative approach which could be implemented in future. In addition, we note that the procedure followed by Pitjeva may be regarded, in a certain sense, as safer for our purposes because it is truly model-independent. Since her goal in estimating $\Delta\dot{\varpi}$ was not the measurement of the Lense-Thirring effect, there is a priori no risk that, knowing in advance the desired answer, something was driven just toward the expected outcome.

The main question to be asked is, at this point, the following: can the result of (2) be explained by other unmodelled/mismodelled canonical or nonconventional dynamical effects? Let us, first, examine some standard candidates like the residual precession due to the still imperfect knowledge of the Sun's quadrupole mass moment $J_2^\odot$ [31] whose action was, in fact, modelled by Pitjeva [26] by keeping it fixed to $J_2^\odot = 2 \times 10^{-7}$ in the global solution in which she estimated the corrections to the perihelion precessions. The answer is negative since the Newtonian secular precession due to the Sun's oblateness, (for an oblate body $J_2 > 0$) whatever magnitude J_2 may have, is positive. Indeed, it is [28, 32]

$$\dot{\varpi}_{J_2} = \frac{3}{2}\frac{nJ_2}{(1-e^2)^2}\left(\frac{R}{a}\right)^2\left(1-\frac{3}{2}\sin^2 i\right), \qquad (3)$$

where $n = \sqrt{GM/a^3}$ is the Keplerian mean motion and R is the Sun's mean equatorial radius. The angle i between the Venus' orbit and the Sun's equator amounts to 3.4 deg only. (Indeed, the orbit of Venus is tilted by 3.7 deg to the mean ecliptic of J2000 (http://ssd.jpl.nasa.gov/txt/aprx_pos_planets.pdf)), while the Carrington's angle between the Sun's equator and the ecliptic is 7.15 deg [33]). For $J_2^\odot = 2 \times 10^{-7}$, the nominal value of the Venus' perihelion precession induced by the solar quadrupole mass moment amounts to $+0.0026\ '' \mathrm{cy}^{-1}$. By assuming an uncertainty of about $\delta J_2 \approx 10\%$. [34], if $\Delta\dot{\varpi}_{\mathrm{Venus}}$ was due to such a mismodelled effect, it should amount to $+0.0002\ '' \mathrm{cy}^{-1}$, which is, instead, ruled out at $6-\sigma$ level. Concerning the precession due to the solar octupole mass moment $J_4^\odot$, it is [32]

$$\dot{\varpi}_{J_4} = -\frac{15}{16} nJ_4\left(\frac{R}{a}\right)^4\left[\frac{3}{(1-e^2)^3} + 7\frac{(1+(3/2)e^2)}{(1-e^2)^4}\right] \times \left(\frac{7}{4}\sin^4 i - 2\sin^2 i + \frac{2}{5}\right). \qquad (4)$$

For Venus, it amounts to $-1.2 J_4^\odot\ '' \mathrm{cy}^{-1}$. Since $J_4^\odot \approx -4 \times 10^{-9}$ [35, 36], we conclude that the second even zonal harmonic of the multipolar expansion of the solar gravitational potential cannot be responsible for (2). More generally, it does not represent a potentially relevant source of systematic error for the measurement of the Lense-Thirring planetary precessions. Similar arguments hold also for other potential sources of systematic errors, for example, the asteroid ring and the Kuiper Belt objects, both modelled in EPM2006. The precessions induced by them are positive. Indeed, a Sun-centered ring of mass m_{ring} and inner and outer radius $R_{\min/\max} \gg a$ induces a perihelion precession [37]:

$$\dot{\varpi}_{\text{ring}} = \frac{3}{4}\sqrt{\frac{Ga^3(1-e^2)}{M}}\frac{m_{\text{ring}}}{R_{\min}R_{\max}(R_{\min}+R_{\max})} > 0. \tag{5}$$

According to (5), the precession induced by the asteroids' ring on the Venus' perihelion amounts to $+0.0007 \pm 0.0001\,''\,\text{cy}^{-1}$ by using $m_{\text{ring}} = (5 \pm 1) \times 10^{-10}\text{M}_{\odot}$ [38]. The lowest value $+0.0006\,''\,\text{cy}^{-1}$ is incompatible with (2) at $10 - \sigma$ level. In the case of the Kuiper belt objects, (5) yields a precession of the order of $+0.00006\,''\,\text{cy}^{-1}$ with $m = 0.052m_{\oplus}$ [37]. Thus, we can rule out such modelled classical features of the Sun and the solar system as explanations of $\Delta\dot{\varpi}_{\text{Venus}}$. General relativistic terms of order $\mathcal{O}(c^{-4})$ were not modelled by Pitjeva. However, the first correction of order $\mathcal{O}(c^{-4})$ to the perihelion precession [39] can be safely neglected because for Venus, it is

$$\dot{\varpi}_{c^4} \propto \frac{n(GM)^2}{c^4a^2(1-e^2)^2} \approx 10^{-7}\,''\,\text{cy}^{-1}. \tag{6}$$

Concerning possible exotic explanations, that is, due to some modifications of the currently known Newton-Einstein laws of gravity, it may have some interest to check some of the recently proposed extra-forces [40] which would be able to phenomenologically accommodate the Pioneer anomaly [41]. All of such hypothetical new forces have not been modelled by Pitjeva, so that if they existed in nature, they would affect $\Delta\dot{\varpi}_{\text{Venus}}$. A central acceleration quadratic in the radial component v_r of the velocity of a test particle [40, 42]

$$A = -v_r^2\mathcal{H}, \quad \mathcal{H} = 6.07 \times 10^{-18}\,\text{m}^{-1} \tag{7}$$

would induce a retrograde perihelion precession according to [43]

$$\dot{\varpi} = \frac{\mathcal{H}na\sqrt{1-e^2}}{e^2}\left(-2+e^2+2\sqrt{1-e^2}\right) < 0. \tag{8}$$

(The quoted numerical value of $\mathcal{H}$ allows to reproduce the Pioneer anomaly). However, (8) predicts a precession of $-0.0016\,''\,\text{cy}^{-1}$ for Venus, which is ruled out by (2) at $12 - \sigma$ level. Another possible candidate considered in [40] is an acceleration linear in the radial velocity

$$A = -|v_r|\mathcal{K}, \quad \mathcal{K} = 7.3 \times 10^{-14}\,\text{s}^{-1}, \tag{9}$$

which yields a retrograde perihelion precession [43]:

$$\dot{\varpi} = -\frac{\mathcal{K}\sqrt{1-e^2}}{\pi}\left[\frac{2e-(1-e^2)\ln\left((1+e)/(1-e)\right)}{e^2}\right] < 0. \tag{10}$$

The prediction of (10) for Venus is $-0.1\,''\,\text{cy}^{-1}$, clearly incompatible with (2). Should one consider a central uniform acceleration with the magnitude of the Pioneer anomalous one, that is, $A = -8.74 \times 10^{-10}\,\text{ms}^{-2}$, the exotic precession induced by it [44, 45] on the perihelion of Venus would be

$$\dot{\varpi}_{\text{Ven}} = A\sqrt{\frac{a(1-e^2)}{GM}} = -16\,''\,\text{cy}^{-1}. \tag{11}$$

Another nonconventional effect which may be considered is the precession predicted by Lue and Starkman [46] in the framework of the DGP multidimensional braneworld model by Dvali et al. [47] which is proposed to explain the cosmic acceleration without invoking dark energy. It is

$$\dot{\varpi}_{\text{LS}} = \mp\frac{3c}{8r_0} + \mathcal{O}(e^2) \approx \mp 0.0005\,''\,\text{cy}^{-1}, \tag{12}$$

where the plus sign is related to the self-accelerated branch, while the minus sign is for the standard, Friedmann-Lemaître-Robertson-Walker (FLRW) branch; $r_0 \approx 5$ Gpc is a threshold characteristic of the DGP model after which gravity would experience neat deviations from the Newtonian-Einsteinian behavior. As can be noted, the self-accelerated branch is ruled out at $9 - \sigma$ level by (2), while the FLRW case is still compatible with (2) ($1 - \sigma$ discrepancy). By the way, apart from the fact that there are theoretical concerns with the DGP model (see, e.g., [48] and references therein), the existence of both the Lue-Starkman FLRW precession and the Lense-Thirring one, implying a total unmodelled effect of $-0.0008\,''\,\text{cy}^{-1}$, would be ruled out by (2) at $4 - \sigma$ level. As a consequence, we can conclude not only that the examined exotic modifications of the standard laws of gravity, not modelled by Pitjeva, are not responsible for the estimated $\Delta\dot{\varpi}_{\text{Venus}}$, but also that their existence in the inner regions of the solar system is falsified by the observations. Moreover, given the magnitudes of the hypothetical effects with the negative sign, it is not possible that reciprocal cancelations with the positive classical mismodelled precessions can explain (2). Indeed, the sum of the latter ones is $+0.0004\,''\,\text{cy}^{-1}$; the sum of, for example, (8) and (12) (FLRW) is $-0.0021\,''\,\text{cy}^{-1}$, while the sum of (8) and (12) (self-accelerated branch) is $-0.0011\,''\,\text{cy}^{-1}$.

Thus, we conclude that the most likely explanation for (2) is just the general relativistic Lense-Thirring effect. However, caution is in order in assessing the realistic uncertainty in such a test because, as already stated, the released error of $0.0001\,''\,\text{cy}^{-1}$ is the formal, statistical one; the realistic uncertainty might be larger. By the way, we can at least firmly conclude that now also in the case of Venus, the general relativistic predictions for the Lense-Thirring effect on $\dot{\varpi}$ are compatible with the observational determinations for the unmodelled perihelion precessions, contrary to the case of [28]. Moreover, future modelling of planetary motions should take into account the relativistic effects of the rotation of the Sun as well. The steady improvement in the planetary ephemerides, which should hopefully benefit of the radiometric data from Messenger and Venus-Express as well, should allow for more accurate and stringent test in the near-mid future. It would be of great significance if also other teams of astronomers would estimate their own corrections to the canonical perihelion (and also node) precessions in order to enhance the statistical significance and robustness of this important direct test of general relativity.

Acknowledgments

The author would like to thank E. V. Pitjeva for useful and important communications. He is also grateful to the referees for their useful and relevant remarks.

References

[1] B. Mashhoon, "Gravitoelectromagnetism," in *Reference Frames and Gravitomagnetism*, J. F. Pascual-Sánchez, L. Floría, A. San Miguel, and F. Vicente, Eds., pp. 121–132, World Scientific, Singapore, 2001.

[2] B. Mashhoon, "Gravitoelectromagnetism: a brief review," in *The Measurement of Gravitomagnetism: A Challenging Enterprise*, L. Iorio, Ed., pp. 29–39, NOVA Science, Hauppauge, NY, USA, 2007.

[3] M. L. Ruggiero and A. Tartaglia, "Gravitomagnetic effects," *Nuovo Cimento della Societa Italiana di Fisica B*, vol. 117, no. 7, pp. 743–767, 2002.

[4] G. Schäfer, "Gravitomagnetic effects," *General Relativity and Gravitation*, vol. 36, no. 10, pp. 2223–2235, 2004.

[5] G. E. Pugh, "Proposal for a satellite test of the coriolis prediction of general relativity," Weapon System Evaluation Group Research Memorandum no. 11, The Pentagon, Washington, DC, USA, 1959.

[6] L. I. Schiff, "Possible new experimental test of general relativity theory," *Physical Review Letters*, vol. 4, no. 5, pp. 215–217, 1960.

[7] C. W. F. Everitt, "The gyroscope experiment I. General description and analysis of gyroscope performance," in *Experimental Gravitation: Proceedings of Course 56 of the International School of Physics "Enrico Fermi"*, B. Bertotti, Ed., pp. 331–360, Academic Press, New York, NY, USA, 1974.

[8] J. Lense and H. Thirring, "Über den einfluss der eigenrotation der zentralkörper auf die bewegung der planeten und monde nach der einsteinschen gravitationstheorie," *Physikalische Zeitschrift*, vol. 19, pp. 156–163, 1918.

[9] I. Ciufolini and E. C. Pavlis, "A confirmation of the general relativistic prediction of the Lens-Thirring effect," *Nature*, vol. 431, no. 7011, pp. 958–960, 2004.

[10] I. Ciufolini and E. Pavlis, "On the measurement of the Lense-Thirring effect using the nodes of the LAGEOS satellites, in reply to "on the reliability of the so-far performed tests for measuring the Lense-Thirring effect with the LAGEOS satellites" by L. Iorio," *New Astronomy*, vol. 10, no. 8, pp. 636–651, 2005.

[11] L. Iorio, "The impact of the new Earth gravity models on the measurement of the Lense-Thirring effect with a new satellite," *New Astronomy*, vol. 10, no. 8, pp. 616–635, 2005.

[12] L. Iorio, "A critical analysis of a recent test of the Lense-Thirring effect with the LAGEOS satellites," *Journal of Geodesy*, vol. 80, no. 3, pp. 128–136, 2006.

[13] L. Iorio, "An assessment of the measurement of the Lense-Thirring effect in the Earth gravity field, in reply to: "on the measurement of the Lense-Thirring effect using the nodes of the LAGEOS satellites, in reply to "on the reliability of the so far performed tests for measuring the Lense-Thirring effect with the LAGEOS satellites" by L. Iorio," by I. Ciufolini and E. Pavlis," *Planetary and Space Science*, vol. 55, no. 4, pp. 503–511, 2007.

[14] D. M. Lucchesi, "The impact of the even zonal harmonics secular variations on the Lense-Thirring effect measurement with the two LAGEOS satellites," *International Journal of Modern Physics D*, vol. 14, no. 12, pp. 1989–2023, 2005.

[15] L. Iorio, "A note on the evidence of the gravitomagnetic field of Mars," *Classical and Quantum Gravity*, vol. 23, no. 17, pp. 5451–5454, 2006.

[16] K. Krogh, "Comment on 'evidence of the gravitomagnetic field of Mars'," *Classical and Quantum Gravity*, vol. 24, no. 22, pp. 5709–5715, 2007.

[17] L. Iorio, "Reply to "Comment on 'Evidence of the gravitomagnetic field of Mars'", by Kris Krogh," *Journal of Gravitational Physics*. In press.

[18] H. Pfister, "On the history of the so-called Lense-Thirring effect," *General Relativity and Gravitation*, vol. 39, no. 11, pp. 1735–1748, 2007.

[19] M. R. Haas and D. K. Ross, "Measurement of the angular momentum of Jupiter and the Sun by use of the Lense-Thirring effect," *Astrophysics and Space Science*, vol. 32, no. 1, pp. 3–11, 1975.

[20] W. de Sitter, "Einstein's theory of gravitation and its astronomical consequences," *Monthly Notices of the Royal Astronomical Society*, vol. 76, pp. 699–728, 1916.

[21] M. Soffel, *Relativity in Astrometry, Celestial Mechanics and Geodesy*, Springer, Berlin, Germany, 1989.

[22] L. Cugusi and E. Proverbio, "Relativistic effects on the motion of Earth's artificial satellites," *Astronomy & Astrophysics*, vol. 69, pp. 321–325, 1978.

[23] F. P. Pijpers, "Helioseismic determination of the solar gravitational quadrupole moment," *Monthly Notices of the Royal Astronomical Society*, vol. 297, no. 3, pp. L76–L80, 1998.

[24] F. P. Pijpers, "Asteroseismic determination of stellar angular momentum," *Astronomy & Astrophysics*, vol. 402, no. 2, pp. 683–692, 2003.

[25] L. Iorio, "Is it possible to measure the Lense-Thirring effect on the orbits of the planets in the gravitational field of the Sun?" *Astronomy & Astrophysics*, vol. 431, no. 1, pp. 385–389, 2005.

[26] E. V. Pitjeva, "Relativistic effects and solar oblateness from radar observations of planets and spacecraft," *Astronomy Letters*, vol. 31, no. 5, pp. 340–349, 2005.

[27] E. V. Pitjeva, "High-precision ephemerides of planets-EPM and determination of some astronomical constants," *Solar System Research*, vol. 39, no. 3, pp. 176–186, 2005.

[28] L. Iorio, "First preliminary tests of the general relativistic gravitomagnetic field of the Sun and new constraints on a Yukawa-like fifth force from planetary data," *Planetary and Space Science*, vol. 55, no. 10, pp. 1290–1298, 2007.

[29] E. V. Pitjeva, "National high-precision ephemerides of planets and the Moon—EPM," in *Proceedings of the Institute for Applied Astronomy, Russian Academy of Sciences*, vol. 17, pp. 42–59, Russian Academy of Sciences, St. Petersburg, Russia, 2007.

[30] E. V. Pitjeva, "A giant step - from milli- to micro-arcsecond astrometry," in *Proceedings of the International Astronomical Union Symposium (IAU '07)*, W. J. Jin, I. Platais, and M. A. C. Perryman, Eds., pp. 20–22, Cambridge University Press, Shanghai, China, October 2007, paper no. 248.

[31] S. Pireaux and J.-P. Rozelot, "Solar quadrupole moment and purely relativistic gravitation contributions to Mercury's

perihelion advance," *Astrophysics and Space Science*, vol. 284, no. 4, pp. 1159–1194, 2003.

[32] M. Capderou, *Satellites: Orbits and Missions*, Springer, Paris, France, 2005.

[33] J. G. Beck and P. Giles, "Helioseismic determination of the solar rotation axis," *The Astrophysical Journal Letters*, vol. 621, no. 2, pp. L153–L156, 2005.

[34] A. Fienga, H. Manche, J. Laskar, and M. Gastineau, "INPOP06: a new numerical planetary ephemeris," *Astronomy & Astrophysics*, vol. 477, no. 1, pp. 315–327, 2008.

[35] I. W. Roxburgh, "Gravitational multipole moments of the Sun determined from helioseismic estimates of the internal structure and rotation," *Astronomy & Astrophysics*, vol. 377, no. 2, pp. 688–690, 2001.

[36] R. Mecheri, T. Abdelatif, A. Irbah, J. Provost, and G. Berthomieu, "New values of gravitational moments J_2 and J_4 deduced from helioseismology," *Solar Physics*, vol. 222, no. 2, pp. 191–197, 2004.

[37] L. Iorio, "Dynamical determination of the mass of the Kuiper Belt from motions of the inner planets of the Solar system," *Monthly Notices of the Royal Astronomical Society*, vol. 375, no. 4, pp. 1311–1314, 2007.

[38] G. A. Krasinsky, E. V. Pitjeva, M. V. Vasilyev, and E. I. Yagudina, "Hidden mass in the asteroid belt," *Icarus*, vol. 158, no. 1, pp. 98–105, 2002.

[39] T. Damour and G. Schäfer, "Higher-order relativistic periastron advances and binary pulsars," *Il Nuovo Cimento B*, vol. 101, no. 2, pp. 127–176, 1988.

[40] E. M. Standish, "Planetary and Lunar Ephemerides: testing alternate gravitational theories," in *Recent Developments in Gravitation and Cosmology: Proceedings of the 3rd Mexican Meeting on Mathematicaland Experimental Physics*, A. Macías, C. Lämmerzahl, and A. Camacho, Eds., vol. 977, pp. 254–263, American Institute of Physics, Mexico City, Mexico, March 2008.

[41] M. M. Nieto, "The quest to understand the Pioneer anomaly," *Europhysics News*, vol. 37, no. 6, pp. 30–34, 2006.

[42] M.-T. Jaekel and S. Reynaud, "Gravity tests in the solar system and the Pioneer anomaly," *Modern Physics Letters A*, vol. 20, no. 14, pp. 1047–1055, 2005.

[43] L. Iorio, "Can the Pioneer anomaly be induced by velocity-dependent forces? Tests in the outer regions of solar system with planetary dynamics," *International Journal of Modern Physics D*. In press.

[44] L. Iorio and G. Giudice, "What do the orbital motions of the outer planets of the Solar System tell us about the Pioneer anomaly?" *New Astronomy*, vol. 11, no. 8, pp. 600–607, 2006.

[45] R. H. Sanders, "Solar system constraints on multifield theories of modified dynamics," *Monthly Notices of the Royal Astronomical Society*, vol. 370, no. 3, pp. 1519–1528, 2006.

[46] A. Lue and G. Starkman, "Gravitational leakage into extra dimensions: probing dark energy using local gravity," *Physical Review D*, vol. 67, no. 6, Article ID 064002, 9 pages, 2003.

[47] G. Dvali, G. Gabadadze, and M. Porrati, "4D gravity on a brane in 5D Minkowski space," *Physics Letters B*, vol. 485, no. 1–3, pp. 208–214, 2000.

[48] K. Izumi, K. Koyama, O. Pujolàs, and T. Tanaka, "Bubbles in the self-accelerating universe," *Physical Review D*, vol. 76, no. 10, Article ID 104041, 9 pages, 2007.