Research Article

Regionalized Lunar South Pole Surface Navigation System Analysis

Bryan W. Welch

Glenn Research Center, National Aeronautics and Space Administration, Cleveland, OH 44135, USA

Correspondence should be addressed to Bryan W. Welch, bryan.w.welch@nasa.gov

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Apollo missions utilized Earth-based assets for navigation, since the landings took place at lunar locations in constant view from the Earth. The new exploration campaign to the lunar South Pole region will have limited Earth visibility, but the extent to which a navigation system comprised solely of Earth-based tracking stations will provide adequate navigation solutions in this region is unknown. This article presents a dilution-of-precision-(DoP-) based stationary surface navigation analysis of the performance of multiple lunar satellite constellations, Earth-based deep space network assets, and combinations thereof. Results show that kinematic and integrated solutions cannot be provided by the Earth-based deep space network stations. Also, the surface stationary navigation system needs to be operated as a two-way navigation system, or as a one-way navigation system with local terrain information, while integrating the position solution over a short duration of time with navigation signals being provided by a lunar satellite constellation.

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1. INTRODUCTION

In support of NASA's vision for space exploration [1], an extension of the position-fixing capability provided by the GPS constellation [2] to the Moon is being analyzed. This extension would be provided by a lunar network of spacecraft orbiting the Moon, Earth-based deep space network assets, and combinations thereof. This study provides a dilution-of-precision-(DoP-) based stationary surface navigation analysis via radiometric navigation signals from the aforementioned systems for users located on the lunar surface. The current study is similar to prior studies on the subject [3–6] with several differences including use of the newly developed DoP technique referred to as “generalized DoP,” combination of radiometric signals from the lunar vicinity and Earth vicinity, and limitations to the regionalization of interest.

Generalized DoP provides the ability to assess the navigational performance associated with a receiver that is able to integrate radiometric measurements over time. Such an analysis method allows one to directly compare the navigational capability associated with sparse constellations with that provided by constellations which support full coverage of an appropriate fold. Estimates of a user state

derived from multiple radiometric measurements collected over a period of time are herein referred to as being “dynamic,” whereas those provided by full constellations that do not employ integration over time in the receiver are referred to as being “kinematic.” As opposed to standard measures of DoP that are restricted to kinematic position-fixing capabilities, the use of generalized DoP further allows assessment of the constellation to be performed in terms of the latency associated with obtaining a specified level of system performance [5, 6].

Several different options for the radiometric navigation signal sources are considered in this study and include equally the Earth-based deep space network (DSN) site locations, two inclined elliptical lunar constellations [7], and combinations thereof. Included in this study are assessments of a number of augmentations to the system, such as two-way mode of operation, good knowledge of the terrain, and the integration of radiometric measurements over periods of time. Comparisons of the system performance under the different system assumptions indicate that system availability performance is significantly improved and latency is reduced by the prescribed augmentations. Results are derived from temporally and spatially averaged system availability

TABLE 1: Lunar network constellations.

Constellation	Satellites no.	Planes no.	SMA (km)	Inclination	Eccentricity	Phasing no.
Ellip 1/1/0 a6541	1	1	6541	62.9°	0.6	0
Ellip 2/1/0 a6541	2	1	6541	62.9°	0.6	0

FIGURE 1: Elliptical 2/1/0 a6541 constellation.

TABLE 2: DSN site locations.

DSN site	Latitude (°N)	Longitude (°E)
Canberra	−35.4	148.967
Madrid	40.383	−4.25
Goldstone	35.33	−116.833

numbers associated with prespecified threshold levels of system availability.

2. SIGNAL SOURCES

Three categories of radiometric signal sources are considered two inclined elliptical lunar-centric constellations, DSN site locations (Canberra, Madrid, and Goldstone), and the combination of the lunar constellations and DSN assets. The notation for the lunar-centric constellations subsequently used, such as Ellip $N/p/f/d$ km, is defined as N the number of satellites, p the number of orbital planes, f the binary answer as to if phasing exists in the mean anomaly between satellites in adjacent planes, and d denotes the semi-major axis (SMA) in kilometers. Table 1 lists the parameters of the two lunar-centric constellations [7] that are considered here. Table 2 lists the latitude and longitude for the three DSN site locations.

Each of the sources in this study was considered for specific reasons. The two inclined elliptical constellations were considered for their providing a focus of coverage over the lunar South Pole region. The Earth-based DSN assets

were considered because of their usefulness and availability for space navigation. The combinations of the two sets were examined to determine additional benefits of combining the services of both assets over their individual performance. Figure 1 shows an illustration of the Ellip 2/1/0 a6541 constellation in orbit around the Moon. Shown in the background of the illustration are the three Earth-based DSN stations. The image was produced from orbital plots in Satellite Orbit Analysis Program.

3. ANALYSIS

3.1. Generalized DoP

The analysis performed is a generalized version of the DoP metric [4–6] of which several forms are subsequently used for analysis. The generalized DoP is derived from the observability grammian, which is obtained by using the navigation user equations of motion and the associated sequence of measurements. The equations of motion and the measurement sequence are given by [4–6]. It is shown that the DoP metric takes the following form, derived in [4–6]. Since the DoP metric is calculated for a stationary user, the state transition matrix is an identity matrix and the measurement partial derivation matrix is utilized directly in (1)

$$\sqrt{\max \left(\text{eig} \left(\left(\sum_{t_0}^{t_n} H_{t_i}^T W H_{t_i} \right)^{-1} \right) \right)}. \quad (1)$$

The variables listed in (1) are as follows:

- (i) $\text{eig}(\cdot)$ is the Matlab function that calculates the eigenvalues of the input matrix;
- (ii) t_0 is time step zero;
- (iii) t_n is the n th time step since time step zero;
- (iv) H_{t_i} is the partial derivative measurement matrix at the i th time step;
- (v) W is the measurement weighting matrix.

3.2. Variations of generalized DoP

In order to relax the constraint of satellite coverage to invert the observability grammian, a number of augmentations to the lunar navigation system are considered in the analysis, as in previous analyses [7]. These augmentations constrain the navigation solution and thereby reduce the number of required satellites in view. These augmentations include clock synchronization and good knowledge of the terrain, and create four forms of DoP. The selected form of DoP used not only affects the required satellites in view, but also the state transition and H matrices used in the calculation. It should also be noted that throughout the analysis, both range and range-rate (Doppler) measurements are used to solve for position and time bias (when appropriate) estimates only. There are no estimates done for velocity or frequency bias, as the users are assumed to be stationary.

The first form of DoP is the geometric dilution of precision (GDoP). It is used in the global positioning system (GPS), where the solution is obtained for position of the user in three dimensions and the time bias, resulting in the requirement of four navigation signals. Since two navigation signals are available from each satellite, then only two satellites are necessary in view to kinematically solve for the user's position. Without two satellites in view, the solution will need to be integrated over time in order to be able to invert the solution and solve for the user's position and time bias. The GDoP metric is used to evaluate a navigation system operating in one-way mode without terrain information. Equation (2) provides the associated H matrix for the GDoP metric

$$H = \begin{bmatrix} \frac{\partial pr_1}{\partial x_1} & \frac{\partial pr_1}{\partial y_1} & \frac{\partial pr_1}{\partial z_1} & \frac{\partial pr_1}{\partial (ct_{\text{bias}_1})} \\ | & | & | & | \\ \frac{\partial pr_m}{\partial x_m} & \frac{\partial pr_m}{\partial y_m} & \frac{\partial pr_m}{\partial z_m} & \frac{\partial pr_m}{\partial (ct_{\text{bias}_m})} \\ \frac{\partial \dot{r}_1}{\partial x_1} & \frac{\partial \dot{r}_1}{\partial y_1} & \frac{\partial \dot{r}_1}{\partial z_1} & \frac{\partial \dot{r}_1}{\partial (ct_{\text{bias}_1})} \\ | & | & | & | \\ \frac{\partial \dot{r}_m}{\partial x_m} & \frac{\partial \dot{r}_m}{\partial y_m} & \frac{\partial \dot{r}_m}{\partial z_m} & \frac{\partial \dot{r}_m}{\partial (ct_{\text{bias}_m})} \end{bmatrix}. \quad (2)$$

The variables listed in (2) are as follows:

- (i) H is the partial derivative measurement matrix;
- (ii) (x_m, y_m, z_m) is the position of the surface user;
- (iii) pr_m is the m th pseudorange measurement;

- (iv) t_{bias_m} is the clock bias of the surface user;
- (v) $\dot{r}_m$ is the m th range rate measurement;
- (vi) c is the speed of light in a vacuum;
- (vii) ∂ is the partial derivative operator.

The second form of DoP is the positional dilution of precision (PDoP). It provides an estimate of user positioning accuracy for the case in which there is no time bias between orbiter clocks and user clocks, such as the case in a two-way mode of operation. PDoP results in the requirement of three navigation signals. Thus, the PDoP metric also requires two satellites in view to kinematically solve for the user's position. The PDoP metric is used to evaluate a navigation system operating in two-way mode without terrain information. Equation (3) provides the associated H matrix for the PDoP metric

$$H = \begin{bmatrix} \frac{\partial r_1}{\partial x_1} & \frac{\partial r_1}{\partial y_1} & \frac{\partial r_1}{\partial z_1} \\ | & | & | \\ \frac{\partial r_m}{\partial x_m} & \frac{\partial r_m}{\partial y_m} & \frac{\partial r_m}{\partial z_m} \\ \frac{\partial \dot{r}_1}{\partial x_1} & \frac{\partial \dot{r}_1}{\partial y_1} & \frac{\partial \dot{r}_1}{\partial z_1} \\ | & | & | \\ \frac{\partial \dot{r}_m}{\partial x_m} & \frac{\partial \dot{r}_m}{\partial y_m} & \frac{\partial \dot{r}_m}{\partial z_m} \end{bmatrix}. \quad (3)$$

The new variable listed in (3) is as follows:

- (i) r_m is the m th pseudorange measurement.

The third form of DoP is the horizontal/time dilution of precision (HTDoP). This form of DoP is applied when a user has knowledge of their altitude above the center of the Moon, but still has a time bias from the source of the navigation signal. This also results in the requirement of three navigation signals, meaning that two satellites must be in view to kinematically solve for the user's topocentric North and East components along with the time bias. The HTDoP metric is used to evaluate a navigation system operating in one-way mode with terrain information. Equation (4) provides the associated H matrix for the HTDoP metric

$$H = \begin{bmatrix} \frac{\partial pr_1}{\partial x_1} & \frac{\partial pr_1}{\partial y_1} & \frac{\partial pr_1}{\partial (ct_{\text{bias}_1})} \\ | & | & | \\ \frac{\partial pr_m}{\partial x_m} & \frac{\partial pr_m}{\partial y_m} & \frac{\partial pr_m}{\partial (ct_{\text{bias}_m})} \\ \frac{\partial \dot{r}_1}{\partial x_1} & \frac{\partial \dot{r}_1}{\partial y_1} & \frac{\partial \dot{r}_1}{\partial (ct_{\text{bias}_1})} \\ | & | & | \\ \frac{\partial \dot{r}_m}{\partial x_m} & \frac{\partial \dot{r}_m}{\partial y_m} & \frac{\partial \dot{r}_m}{\partial (ct_{\text{bias}_m})} \end{bmatrix}. \quad (4)$$

Finally, the fourth form of DoP is the Horizontal Dilution of Precision (HDoP). It provides an estimate of user positioning accuracy when both time and user altitude are known, only requiring two navigation signals, such as the case of a two-way mode of operation with good knowledge of terrain. This requires that only one satellite be in view to kinematically solve for the user's topocentric North and East components. The HDoP metric is used to evaluate a navigation system operating in two-way mode with terrain information. Equation (5) provides the associated H matrix for the HDoP metric

$$H = \begin{bmatrix} \frac{\partial r_1}{\partial x_1} & \frac{\partial r_1}{\partial y_1} \\ | & | \\ \frac{\partial r_m}{\partial x_m} & \frac{\partial r_m}{\partial y_m} \\ \frac{\partial \dot{r}_1}{\partial x_1} & \frac{\partial \dot{r}_1}{\partial y_1} \\ | & | \\ \frac{\partial \dot{r}_m}{\partial x_m} & \frac{\partial \dot{r}_m}{\partial y_m} \end{bmatrix}. \quad (5)$$

3.3. System availability

The underlying figure of merit (FOM) used for evaluating the performance associated with a navigation system is system availability. System availability is defined here as the proportion of time that the navigation system is predicted to provide performance at or below a specified level of DoP. In other words, the navigation system is defined as “available” when the appropriately chosen version of DoP falls below a certain threshold. For this study, as in previous studies, the threshold is set at 10 [4–6]. Furthermore, a DoP of 10, coupled with a 1 meter user range error, implies a user state uncertainty of 10 meters. Results provided are in terms of system availability for a given latency, whether the solution have zero latency (kinematic) or dynamic solutions of 15 minutes or one hour. Equation (6) describes how the system availability FOM is calculated, where SA represents system availability. This results in an estimate of the percentage of time that the system availability condition has been satisfied

$$SA = 100 * \frac{\sum_{m=1}^{t_n} \cos(\text{lat}_m) * \sum_{n=1}^{t_f} (\text{DoP}_{n,m} \leq \text{threshold})}{t_f * n_{\text{long}} * \sum_{m=1}^{n_{\text{lat}}} \cos(\text{lat}_m)}. \quad (6)$$

The variables listed in (6) are as follows:

- (i) n_{lat} is the number of latitude points in the simulation;
- (ii) n_{long} is the number of longitude points in the simulation;
- (iii) lat_m is the latitude value of the m th simulation point of interest;
- (iv) t_f is the number of time epochs in the simulation;
- (v) t_n is the total number of points in the simulation.

TABLE 3: Navigation signal assumptions.

Frequency used for Doppler measurements	GPS L1 (1.57545 GHz)
URE (user range error)	1 m
URRE (user range rate error)	0.1 mm/sec

FIGURE 2: Surface sample points.

3.4. Navigation signal

The navigation signal requirements used in this study are outlined in Table 3.

3.5. Simulation

The lunar South Pole region is taken as a set of 721 points on the surface, spaced evenly in latitude and longitude. The longitudes for the points go from -175°E to 180°E in 5° increments, and the latitudes of the points go from -89°N to -80°N in 1° increments. There is an additional point of interest that is added to the set of data points which is located at the exact South Pole of -90°N . The analysis is performed over the duration of 1 lunar sidereal month (27.3 Earth days), where DoPs are calculated at an epoch rate of 15 minutes. The starting epoch for the simulations is Jan 1, 2020 12:00:00.000 UTC. Visibility to the constellations from the surface points is computed based on a 10° minimum user elevation angle. Figure 2 plots the surface sample points used for the simulation along an orthographic projection of the lunar surface.

3.6. User burden

Receivers that support a reduced number of satellites will have an increased level of processing or other sensing equipment associated with them. This situation leads to increased user burden in terms of the mass and power the host platform must provide to the navigation receiver. To provide

TABLE 4: System availability—“one-way, no terrain” results.

Configuration	Kinematic	Dynamic (15 min)	Dynamic (1 h)
Ellip 1/1/0 a6541	0.00	0.00	8.74
Ellip 2/1/0 a6541	0.00	34.18	61.50
DSN	0.00	0.00	0.00
Ellip 1/1/0 a6541 + DSN	2.83	3.84	12.53
Ellip 2/1/0 a6541 + DSN	4.91	37.67	63.62

TABLE 5: System availability—“one-way, good terrain” results.

Configuration	Kinematic	Dynamic (15 min)	Dynamic (1 h)
Ellip 1/1/0 a6541	0.00	71.43	77.12
Ellip 2/1/0 a6541	46.31	100.00	100.00
DSN	0.00	0.00	0.06
Ellip 1/1/0 a6541 + DSN	3.75	71.62	77.35
Ellip 2/1/0 a6541 + DSN	49.12	100.00	100.00

knowledge sufficient to infer user altitude given a horizontal location, a large digital elevation map would have to be available to the user. To provide an error comparable to the 1-m user range error (URE) assumed for the system, the user is required to store approximately 1 TB of terrain data for global coverage. For the user to have knowledge of terrain within a 30-km radius of a starting point, approximately 100 MB is required for storage.

For a navigation system using two-way radiometric signals as a mode of operation, the two-way radiometric assumption implies that the user would have to be able to transpond the ranging signal which is initialized from the lunar-centric constellation or Earth-based DSN assets. The clock synchronization, which is necessary for one-way radiometric navigation systems, is not a requirement when using two-way radiometric navigation signals for the system’s mode of operation.

4. RESULTS

Results are reported as the system availability FOM and are presented in tabular form for the lunar South Pole regions. The term “no terrain” indicates that there is no detailed cartography of the terrain that would allow determining the altitude of the user. The term “good terrain” indicates that there is such knowledge and that an accurate estimate of user altitude above the lunar datum is available to the navigation receiver. The term “one-way” indicates that the mode of operation for the navigation system is such that signals are transmitted via the lunar-centric constellation or Earth-based DSN assets and received by the lunar stationary surface user. The term “two-way” indicates that the mode of operation for the navigation system is such that signals are transmitted via the lunar-centric constellation or Earth-based DSN assets and transponded by the lunar stationary surface user back to the original transmitter, which then

sends data back to the lunar stationary surface user for position determination. Tables 4 through 7 provide the system availability results for the four navigation system types previously described.

These results are also summarized in a stoplight chart, which shows the performance of each of the configurations proposed herein in terms of the latency required to achieve 80% system availability over the lunar South Pole region. The correlation between color and latency is as follows.

- (i) A box shaded green indicates that the configuration meets the 80% system availability kinematically.
- (ii) A box shaded yellow indicates that the configuration meets the 80% system availability dynamically in 15 minutes.
- (iii) A box shaded red indicates that the configuration meets the 80% system availability dynamically in one hour.
- (iv) A box shaded gray indicates that the configuration does not meet the 80% system availability within one hour. It does not mean that the system does not meet the 80% system availability at all, but rather it indicates that it could take more than one hour to do so if it will do so.

Table 8 illustrates the performance of the five configurations in the stoplight chart form. This form will be useful in making comparisons between configurations.

Inspection of the latency result summary provided in Table 8 and the system availability summaries provided in Tables 4 through 7 reveal four general trends apparent in the lunar South Pole region overall. The first general trend is that system availability improves for a given constellation as the solution is integrated over time. Also, there are larger improvements when comparing the transition to the 15-minute dynamic solution (from the kinematic solution) as compared with the transition to the one-hour dynamic

TABLE 6: System availability—“two-way, no terrain” results.

Configuration	Kinematic	Dynamic (15 min)	Dynamic (1 h)
Ellip 1/1/0 a6541	0.00	71.42	77.12
Ellip 2/1/0 a6541	46.31	100.00	100.00
DSN	0.45	0.59	1.12
Ellip 1/1/0 a6541 + DSN	3.86	71.78	77.51
Ellip 2/1/0 a6541 + DSN	49.41	100.00	100.00

TABLE 7: System availability—“two-way, good terrain” results.

Configuration	Kinematic	Dynamic (15 min)	Dynamic (1 h)
Ellip 1/1/0 a6541	73.31	75.20	80.90
Ellip 2/1/0 a6541	100.00	100.00	100.00
DSN	4.80	4.94	5.24
Ellip 1/1/0 a6541 + DSN	74.65	76.49	81.94
Ellip 2/1/0 a6541 + DSN	100.00	100.00	100.00

solution (from the 15-minute dynamic solution). It appears that with these sparse constellations the system availability curve plotted against integrated time levels off to a non-100% value. Further analysis would be needed to validate that observation.

The second general trend observed for the lunar South Pole region is that the system availability improves for either a one-way or two-way navigation system with the inclusion of the local terrain information. This is due to the need for fewer signal sources necessary to solve for the reduced number of states being solved for. In a one-way navigation system, four states are solved for by the DoP technique, which are the local x , y , and z topocentric coordinates and the time bias. Providing local terrain information removes the need to solve for the local topocentric z coordinate. Similar statements can be made regarding a two-way navigation system.

The third general trend observed between the one-way and two-way navigation system modes is that the two-way mode is better than the one-way mode in providing a navigation solution for all the configurations examined. The results in Table 8 show improvements in system availability above the 80% threshold in the stoplight chart for the two-way system over that of the one-way system. This is true for cases with and without local terrain information. Results show that the Elliptical 2/1/0 constellation can provide 100% system availability with a dynamic solution of 15 minutes in the two-way mode, compared with a system availability of 34% in one-way mode with a dynamic solution of 15 minutes. Only the Elliptical 2/1/0 constellation (with and without the DSN augmentation) can provide the 80% system availability kinematically, when operating in two-way mode with local terrain information.

The fourth general trend observed is that the combinations of the elliptical lunar-centric constellations and the Earth-based DSN ground stations did not significantly improve system availabilities for all the configurations. Data

provided in Tables 4 through 7 tabulate the system availabilities for all of the configurations for the different systems schemes. It is observed that the system availabilities obtained for the Ellip 1/1/0 a6541 + DSN and Ellip 2/1/0 a6541 + DSN configurations are not much larger than for the Ellip 1/1/0 a6541 or Ellip 2/1/0 a6541 configurations, respectively. Therefore, it can be determined that the geometry added via the DSN measurements does not improve the geometry from the lunar-centric measurements by a significant factor.

It is important to note that the performance of the DSN configuration should not be expected to be above a system availability of 50%. The reason for this is that half of the points in the simulation are on the lunar far side, the side that does not face toward the Earth. It is noted that due to the wobble of the Moon in orbit, that some points near the lunar South Pole could face the Earth when the Moon is tilted away from the Earth in the Moon’s northern hemisphere. However, those points would still require a minimum user elevation angle of 10° to consider measurements visible. Therefore, at most, only the lunar nearside points could have access to the Earth-based DSN ground stations.

5. CONCLUSIONS

Generalized DoP allows the effects of multiple radiometric measurements to be assessed in the same manner that standard measures of DoP are used. In the current case, the effect of integrating multiple radiometric measurements in time is assessed to allow the performance of sparse constellations around the Moon to be analyzed and compared with kinematic only solutions. Using this innovation, the basis of comparison can be changed to a domain that is more closely aligned with user requirements, namely, the latency associated with achieving a particular level of precision in the state estimate.

A restriction to the use of kinematic solutions, as is done with analysis based on static DoP, biases the selection

TABLE 8: System availability—“two-way, good terrain” results.

Configuration	“One-way, no terrain”	“One-way, good terrain”	“Two-way, no terrain”	“Two-way, good terrain”
Ellip 1/1/0 a6541				
Ellip 2/1/0 a6541				
DSN				
Ellip 1/1/0 a6541 + DSN				
Ellip 2/1/0 a6541 + DSN				

of a constellation to those with more satellites. The use of dynamic solutions allows for integrating radiometric signals over a period of time to improve the system availability and thus allows for the consideration of constellations with fewer satellites. The application of generalized DoP for the evaluation of inherent navigation capability of constellations of lunar-centric satellites, Earth-based DSN assets, and combinations thereof has eliminated this bias.

Inspection of the results summaries provided in Tables 4 through 8 reveal several trends. First, time integration of the solution will improve the system availability metric and lower the estimated solution error. Second, use of a local terrain map can improve system performance for both one-way and two-way navigation systems. Third, use of a two-way navigation system has improved performance over the use of a one-way navigation system due to the lack of needing to solve for the user’s time bias. Finally, augmenting the lunar-centric constellations with the Earth-based DSN assets provides minimal improvements in system availability performance. It should also be noted that the Earth-based DSN configuration exhibits system availabilities of a maximum of 5% during the case of operating as a two-way navigation system with local terrain information while integrating the solution for one hour.

From this list of possible configurations using the stated assumptions regarding visibility, the recommended constellation would be the Ellip 2/1/0 a6541. The navigation system should operate in two-way mode, collecting range and range-rate measurements while be augmented via a local terrain map. Under this mode of operation for the Ellip 2/1/0 a6541 satellite constellation, the 80% system availability metric can be met kinematically with a predicted system availability of 100%. The Ellip 2/1/0 a6541 + DSN constellation also provides this level of system availability, but is therefore not necessary to meet a DoP threshold of 10, corresponding to an error on the order of 10 meters.

REFERENCES

- [1] G. W. Bush, New Vision for Space Exploration Program. Today’s Presidential Action, January 2004.
- [2] B. Parkinson and J. Spilker Jr., *The Global Positioning System: Theory and Applications*, AIAA, Washington, DC, USA, 1996.
- [3] J. H. MacNicol and J. F. Raquet, “A study of satellite navigation, dilution of precision, and positioning techniques for use on and around the moon,” in *Proceedings of the ION 58th Annual Meeting/CIGTF 21st Guidance Test Symposium*, p. 506, Albuquerque, NM, USA, June 2002.
- [4] O. S. Sands, J. W. Connolly, B. W. Welch, J. R. Carpenter, T. A. Ely, and K. Berry, “Dilution of precision-based lunar navigation assessment for dynamic position fixing,” in *Proceedings of the ION National Technical Meeting (NTM ’06)*, Monterey, Calif, USA, January 2006.
- [5] B. W. Welch, J. W. Connolly, and O. S. Sands, “Dilution of precision-based lunar surface navigation system analysis utilizing earth-based assets,” Tech. Rep. NASA/TP-2007-214407, National Aeronautics and Space Administration, Washington, DC, USA, 2007.
- [6] B. W. Welch, J. W. Connolly, and O. S. Sands, “Dilution of precision-based lunar surface navigation system analysis utilizing lunar orbiters,” Tech. Rep. NASA TP 2007-214427, National Aeronautics and Space Administration, Washington, DC, USA, 2007.
- [7] T. A. Ely, “Stable constellations of frozen elliptical inclined lunar orbits,” *Journal of the Astronautical Sciences*, vol. 53, no. 3, pp. 301–316, 2005.