

Research Article

Dynamics and Stability of Stepped Gun Barrels with Moving Bullets

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The stability of an Euler-Bernoulli beam under the effect of a moving projectile will be reintroduced using simple eigenvalue analysis of a finite element model. The eigenvalues of the beam change with the mass, speed, and position of the projectile, thus, the eigenvalues are evaluated for the system with different speeds and masses at different positions until the lowest eigenvalue reaches zero indicating the instability occurrence. Then a map for the stability region may be obtained for different boundary conditions. Then the dynamics of the beam will be investigated using the Newmark algorithm at different values of speed and mass ratios. Finally, the effect of using stepped barrels on the stability and the dynamics is going to be investigated. It is concluded that the technique used to predict the stability boundaries is simple, accurate, and reliable, the mass of the barrel on the dynamics of the problem cannot be ignored, and that using the stepped barrels, with small increase in the diameter, enhances the stability and the dynamics of the barrel.

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1. INTRODUCTION

The problem of the dynamics and stability of beams carrying moving masses drew a lot of attention in the past half century due to the applications that require it such as fast trains, motion on bridges, and light guns mounted on aircraft. In 1971, Nelson and Conover [1] presented a study of the problem of an infinite thin beam with periodically distributed simple supports resting on elastic foundation with a train of masses moving on it at constant speed. They applied Galerkin method to the proposed approximate solution to get the system equations then applied the Floquet theorem to get the stability boundaries for the periodically repeating system. Simultaneously, Benedetti [2, 3] used a similar approach and could present an analytical relation between the mass parameter and the critical speed parameter using classical techniques.

Since most of the research was directed to civil structures and lathe-machined work pieces, the stability regions were not of major interest for general structures. Rather, the response of the structure to moving loads or masses presented a more practical problem. Further, most of the studies were interested in problems with periodically supported beams that simulate train rails (see [4–7] as examples for such studies).

Recently, the emergence of the need for very light guns that are mounted on combat aircraft reintroduced the stability problem with new conditions. The motion of the bullets inside the gun barrels introduces compression force on the shell walls, in turn, this compression may cause dynamic buckling [2] and excessive vibration in the shell wall [8]. This type of instability, though of major importance, will not be covered in this study.

The problem of projectiles inside gun barrels, though of important application, was not the subject of many researches. In [9, 10], the stability problem was studied using the impulsive parametric excitation theory [11, 12] for a thin beam with periodically distributed controllers. The study ignored the effect of the dynamics of the barrel shell. The problem of the dynamics of the barrel shell under the effect of the moving projectile with shock and expansion waves was studied in [8] but the study did not tackle the stability problem. In [13], the author presented one of the very rare studies that handled finite beams. In that study, the beam under investigation was modeled as a Timoshenko beam with simple supports and elastic foundation. The results presented different cases of multiple masses and foundation stiffness but did not present any comparison with published or experimental results.

In this study, the stability of an Euler-Bernoulli beam under the effect of a moving projectile will be reintroduced using simple eigenvalue analysis of a finite element model. The eigenvalues of the beam change with the mass, speed, and position of the projectile, thus the eigenvalues are evaluated for the system with different speeds and masses at different positions until the lowest eigenvalue reaches zero indicating the instability occurrence. Then a map for the stability region may be obtained for different boundary conditions. Then the dynamics of the beam will be investigated using the Newmark algorithm at different values of speed and mass ratios. Finally, the effect of using stepped barrels on the stability and the dynamics is going to be investigated.

2. MODEL

The model derived in this section will have the following assumptions: the gun barrel will be modeled as thin beam that follows the Euler-Bernoulli theorem, the barrel deflections are small, and the effect of elastic foundation will be included for the purpose of comparison with published data.

The Hamilton principle states that

$$\int_{t_1}^{t_2} \delta \Pi dt = \int_{t_1}^{t_2} \delta (T - U + W_{\text{bullet}}) dt = 0, \quad (1)$$

where T is the kinetic energy, U is the potential energy, and W_{bullet} is the work done by the bullet on the beam. The kinetic energy of the system may be written as

$$T = \frac{1}{2} \int_0^l \rho A \dot{w}^2 dx + \frac{1}{2} m \left(\dot{w}^2 + 2V \dot{w} \frac{\partial w}{\partial x} + V^2 \right)_{x=x_m}, \quad (2)$$

where ρ is the beam mass density, A is the beam cross section area, m is the bullet mass, V is the bullet speed, w is the beam transverse displacement, and x_m is the position of the bullet. The potential energy of the system may be written as

$$U = \frac{1}{2} \int_0^l \left(EI \left(\frac{\partial^2 w}{\partial x^2} \right)^2 + k w^2 - \rho A g w \right) dx - mg(w)_{x=x_m}, \quad (3)$$

where E is the beam modulus of elasticity, I is the beam second moment of area, k is the foundation stiffness, and g is the gravitational acceleration. And the work done by the bullet may be written as

$$W_{\text{bullet}} = \frac{1}{2} \left(m V^2 \left(\frac{\partial w}{\partial x} \right)^2 \right)_{x=x_m}. \quad (4)$$

The variation of the kinetic energy may be written as

$$\delta T = \int_0^l \rho A \delta \dot{w} \cdot \dot{w} dx + \frac{1}{2} m \left(\delta \dot{w} \cdot \dot{w} + 2V \delta \dot{w} \frac{\partial w}{\partial x} + 2V \dot{w} \frac{\partial \delta w}{\partial x} \right)_{x=x_m}. \quad (5)$$

And the variation of the potential energy may be written as

$$\delta U = \int_0^l \left(EI \left(\frac{\partial^2 \delta w}{\partial x^2} \right) \left(\frac{\partial^2 w}{\partial x^2} \right) + k \delta w \cdot w - \rho A g \delta w \right) dx - mg(\delta w)_{x=x_m}, \quad (6)$$

while the variation of the work done by the bullet may be written as

$$\delta W_{\text{bullet}} = \left(m V^2 \frac{\partial \delta w}{\partial x} \frac{\partial w}{\partial x} \right)_{x=x_m}. \quad (7)$$

Applying the Hamilton principle and using the standard beam finite element interpolation function, we may write the element equation of motion as

$$([M] + [M_{\text{bullet}}]) \{\ddot{w}\} + [C_{\text{bullet}}] \{\dot{w}\} + ([K] + [K_s] - [K_{\text{bullet}}]) \{w\} = \{f_w\} + \{f_{\text{bullet}}\}, \quad (8)$$

where the beam mass matrix is evaluated by

$$[M] = \int_0^l \rho A \{N(x)\}^T \{N(x)\} dx, \quad (9)$$

where $N(x)$ are the beam interpolation polynomials. The effective bullet mass matrix is evaluated at the bullet position as

$$[M_{\text{bullet}}] = m \{N(x)\}^T \{N(x)\} |_{x=x_m}. \quad (10)$$

While the beam stiffness matrix is evaluated by

$$[K] = \int_0^l EI \{N_{xx}(x)\}^T \{N_{xx}(x)\} dx. \quad (11)$$

The foundation stiffness matrix is evaluated by

$$[K_s] = \int_0^l \gamma EI \{N(x)\}^T \{N(x)\} dx, \quad (12)$$

where γ is the foundation stiffness ratio given by $\gamma = k/EI$. The effective geometric bullet stiffness matrix is evaluated at the bullet position as

$$[K_{\text{bullet}}] = m V^2 \{N_{xx}(x)\}^T \{N_{xx}(x)\} |_{x=x_m}. \quad (13)$$

The effective bullet Coriolis matrix is evaluated by

$$[C_{\text{bullet}}] = 2mV \{N(x)\}^T \{N_x(x)\} |_{x=x_m}. \quad (14)$$

The forces due to the barrel and bullet weights are evaluated, respectively, by

$$\begin{aligned} \{f_w\} &= - \int_0^l \{N(x)\}^T \rho A g dx, \\ \{f_{\text{bullet}}\} &= - \{N(x)\}^T mg |_{x=x_m}. \end{aligned} \quad (15)$$

2.1. Stability boundaries

To obtain the eigenvalues of the system, including the effect of all components, we will need to transform the system into a first-order system by the standard transformation,

$$z = \dot{w}. \quad (16)$$

Using the above transformation, we will obtain the homogeneous equation of motion in the form

$$\begin{Bmatrix} \dot{w} \\ \dot{z} \end{Bmatrix} = \begin{bmatrix} 0 & I_{n \times n} \\ MK & MC \end{bmatrix} \begin{Bmatrix} w \\ z \end{Bmatrix}, \quad (17)$$

where MK denotes $-([M] + [M_{\text{bullet}}])^{-1} ([K] + [K_s] - [K_{\text{bullet}}])$ and MC denotes $-([M] + [M_{\text{bullet}}])^{-1} [C_{\text{bullet}}]$.

For the above system, the eigenvalues should represent the natural frequencies of oscillation of the beam with the bullet. The eigenvalues should all be complex with nonzero imaginary parts for all values of the speed that are below the critical speed. As the bullet speed reaches the critical speed, the smallest complex pair will have zero imaginary parts.

The search for the critical values of the speed may be done using the following algorithm.

- (1) Select the bullet mass.
- (2) Select the bullet speed.
- (3) Change the value of the x -location of the bullet and evaluate the eigenvalues of the system.
- (4) If all eigenvalues have nonzero imaginary parts then increase speed and go to step (3), else go to step (6).
- (5) If all speed values did not reach the critical value, then reset speed value and increase mass up to a given limit and proceed to step (3). If mass limit is reached, terminate.
- (6) Store the values of the critical speed and the mass.
- (7) Increase speed up to a given limit and proceed to step (3). If mass limit is reached, terminate.

The above procedure may be repeated for all values of mass, speed, and boundary conditions and different information may be compiled out of the extracted data.

2.2. Time response

The time response of the system presented by (8) may be obtained using the Newmark algorithm as presented in [14]. The algorithm may be presented as follows.

For the system

$$[M]\{\ddot{w}\} + [C]\{\dot{w}\} + [K]\{w\} = \{f_w\}, \quad (18)$$

- (1) evaluate the acceleration of the system using

$$\begin{aligned} & ([M] + \delta\Delta t[C] + \mu\Delta t^2[K])\{\ddot{w}\}_{t+\Delta t} \\ & = \{f_w\}_{t+\Delta t} - [C](\{\dot{w}\}_t + (1 - \delta)\Delta t\{\ddot{w}\}_t) \\ & \quad - [K]\left(\{w\}_t + \Delta t\{\dot{w}\}_t + \left(\frac{1}{2} - \mu\right)\Delta t^2\{\ddot{w}\}_t\right); \end{aligned} \quad (19)$$

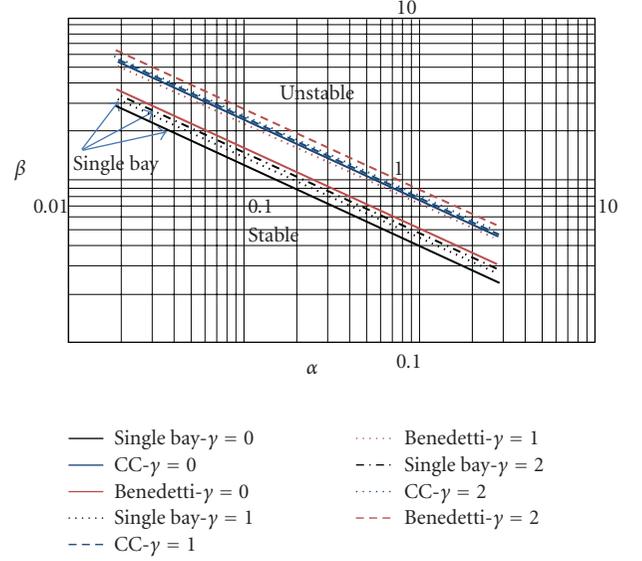


FIGURE 1: Stability boundaries for different cases.

- (2) then the velocity and displacement may be evaluated using

$$\begin{aligned} \{\dot{w}\}_{t+\Delta t} &= \{\dot{w}\}_t + (1 - \delta)\Delta t\{\ddot{w}\}_t + \mu\Delta t\{\ddot{w}\}_{t+\Delta t}, \\ \{w\}_{t+\Delta t} &= \{w\}_t + \Delta t\{\dot{w}\}_t + \left(\frac{1}{2} - \mu\right)\Delta t^2\{\ddot{w}\}_t + \mu\Delta t^2\{\ddot{w}\}_{t+\Delta t}. \end{aligned} \quad (20)$$

In the above algorithm, δ and μ are parameters that have to obey the constraints $\delta \geq 0.5$ and $\mu \geq (1/4)(\delta + (1/2))^2$.

3. RESULTS AND DISCUSSION

3.1. Stability boundaries

A program was written using MATLAB to perform the calculations of the problem using the following data: Beam modulus of elasticity 71 GPa, Density 2840 kg/m³, inner radius 0.007 m, and outer radius 0.008 m. The projectile mass ranged from 0.005 to 0.75 Kg, and its speed ranged from 2.5 to 600 m/s. Two cases were used to demonstrate the validity of the procedure, simply supported and clamped at both sides. Each case was run with three values of foundation stiffness $\gamma = 0, 1, 2$. The program used 10 3-node beam elements, with 5th-order polynomial, (see [15]), and checked for the eigenvalues at 20 equidistant points in each element. Note that the 2-node element, 3rd-order polynomial, was checked for accuracy and the results were not of any considerable difference, rather, the program was already written for 3-node elements and thus used. The convergence of the solution was checked and it was found that the 10 elements with 20 internal points gave adequate accuracy.

Figure 1 shows the results of the two cases with the three foundation stiffness values compared to those obtained from the equation given by Benedetti [2]. It can be obviously

seen in that graph that the results obtained for the clamped-clamped beam are almost identical for the three values of γ . Meanwhile, all the results obtained using the simply supported beam gave lower values for the critical speed. The results obtained from the equation by Benedetti [2], however, varied from nearest to single bay (simply supported) ($\gamma = 0$) to almost identical to clamped-clamped case ($\gamma = 1$) ending with just being higher than the clamped-clamped case with $\gamma = 2$.

Note that the fundamental solution of Benedetti used above gives the relation between the critical speed factor, β_{cr} , and the mass factor, α , as

$$\beta_{cr} = \frac{1}{2}\sqrt{1 + \gamma\alpha^{-0.5}}, \quad (21)$$

where $\alpha = m/\rho AL$ and $\beta = VL/(2\pi\sqrt{\rho A/EI})$. It has to be noted at this point that the results that were presented by Benedetti [2, 3] and Nelson and Conover [1] were obtained for the case of an infinite beam that is periodically simply supported. Thus such a structure should be expected to be more stable than a single bay of simply supported beam. However, the results showed that the clamped-clamped beam (single bay) showed very little change with the foundation stiffness. Finally, it is clear that the results of Benedetti's formula gave values that are much like those of a clamped-clamped beam.

Now the most important observation that may be taken from Figure 1 is that when the stability boundaries are all drawn on log-log scale the results all showed linear trends, further, all lines are parallel with a slope of -0.5 (note that the formula presented by Benedetti had a relation between α and β in the form of $\beta_{cr} = a\alpha^{-0.5}$, see the relation above). Thus it may be concluded that the procedure presented in this paper can accurately predict the stability boundaries of the problem with different boundary conditions. Further, using regression techniques, we may write down a relation for the critical speed factor as

$$\begin{aligned} \beta_{cr}|_{S-S} &\approx 0.195\sqrt{4 + \gamma}\alpha^{-0.5}, \\ \beta_{cr}|_{C-C} &\approx 0.77\alpha^{-0.5}. \end{aligned} \quad (22)$$

The main aim of this study is to determine the stability boundaries for gun barrels. Such structures may be modeled by a cantilever beam. Using the cantilever boundary conditions in the above procedure, and setting the foundation stiffness to zero, we get the critical speed boundaries as presented in Figure 2. The relation may be approximated by the formula

$$\beta_{cr}|_{\text{Cantilever}} \approx 0.21\alpha^{-0.5}. \quad (23)$$

Another common configuration of the gun barrel is the clamped-pinned configuration. This configuration is used to support the tip of the gun barrel and, hence, increases its stability. When that configuration was used in the program, the results obtained were similar to all the other cases. The relation between the mass parameter and the critical velocity parameters may be given by

$$\beta_{cr}|_{\text{Clamped-Pinned}} \approx 0.45\alpha^{-0.5}. \quad (24)$$

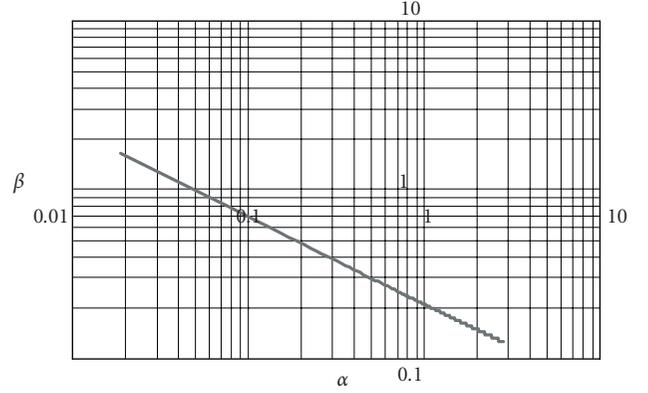


FIGURE 2: Stability boundaries for cantilever beam.

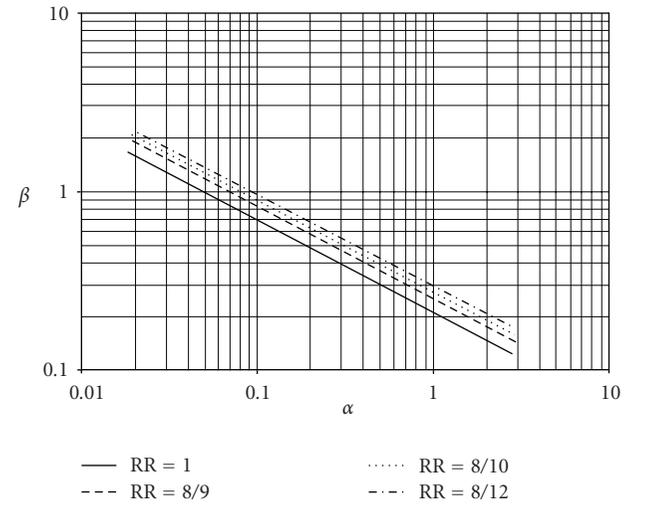


FIGURE 3: Stability boundaries for stepped cantilever beam.

The above relation reflects the expected increase in the stability of the gun barrel when supported at the tip.

Now that the stability boundaries for beams with different boundary conditions are realized to be a simple relation between the mass and velocity parameters, we need to investigate the effect of creating a stepped gun barrel on the stability boundaries. It may be realized that increasing the radius in parts of the gun barrel will automatically increase the stability range as the stiffness increases. Rather, the study would be on how much that increase in radius should be so as not to add unnecessary weight. In the following, the cases, the 8 mm outer radius of the barrel will be the base for the comparison. The barrel will be divided into twelve equal-length parts six of which have 8 mm radius and the other six will have the same radius but with a value more than 8 mm. Figure 3 presents stability boundaries for barrels with different radius ratios (RR). As clearly evident, the barrels with steps of more than 8 mm have higher stability boundaries. But it is also evident that 8 to 9 ratio achieved the highest change in the stability boundaries compared to the changes occurring when increasing the radius ratio from 8 to 9 to 8 to 10. Thus the highest achievement, in terms of gain in stability boundaries, was by increasing the radius by only 12.5%.

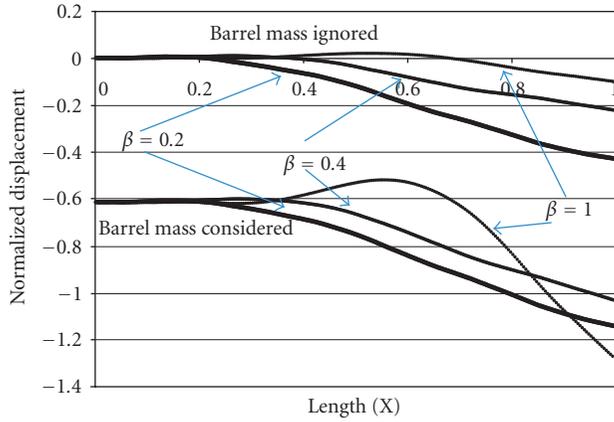


FIGURE 4: Response of the barrel’s tip versus bullet position with and without considering the barrels weight for $\alpha = 0.2$ and different values of β .

3.2. Time response

A program was developed for the time response of the barrel to moving bullets using the Newmark technique presented earlier and the results were compared to response presented in [16] with the consideration that the reference did not include the effect of the external work done by the bullet on the barrel. The values used for the algorithm parameters were $\delta = 0.52$ and $\mu = 0.27$. In all numerical results for the dynamics, presented in this section, the time step used was $1/2400$ of the total time required for the bullet to transverse the barrel. Convergence of the solution was tested using different values of the time step and it was found that this value was accurate and convenient.

The response of the barrel to the motion of the bullet is one of the important aspects that should be considered when designing a gun barrel. As the instability described in the previous section is a static instability, pitchfork bifurcation, it is reflected in higher dynamic response to the external excitation, rather than self-excited vibrations that are associated with Hopf bifurcations. The higher the response becomes, the more the time between the bullets should be to ensure the accuracy of target hits.

One main observation from the literature that studied the response of beams to moving masses or loads is that most of them ignored the deflection of the beam due to its own weight. That type of deflection was ignored mostly because such studies were directed to cases where the load is much more than the beam weight, as in the case of train moving on a railroad. In our problem, the bullet mass, in most guns, is much less than the barrel mass. Hence, ignoring the initial deflection due to mass may introduce much difference in the response. To illustrate the effect of the initial deflections, the response of the barrel to the moving mass is plot in the cases where the barrel mass was not considered. Figure 4 shows clearly that the response, while ignoring the mass, is not a mere shift downwards for the curves, rather, the response, especially at high values of the speed parameter, has a completely different pattern. The normalized deflection is

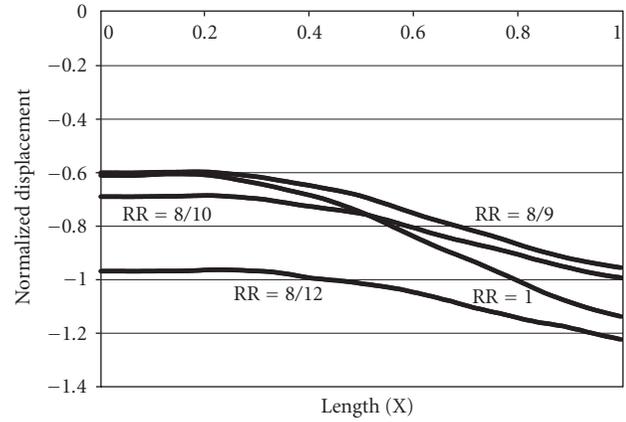


FIGURE 5: Response of the barrel’s tip versus bullet position for stepped beams with different radius ratios with $\alpha = 0.2$ and $\beta = 0.2$.

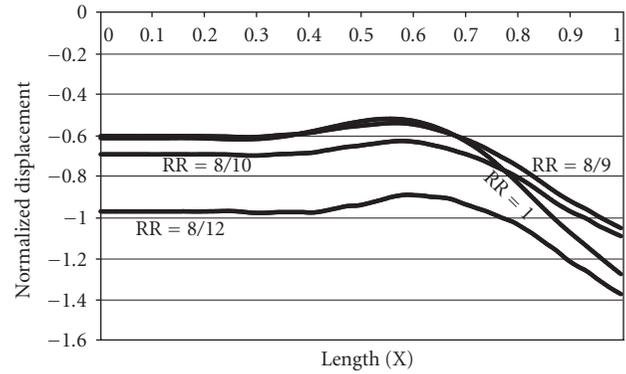


FIGURE 6: Response of the barrel’s tip versus bullet position for stepped beams with different radius ratios with $\alpha = 0.2$ and $\beta = 1.0$.

calculated as

$$w_{\text{normalized}} = \frac{w}{\rho ALg/(EI/L^3)}. \quad (25)$$

This normalization of the deflection will ensure the same response curve for the same values of α and β regardless of the beam properties and geometry.

Investigating the effect of the stepping of the gun barrel, as in the previous section, on the response to the motion of the bullet was the following step. Figure 5 presents the response of the barrel motion of the bullet with $\alpha = 0.2$ and $\beta = 0.2$, while Figure 6 presents the response with $\alpha = 0.2$ and $\beta = 1.0$.

The results in Figures 5 and 6 both agree on that the response of the plain, less stiff, beam is higher than that for stepped beams in general. It may be also observed that the gain, in terms of tip vibration reduction, obtained by increasing the step ratio to 8 to 9 is very good especially that it presents the least increase in weight.

4. CONCLUSIONS

In this study, a finite element model was used to predict the stability boundaries for beams with moving masses subject to different boundary conditions. Because of the lack of literature, the results were compared to classical solutions presented for infinite beams simply supported at equal intervals. As predicted, the solution of SS beams underpredicted the stability boundaries compared to classical solution. Meanwhile, clamped-clamped beams showed, almost, no change with foundation stiffness.

An empirical relation between the mass parameter and critical speed parameter could be obtained for simply supported, clamped-clamped, cantilever, and clamped-pinned beams. Generally, the relation was given by the relation $\beta_{cr} \approx c\alpha^{-0.5}$, where c is a constant that is determined by the boundary conditions of the beam.

Also the effect of using stepped barrels was studied to investigate the feasibility of such techniques. It was found that with, considerably, small increase in the radius of the barrel in some parts, a significant increase in the stability boundaries was obtained.

In this study, for the first time in literature, the stability boundaries were predicted using eigenvalues of the system rather than using time marching techniques. The results presented in this paper are, to the extent of the authors knowledge, the first in the literature to present accurately the relation between the mass parameter and critical speed parameter for beams with general boundary conditions.

Further, the model was used to predict the response of the tip of the gun barrel to the motion of the bullet using Newmark algorithm. The results emphasized that the weight of the barrel should be included in the calculations as constant force distributed on the beam. Also the results showed that the response may also be reduced using the stepped barrels.

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