Research Letter

Differential Gain in InGaAsN/GaAs Double Quantum Well Structures by Numerical Simulations

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We performed numerical studies of differential gain in a coupled quantum well structures built from InGaAsN. Differential gain in In0.38Ga0.62AsN/GaAs quantum well structures was determined and analyzed. A 10-band k.p Hamiltonian matrix was used in the calculations and solved self-consistently with Poisson’s equation. The effect of Nitrogen composition and barrier thickness on differential gain has been determined. The influence of Nitrogen composition on differential gain is significant whereas barrier effects are modest.

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1. INTRODUCTION

A new material system InGaAsN is currently under intensive investigations. Several properties of those systems have been analyzed [1–3]. InGaAsN is a practical alternative to InP-based materials for 1.3 μm operation of quantum-well-based semiconductor lasers. The advantages over InP systems include lower cost, higher characteristic temperature T0, higher output power, better heat conduction, and applicability to use in vertical cavity surface emitting lasers.

For this new material system there is a need to analyze its properties versus different parameters. One of those parameters is coupling between two quantum wells. Recently, we addressed the question of the effect of well coupling in InGaAsN on the effective masses and optical gain [4, 5]. The optical gain spectra were obtained for various values of Nitrogen composition and barrier widths. They show optimum values for particular barrier width, although the variations are not significant. However, the magnitude of gain peak undergoes significant changes when varying Nitrogen composition.

We have also recently reported on the results of differential gain for single quantum well [6] fabricated using InGaAsN/GaAs material system. It was predicted that spectra of differential gain show significant variation versus Nitrogen composition, well width, and carrier density.

In order to properly manipulate differential gain and optical gain in achieving prescribed engineering goals, we introduced here one more design parameter, namely barrier width of a double-well system. As it is known, for a GaAs/Al0.2Ga0.8As double quantum well system [7], barrier thickness has a significant effect on differential gain. The differential gain at the gain peak is not a monotonic function of the barrier thickness. For relatively thick barriers inclusion of coupling reduces the differential gain but as the barrier gets thinner the differential gain can be significantly enhanced or suppressed compared to its uncoupled value.

In the present paper, we used the theory leading to the determination of differential gain of two wells consisting of InGaAsN. The details of our approach are described in [5, 6]. The gain formulation incorporates non-Markovian effects, Coulombic interactions, and also bandgap renormalization. In the determination of electron and hole band structures we also incorporated self-consistent effects by solving the relevant matrix Schrödinger equation simultaneously with Poisson’s equation.

In Section 2, we outline briefly our formalism and in Section 3 present our results and conclusions.

2. FORMALISM

The formalism used in our paper has been recently described by us [5]. We use a plane-wave expansion method to calculate
the electron and hole band structures of the quantum well and a 10 × 10 Hamiltonian as presented by Tomić et al. [8], where the effect of adding nitrogen to the structure is modeled perturbatively. The Hamiltonian has been also analyzed by O’Reilly et al. [9] and Hader et al. [10]. It can be considered as a good model of InGaAsN system. The selection of parameters has been discussed extensively by Choulis et al. [11]. Specific form of this Hamiltonian is

\[
H = \begin{pmatrix}
E_N & V_N & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
E_C & -\sqrt{3}T_r & \sqrt{2}U & -U & 0 & 0 & 0 & -T_r & -\sqrt{2}T_r & 0 \\
E_{HH} & \sqrt{2}S & -S & 0 & 0 & -R & -\sqrt{2}R & 0 & 0 & 0 \\
E_L & 0 & T_r^* & R & 0 & 0 & 0 & 0 & 0 & 0 \\
E_{SO} & \sqrt{2}T_r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
E_N & V_N & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
E_C & -\sqrt{3}T_r & \sqrt{2}U & -U & 0 & 0 & 0 & -T_r & -\sqrt{2}T_r & 0 \\
E_{HH} & \sqrt{2}S & -S & 0 & 0 & -R & -\sqrt{2}R & 0 & 0 & 0 \\
E_L & 0 & T_r^* & R & 0 & 0 & 0 & 0 & 0 & 0 \\
E_{SO} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

(1)

Here the subscripts N, C, HH, LH, and SO stand for nitrogen, conduction, heavy-hole, light-hole, and split-off bands, respectively.

The self-consistent method is similar to the one which we used before [12] which consists in a self-consistent solution of the Poisson equation with the matrix Schroedinger equation described by Hamiltonian (1). The Poisson equation is

\[
d \frac{d}{dz} \left[ \varepsilon(z) \frac{d}{dz} \phi(z) \right] = -e \left[ \rho_{HH}(z) + \rho_{LH}(z) - \rho_C(z) \right],
\]

(2)

where \(\varepsilon(z)\) is the fundamental permittivity, \(\rho_C(z)\), \(\rho_{HH}(z)\) and \(\rho_{LH}(z)\) are the position-dependent electron and hole band density distributions, respectively. The function \(\phi(z)\) is the electrostatic potential. The density distributions are [12]

\[
\rho_n(z) = \frac{k_B T}{\pi \hbar^2} \left\{ \sum_m \left| F_{n,m}(z) \right|^2 \ln \left[ 1 + \exp \left( \frac{E_{n,m} - E_{n,m}^\parallel}{k_B T} \right) \right] \right\},
\]

(3)

The symbol \(\alpha\) represents the conduction (C), heavy-hole (HH) and light-hole (LH) bands and \(n\) is an index over the subbands, \(E_n^\parallel\) is the conduction band Fermi level and \(E_{HH}^\parallel, E_{LH}^\parallel\) is the valence band Fermi level. Their values are determined by the standard methods [13]. The symbol \(k_B\) is Boltzmann’s constant, \(T\) is temperature, \(m_n^*\) is the average effective mass for the particular band which is approximated as the effective mass in the well since most of the carriers are confined there. Here, \(F_{n,m}(z)\) and \(E_{n,m}^\parallel\) are the respective envelope eigenfunctions and eigenvalues of the various subbands.

We use our previous relation for optical gain quantity [5, 6], which incorporates band-gap renormalization, Coulombic, and non-Markovian effects [14–19]. The TE material gain is given by

\[
g(N, \omega) = \frac{\omega \mu c}{n_i V \sum_{\ell m} \sum_{k_i} |\hat{\epsilon} \cdot \mathbf{M}_{\ell m}^n(k_i)|^2}
\times \left[ f_i^\parallel(N, k_i) - f_i^\perp(N, k_i) \right] \Re \left[ \frac{\mathbf{q}_{\ell m}^n(0, \Delta_{\ell m}^n(k_i))}{1 - \Re \mathbf{q}_{\ell m}^n(N, k_i)} \right],
\]

(4)

The details of this equation can be found in [19]. We have added the functional dependence on carrier density \(N\) for clarity. The term \(\mathbf{M}_{\ell m}^n(k_i)\) represents the dipole matrix elements for which the calculation of electron and hole wavefunctions was required. For this case, the holes are solved in the LK approximation. The superscript \(\nu\) on the variables represents the heavy and light hole bands and \(l\) and \(m\) represent the subband indices of the conduction and hole subbands.

The calculations are performed in a self-consistent way as described before [12]. Optical gain is determined using (4). Finally, the differential gain is determined by numerically differentiating expression for TE gain as

\[
g'(E, N) = g(E, N + \delta N) - g(E, N),
\]

(5)

where \(\delta N\) is a small increment in the average carrier density in the wells.

3. RESULTS AND CONCLUSIONS

We performed our analysis for a double quantum well system containing In0.38Ga0.62As1−yNy wells and In0.03Ga0.95As0.985N0.015 barriers. In our calculations, the well widths are 7 nm and composition of nitrogen in the wells varies from 0.5% to 1.5%. The barrier was varied between 2 nm and 6 nm.

Figure 1 shows differential gain calculated at optical gain peak versus nitrogen composition for several values of barrier thickness.

![Figure 1: Differential gain calculated at optical gain peak versus nitrogen composition for several values of barrier thickness.](image-url)
To analyze barrier dependence, in Figure 2 we showed differential gain calculated at optical gain peak versus barrier thickness for several values of nitrogen composition. The strongest dependence on barrier thickness is for 0.5% of nitrogen composition. For 1.5% of nitrogen the value of differential gain for 2 nm to 6 nm barrier’s thicknesses is almost flat.

In Figure 3 we analyze spectra of differential gain for the case of 0.5% of nitrogen. The peak of differential gain is around 1080 nm range and shifts towards smaller wavelengths when barrier width increases. It is far from the peak value of optical gain which for 0.5% of nitrogen is around 1190 nm wavelength, see Figure 4. A similar situation can be observed for 1.0% and 1.5% of nitrogen where also peak of differential gain lies in a different range of wavelengths than peak of optical gain.

Finally, in Figure 4 we showed the results of the wavelength dependence of maximum gain and differential gain versus nitrogen composition for 2 nm barrier thickness. One can observe linear dependence of nitrogen composition in both cases. The differential gain spectrum is a direct implication of the conduction band nonparabolicity of GaInNAs alloys which results in the high electron effective mass.

In conclusion, we have analyzed the effects of nitrogen composition and barrier width in a double quantum well system consisting of InGaAsN/GaAs. A 10-band $k \cdot p$ Hamiltonian matrix was used in the calculations and solved self-consistently with Poisson’s equation. The effect of nitrogen composition and barrier thickness on differential gain has been determined. The influence of nitrogen composition on differential gain is significant whereas barrier effects are modest.

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