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Research Letter

Differential Gain in InGaAsN/GaAs Double Quantum Well Structures by Numerical Simulations

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We performed numerical studies of differential gain in a coupled quantum well structures built from InGaAsN. Differential gain in $In_{0.38}Ga_{0.62}As_{1-y}N_y$/GaAs quantum well structures was determined and analyzed. A 10-band k_p Hamiltonian matrix was used in the calculations and solved self-consistently with Poisson's equation. The effect of Nitrogen composition and barrier thickness on differential gain has been determined. The influence of Nitrogen composition on differential gain is significant whereas barrier effects are modest.

1. INTRODUCTION

A new material system $In_xGa_{1-x}As_{1-y}N_y$ is currently under intensive investigations. Several properties of those systems have been analyzed [1–3]. $In_xGa_{1-x}As_{1-y}N_y$ is a practical alternative to InP-based materials for 1.3 μm operation of quantum-well-based semiconductor lasers. The adventages over InP systems include lower cost, higher characteristic temperature T_0, higher output power, better heat conduction, and applicability to use in vertical cavity surface emitting lasers.

For this new material system there is a need to analyze its properties versus different parameters. One of those parameters is coupling between two quantum wells. Recently, we addressed the question of the effect of well coupling in $In_xGa_{1-x}As_{1-y}N_y$ on the effective masses and optical gain [4, 5]. The optical gain spectra were obtained for various values of Nitrogen composition and barrier widths. They show optimum values for particular barrier width, although the variations are not significant. However, the magnitude of gain peak undergoes significant changes when varying Nitrogen composition.

We have also recently reported on the results of differential gain for single quantum well [6] fabricated using InGaAsN/GaAs material system. It was predicted that spectra of differential gain show significant variation versus Nitrogen composition, well width, and carrier density.

In order to properly manipulate differential gain and optical gain in achieving prescribed engineering goals, we introduced here one more design parameter, namely barrier width of a double-well system. As it is known, for a GaAs/$Al_{0.2}Ga_{0.8}As$ double quantum well system [7], barrier thickness has a significant effect on differential gain. The differential gain at the gain peak is not a monotonic function of the barrier thickness. For relatively thick barriers inclusion of coupling reduces the differential gain but as the barrier gets thinner the differential gain can be significantly enhanced or suppressed compared to its uncoupled value.

In the present paper, we used the theory leading to the determination of differential gain of two wells consisting of $In_xGa_{1-x}As_{1-y}N_y$. The details of our approach are described in [5, 6]. The gain formulation incorporates non-Markovian effects, Coulombic interactions, and also band-gap renormalization. In the determination of electron and hole band structures we also incorporated self-consistent effects by solving the relevant matrix Schroedinger equation simultaneously with Poisson's equation.

In Section 2, we outline briefly our formalism and in Section 3 present our results and conclusions.

2. FORMALISM

The formalism used in our paper has been recently described by us [5]. We use a plane-wave expansion method to calculate

the electron and hole bandstructures of the quantum well and a 10×10 Hamiltonian as presented by Tomić et al. [8], where the effect of adding nitrogen to the structure is modeled perturbatively. The Hamiltonian has been also analyzed by O'Reilly et al. [9] and Hader et al. [10]. It can be considered as a good model of InGaAsN system. The selection of parameters has been discussed extensively by Choulis et al. [11]. Specific form of this Hamiltonian is

$$H=\begin{pmatrix} E_{\mathrm{N}} & V_{Nc} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & E_{\mathrm{C}} & -\sqrt{3}T_{+} & \sqrt{2}U & -U & 0 & 0 & 0 & -T_{-} & -\sqrt{2}T_{-} \\ & & E_{\mathrm{HH}} & \sqrt{2}S & -S & 0 & 0 & 0 & -R & -\sqrt{2}R \\ & & & E_{\mathrm{LH}} & Q & 0 & T_{+}^{*} & R & 0 & \sqrt{3}S \\ & & & & E_{\mathrm{SO}} & 0 & \sqrt{2}T_{+}^{*} & \sqrt{2}R & -\sqrt{3}S & 0 \\ & & & & & E_{\mathrm{N}} & V_{Nc} & 0 & 0 & 0 \\ & & & & & & E_{\mathrm{C}} & -\sqrt{3}T_{-} & \sqrt{2}U & -U \\ & & & & & & & E_{\mathrm{HH}} & \sqrt{2}S^{*} & -S^{*} \\ & & & & & & & & E_{\mathrm{LH}} & Q \\ & & & & & & & & & E_{\mathrm{SO}} \end{pmatrix}. \tag{1}$$

Here the subscripts N, C, HH, LH, and SO stand for nitrogen, conduction, heavy-hole, light-hole, and split-off bands, respectively.

The self-consistent method is similar to the one which we used before [12] which consists in a self-consistent solution of the Poisson equation with the matrix Schroedinger equation described by Hamiltonian (1). The Poisson equation is

$$\frac{d}{dz}\left[\varepsilon(z)\frac{d}{dz}\varphi(z)\right] = -e\left[\rho_{\mathrm{HH}}(z)+\rho_{\mathrm{LH}}(z)-\rho_{\mathrm{C}}(z)\right], \tag{2}$$

where e is the fundamental charge, $\varepsilon(z)$ is the position-dependent permittivity, $\rho_{\mathrm{C}}(z)$, $\rho_{\mathrm{HH}}(z)$ and $\rho_{\mathrm{LH}}(z)$ are the position-dependent electron and hole band density distributions, respectively. The function $\varphi(z)$ is the electrostatic potential. The density distributions are [12]

$$\rho_{\alpha}(z) = \frac{k_B T}{\pi\hbar^2}\left\{\sum_n \overline{m}_{\alpha}^{*}\,|F_{n,\alpha}^{p}(z)|^2 \ln\left[1+\exp\left(\frac{E_{\alpha}^{f}-E_{n,\alpha}^{p}}{k_B T}\right)\right]\right\}. \tag{3}$$

The symbol α represents the conduction (C), heavy-hole (HH) and light-hole (LH) bands and n is an index over the subbands. E_c^f is the conduction band Fermi level and $E_{\mathrm{HH}}^f = E_{\mathrm{LH}}^f$ is the valence band Fermi level. Their values are determined by the standard methods [13]. The symbol k_B is Boltzmann's constant, T is temperature, $\overline{m}_{\alpha}^{*}$ is the average effective mass for the particular band which is approximated as the effective mass in the well since most of the carriers are confined there. Here, $F_{n,\alpha}^{p}(z)$ and $E_{n,\alpha}^{p}$ are the respective envelope eigenfunctions and eigenvalues of the various subbands.

We use our previous relation for optical gain quantity [5, 6], which incorporates band-gap renormalization, Coulombic, and non-Markovian effects [14–19]. The TE material gain is given by

$$g(N,\omega) = \frac{\omega\mu c}{n_r V}\sum_{vlm}\sum_{\mathbf{k}_{\parallel}} |\hat{\varepsilon}\cdot\mathbf{M}_{lm}^{v}(\mathbf{k}_{\parallel})|^2 \times \frac{(f_l^c(N,\mathbf{k}_{\parallel}) - f_m^v(N,\mathbf{k}_{\parallel}))\Re\Xi_{lm}^{v}(0,\Delta_{lm}^{v}(\mathbf{k}_{\parallel}))}{1-\Re q_{1lm}^{v}(N,\mathbf{k}_{\parallel})}. \tag{4}$$

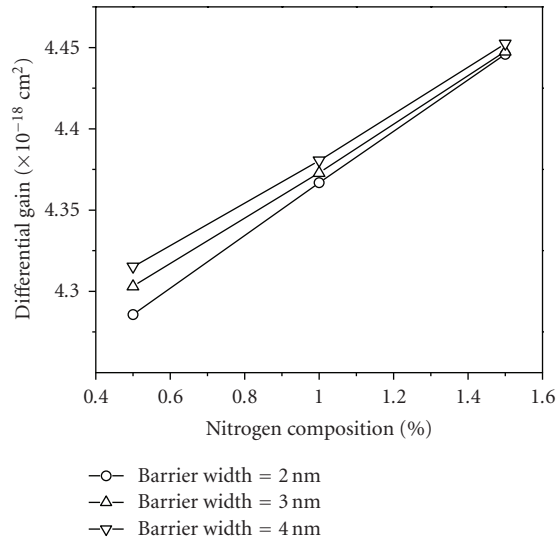

Figure 1: Differential gain calculated at optical gain peak versus nitrogen composition for several values of barrier thickness.

The details of this equation can be found in [19]. We have added the functional dependence on carrier density N for clarity. The term $\mathbf{M}_{lm}^{v}(\mathbf{k}_{\parallel})$ represents the dipole matrix elements for which the calculation of electron and hole wavefunctions was required. For this case, the holes are solved in the LK approximation. The superscript v on the variables represents the heavy and light hole bands and l and m represent the subband indices of the conduction and hole subbands.

The calculations are performed in a self-consistent way as described before [12]. Optical gain is determined using (4). Finally, the differential gain is determined by numerically differentiating expression for TE gain as

$$g'(E,N) \approx \frac{g(E,N+\delta N)-g(E,N)}{\delta N}, \tag{5}$$

where δN is a small increment in the average carrier density in the wells.

3. RESULTS AND CONCLUSIONS

We performed our analysis for a double quantum well system containing $In_{0.38}Ga_{0.62}As_{1-y}N_y$ wells and $In_{0.05}Ga_{0.95}As_{0.985}N_{0.15}$ barriers. In our calculations, the well widths are 7 nm and composition of nitrogen in the wells varies from 0.5% to 1.5%. The barrier was varied between 2 nm and 6 nm.

Figure 1 shows differential gain calculated at optical gain peak versus nitrogen composition for several values of barrier thickness. One observes significant dependence of the differential gain on nitrogen composition. For the considered cases, it attains the largest value of 4.45×10^{-18} cm^2 for 1.5% of nitrogen, which fits well in the 1.3 μm telecommunication window. At the same time, the dependence on the barrier width is relatively weak.

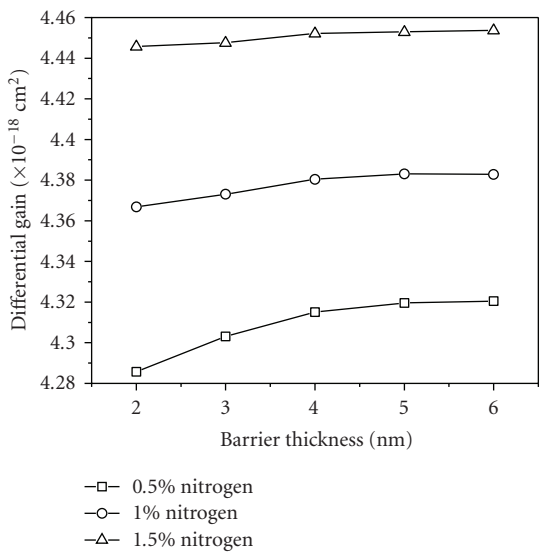

Figure 2: Differential gain calculated at optical gain peak versus barrier thickness for several values of Nitrogen composition.

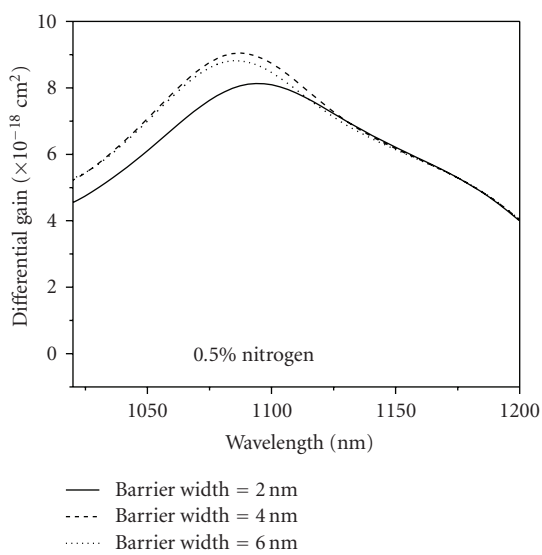

Figure 3: Differential gain spectra for several values of barrier thickness and nitrogen composition of 0.5%.

To analyze barrier dependence, in Figure 2 we showed differential gain calculated at optical gain peak versus barrier thickness for several values of nitrogen composition. The strongest dependence on barrier thickness is for 0.5% of nitrogen composition. For 1.5% of nitrogen the value of differential gain for 2 nm to 6 nm barrier's thicknesses is almost flat.

In Figure 3 we analyze spectra of differential gain for the case of 0.5% of nitrogen. The peak of differential gain is around 1080 nm range and shifts towards smaller wavelengths when barrier width increases. It is far from the peak value of optical gain which for 0.5% of nitrogen is around 1190 nm wavelength, see Figure 4. A similar situation

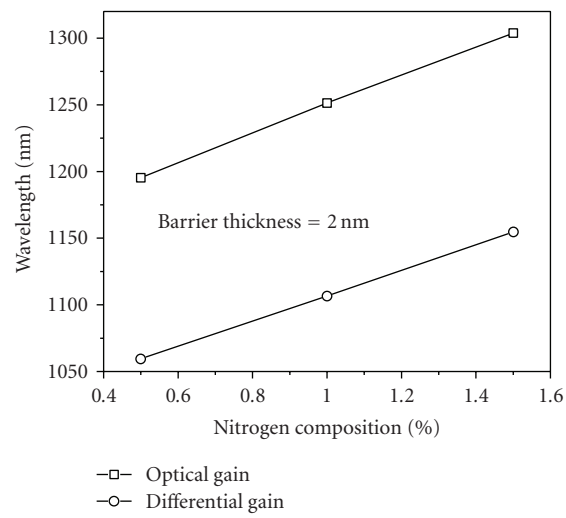

Figure 4: Wavelength of maximum gain and differential gain versus nitrogen composition for 2 nm barrier thickness.

can be observed for 1.0% and 1.5% of nitrogen where also peak of differential gain lies in a different range of wavelengths than peak of optical gain.

Finally, in Figure 4 we showed the results of the wavelength dependence of maximum gain and differential gain versus nitrogen composition for 2 nm barrier thickness. One can observe linear dependence of nitrogen composition in both cases. The differential gain spectrum is a direct implication of the conduction band nonparabolicity of GaInNAs alloys which results in the high electron effective mass.

In conclusion, we have analyzed the effects of nitrogen composition and barrier width in a double quantum well system consisting of InGaAsN/GaAs. A 10-band $k \cdot p$ Hamiltonian matrix was used in the calculations and solved self-consistently with Poisson's equation. The effect of nitrogen composition and barrier thickness on differential gain has been determined. The influence of nitrogen composition on differential gain is significant whereas barrier effects are modest.

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REFERENCES

[1] M. Kondow, K. Uomi, A. Niwa, T. Kitatani, S. Watahiki, and Y. Yazawa, "GaInNAs: a novel material for long-wavelength-range laser diodes with excellent high-temperature performance," *Japanese Journal of Applied Physics*, vol. 35, no. 2, pp. 1273–1275, 1996.

[2] M. Kondow and T. Kitatani, "Progress in research into mixed group-V nitride alloys," *IEE Proceedings: Optoelectronics*, vol. 150, no. 1, pp. 9–11, 2003.

[3] P. J. Klar, "Recent developments in metastable dilute-N III-V semiconductors," *Progress in Solid State Chemistry*, vol. 31, no. 4, pp. 301–349, 2003.

[4] M. S. Wartak and P. Weetman, "The effect of well coupling on effective masses in the InGaAsN material system," *Journal of Physics: Condensed Matter*, vol. 19, no. 27, Article ID 276202, 8 pages, 2007.

[5] M. S. Wartak and P. Weetman, "Numerical studies of optical gain of coupled quantum wells based on InGaAsN material system," *Physica Status Solidi (B)*, vol. 245, pp. 726–730, 2008.

[6] M. S. Wartak and P. Weetman, "Spectra of differential gain and linewidth enhancement factor in InGaAsN/GaAs single quantum well structures," *Physica Status Solidi (A)*, vol. 204, no. 7, pp. 2487–2494, 2007.

[7] A. I. Akhtar and J. M. Xu, "Differential gain in coupled quantum well lasers," *Journal of Applied Physics*, vol. 78, no. 5, pp. 2962–2969, 1995.

[8] S. Tomić, E. P. O'Reilly, R. Fehse, et al., "Theoretical and experimental analysis of 1.3 μm InGaAsN/GaAs lasers," *IEEE Journal on Selected Topics in Quantum Electronics*, vol. 9, no. 5, pp. 1228–1238, 2003.

[9] E. P. O'Reilly and A. Lindsay, "$k\cdot p$ model of ordered GaN_xAs_{1-x}," *Physica Status Solidi (B)*, vol. 216, no. 1, pp. 131–134, 1999.

[10] J. Hader, S. W. Koch, J. V. Moloney, and E. P. O'Reilly, "Gain in 1.3 μm materials: InGaNAs and InGaPAs semiconductor quantum-well lasers," *Applied Physics Letters*, vol. 77, no. 5, pp. 630–632, 2000.

[11] S. A. Choulis, T. J. C. Hosea, S. Tomić, et al., "Electronic structure of $In_yGa_{1-y}As_{1-x}N_x$/GaAs multiple quantum wells in the dilute-N regime from pressure and $k\cdot p$ studies," *Physical Review B*, vol. 66, no. 16, Article ID 165321, 9 pages, 2002.

[12] P. Weetman, M. S. Wartak, and M. Kucharczyk, "Modeling the electrostatic and band-mixing effects on gain for double-quantum-well lasers," *Microwave and Optical Technology Letters*, vol. 33, no. 1, pp. 35–37, 2002.

[13] S.-L. Chuang, *Physics of Optoelectronic Devices*, John Wiley & Sons, New York, NY, USA, 1995.

[14] S. Seki, K. Yokoyama, and P. Sotirelis, "Theoretical analysis of high-temperature characteristics of 1.3 μm InP-based quantum-well lasers," *IEEE Journal on Selected Topics in Quantum Electronics*, vol. 1, no. 2, pp. 264–274, 1995.

[15] C.-F. Hsu, P. S. Zory, C.-H. Wu, and M. A. Emanuel, "Coulomb enhancement in InGaAs-GaAs quantum-well lasers," *IEEE Journal on Selected Topics in Quantum Electronics*, vol. 3, no. 2, pp. 158–165, 1997.

[16] M. J. Connelly, "Wideband semiconductor optical amplifier steady-state numerical model," *IEEE Journal of Quantum Electronics*, vol. 37, no. 3, pp. 439–447, 2001.

[17] W. W. Chow, S. W. Koch, and M. Sargent III, *Semiconductor-Laser Physics*, Springer, Berlin, Germany, 1997.

[18] J. Minch, S. H. Park, T. Keating, and S. L. Chuang, "Theory and experiment of $In_{1-x}Ga_xAs_yP_{1-y}$ and $In_{1-x-y}Ga_xAl_yAs$ long-wavelength strained quantum-well lasers," *IEEE Journal of Quantum Electronics*, vol. 35, no. 5, pp. 771–782, 1999.

[19] D. Ahn, "Optical gain of a quantum-well laser with non-Markovian relaxation and many-body effects," *IEEE Journal of Quantum Electronics*, vol. 32, no. 6, pp. 960–965, 1996.