

## Research Article

# Solar System Tests of Some Models of Modified Gravity Proposed to Explain Galactic Rotation Curves without Dark Matter

Lorenzo Iorio<sup>1</sup> and Matteo Luca Ruggiero<sup>2,3</sup>

<sup>1</sup> *Istituto Nazionale di Fisica Nucleare (INFN), Sezione di Pisa, 3 - 56127 Pisa, Italy*

<sup>2</sup> *Dipartimento di Fisica, Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy*

<sup>3</sup> *Istituto Nazionale di Fisica Nucleare (INFN), Sezione di Torino, Via Pietro Giuria, 1 10125 Torino, Italy*

Correspondence should be addressed to Lorenzo Iorio, [lorenzo.iorio@libero.it](mailto:lorenzo.iorio@libero.it)

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We consider the recently estimated corrections  $\Delta\bar{\omega}$  to the standard Newtonian/Einsteinian secular precessions of the longitudes of perihelia  $\bar{\omega}$  of several planets of the Solar System in order to evaluate whether they are compatible with the predicted anomalous precessions due to models of long-range modified gravity put forth to account for certain features of the rotation curves of galaxies without resorting to dark matter. In particular, we consider a logarithmic-type correction and a  $f(R)$  inspired power-law modification of the Newtonian gravitational potential. The results obtained by taking the ratio of the predicted apsidal rates for different pairs of planets show that the modifications of the Newtonian potentials examined in this paper are not compatible with the secular extra-precessions of the perihelia of the Solar System's planets estimated by E. V. Pitjeva as solve-for parameters processing almost one century of data with the latest EPM ephemerides.

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## 1. Introduction

Dark matter and dark energy are, possibly, the most severe theoretical issues that modern astrophysics and cosmology have to face, since the available observations seem to question the model of gravitational interaction on a scale larger than the Solar System. In fact, the data coming from the galactic rotation curves of spiral galaxies [1] cannot be explained on the basis of Newtonian gravity or general relativity (GR) if one does not introduce invisible dark matter compensating for the observed inconsistency of the Newtonian dynamics of stars. Likewise, since 1933, when Zwicky [2] studied the velocity dispersion in the Coma cluster, there is the common agreement on the fact that the dynamics of cluster of galaxies is poorly understood on the basis of Newtonian gravity or GR [3, 4] if the dark matter is not taken into account. In order to reconcile theoretical models with observations, the existence of a peculiar form of matter is postulated, the so-called *dark matter*, which is supposed to be a cold and pressureless medium, whose distribution is that of a spherical halo around the galaxies. On the other hand, a lot of observations, such as the light curves of the type Ia supernovae and the cosmic microwave background (CMB) experiments [5–8],

firmly state that our Universe is now undergoing a phase of accelerated expansion. Actually, the present acceleration of the Universe cannot be explained within GR, unless the existence of a cosmic fluid having exotic equation of state is postulated, the so-called *dark energy*.

The main problem dark matter and dark energy bring about is understanding their nature, since they are introduced as ad hoc gravity sources in a well-defined model of gravity, that is, GR (or Newtonian gravity). Of course, another possibility exists; GR (and its approximation, Newtonian gravity) is unfit to deal with gravitational interaction at galactic, intergalactic, and cosmological scales. The latter viewpoint led to the introduction of various modifications of gravity [9, 10].

However, we must remember that, even though there are problems with galaxies and clusters dynamics and cosmological observations, GR is in excellent agreement with the Solar System experiments (see [11, 12]). Hence, every theory that aims at explaining the large-scale dynamics and the accelerated expansion of the Universe should reproduce GR at the Solar System scale, that is, in a suitable weak field limit. So, the viability of these modified theories of gravity at Solar System scale should be studied with great care.

In this paper, we wish to quantitatively deal with such a problem by means of the secular (i.e., averaged over one orbital revolution) precessions of the longitudes of the perihelia  $\hat{\omega}$  of some planets of the Solar System (see also [13, 14]) in the following way. Generally speaking, let long-range modified model of gravity (LRMOG) be a given exotic model of modified gravity parameterized in terms of, say,  $K$ , in such a way that  $K = 0$  would imply no modifications of gravity at all. Let  $\mathcal{P}$ (LRMOG) be the prediction of a certain effect induced by such a model like the secular precession of the perihelion of a planet. For all the exotic models considered, it turns out that

$$\mathcal{P}(\text{LRMOG}) = Kg(a, e), \quad (1)$$

where  $g$  is a function of the system's orbital parameters  $a$  (semimajor axis) and  $e$  (eccentricity); such  $g$  is a peculiar consequence of the model LRMOG (and of all other models of its class with the same spatial variability). Now, let us take the ratio of  $\mathcal{P}$ (LRMOG) for two different systems A and B, for example, two Solar System's planets:  $\mathcal{P}_A(\text{LRMOG})/\mathcal{P}_B(\text{LRMOG}) = g_A/g_B$ . The model's parameter  $K$  is now disappeared, but we still have a prediction that retains a peculiar signature of that model, that is,  $g_A/g_B$ . Of course, such a prediction is valid if we assume that  $K$  is not zero, which is just the case both theoretically (If  $K$  was zero, no modifications at all would occur) and observationally because  $K$  is usually determined by other independent long-range astrophysical/cosmological observations. Otherwise, one would have the meaningless prediction  $0/0$ . The case  $K = 0$  (or  $K \leq \bar{K}$ ) can be, instead, usually tested by taking one perihelion precession at a time, as already done, for example, in [13, 14].

If we have observational determinations  $\mathcal{O}$  for A and B of the effect considered above such that they are affected also by LRMOG (if they are differential quantities constructed by contrasting observations to predictions obtained by analytical force models of canonical effects,  $\mathcal{O}$  are, in principle, affected also by the mismodelling in them) (it is just the case for the purely phenomenologically estimated corrections to the standard Newton-Einstein perihelion precessions, since LRMOG has not been included in the dynamical force models of the ephemerides adjusted to the planetary data in the least-square parameters' estimation process by Pitjeva [15–17]), we can construct  $\mathcal{O}_A/\mathcal{O}_B$  and compare it with the prediction for it by LRMOG, that is, with  $g_A/g_B$ . Note that  $\delta\mathcal{O}/\mathcal{O} > 1$  only means that  $\mathcal{O}$  is compatible with zero, being possible a nonzero value smaller than  $\delta\mathcal{O}$ . Thus, it is perfectly meaningful to construct  $\mathcal{O}_A/\mathcal{O}_B$ . Its uncertainty will be conservatively evaluated as  $|1/\mathcal{O}_B|\delta\mathcal{O}_A + |\mathcal{O}_A/\mathcal{O}_B^2|\delta\mathcal{O}_B$ . As a result,  $\mathcal{O}_A/\mathcal{O}_B$  will be compatible with zero. Now, the question is: Is it the same for  $g_A/g_B$  as well? If yes, that is, if

$$\frac{\mathcal{O}_A}{\mathcal{O}_B} = \frac{\mathcal{P}_A(\text{LRMOG})}{\mathcal{P}_B(\text{LRMOG})} \quad (2)$$

within the errors, or, equivalently, if

$$\left| \frac{\mathcal{O}_A}{\mathcal{O}_B} - \frac{\mathcal{P}_A(\text{LRMOG})}{\mathcal{P}_B(\text{LRMOG})} \right| = 0 \quad (3)$$

within the errors, LRMOG survives (and the use of the single perihelion precessions can be used to put upper bounds on  $K$ ). Otherwise, LRMOG is challenged.

The paper is organized as follows. In Section 2, we apply this approach to a test particle in motion around a central mass  $M$  whose Newtonian gravitational potential exhibits a logarithmic-type correction. Then, in Section 3, we consider the power-law modification of the gravitational potential inspired by  $f(R)$  extended theories of gravity. Comments and conclusions are outlined in Section 4.

## 2. The Perihelion Precession Due to a $1/r$ Force

Logarithmic corrections to the Newtonian gravitational potential have been recently used to phenomenologically tackle the problem of dark matter in galaxies [18–23]. For example, Fabris and Campos [22] used

$$V_{\text{ln}} = -\alpha GM \ln\left(\frac{r}{r_0}\right), \quad (4)$$

where  $\alpha$  has the dimension of  $L^{-1}$ , to satisfactorily fit the rotation curves of 10 spiral galaxies getting

$$\alpha \approx -0.1 \text{ kpc}^{-1}. \quad (5)$$

The extra potential of (4) yields an additional  $1/r$  radial force (for another example of a  $1/r$  extra force and its connection with galaxy rotation curves see [24]):

$$\mathbf{A}_{\text{ln}} = \frac{\alpha GM}{r} \hat{\mathbf{r}}. \quad (6)$$

Various theoretical justifications have been found for a logarithmic-like extra potential. For example, according to Kirillov [21], it would arise from large-scale discrepancies of the topology of the actual Universe from the Friedmann space; Sobouti [23] obtained it in the framework of the  $f(R)$  modifications of general relativity getting preliminarily flat rotation curves, the Tully-Fisher relation (admittedly with some reservations), and a version of MOND, while Fabris and Campos [22] pointed out that string-like objects [25, 26] would yield a logarithmic-type potential with  $\alpha$  related to the string tension.

Alternative tests of such proposed correction to the Newtonian potential, independent of the dark matter issues themselves, would be, of course, highly desirable and could be, in principle, conducted in the Solar System. This is, indeed, considered one of the tasks to be implemented in further investigations by Sobouti [23]; Fabris and Campos [22] argue that, in view of their extreme smallness due to (5), no detectable effects induced by (6) would be possible at the level of Solar System.

Pitjeva has recently processed almost one century of data of different types for the major bodies of the Solar System in the effort of continuously improving the EPM2004/EPM2006 planetary ephemerides [15, 17]. Among other things, she also simultaneously estimated corrections  $\Delta\hat{\omega}$  to the secular rates of the longitudes of perihelia  $\hat{\omega}$  of the inner [16] and of some of the outer [17, 27] planets of the

TABLE 1: Inner planets. First row: estimated perihelion extra precessions, from [16, Table 3]. The quoted errors are not the formal ones but are realistic. The units are arcseconds per century ( $''\text{cy}^{-1}$ ). Second row: semimajor axes, in Astronomical Units (AU). Their formal errors are in [15, Table IV] in meter. Third row: eccentricities. Fourth row: orbital periods in years.

|  | Mercury              | Earth                | Mars                |
|--|----------------------|----------------------|---------------------|
| $\langle \Delta \hat{\omega} \rangle (''\text{cy}^{-1})$ | $-0.0036 \pm 0.0050$ | $-0.0002 \pm 0.0004$ | $0.0001 \pm 0.0005$ |
| $a$ (AU)   | 0.387                | 1.000                | 1.523               |
| $e$  | 0.2056               | 0.0167               | 0.0934              |
| $P$ (yr)   | 0.24                 | 1.00                 | 1.88                |

TABLE 2: Outer planets. First row: estimated perihelion extra precessions [17, 27]. The quoted uncertainties are the formal, statistical errors rescaled by a factor 10 in order to get the realistic ones. The units are arcseconds per century ( $''\text{cy}^{-1}$ ). Second row: semimajor axes in Astronomical Units (AU). Their formal errors are in [15, Table IV] in meter. Third row: eccentricities. Fourth row: orbital periods in years.

|  | Jupiter            | Saturn          | Uranus          |
|--|--------------------|-----------------|-----------------|
| $\langle \Delta \hat{\omega} \rangle (''\text{cy}^{-1})$ | $0.0062 \pm 0.036$ | $-0.92 \pm 2.9$ | $0.57 \pm 13.0$ |
| $a$ (AU)   | 5.203              | 9.537           | 19.191          |
| $e$  | 0.0483             | 0.0541          | 0.0471          |
| $P$ (yr)   | 11.86              | 29.45           | 84.07           |

Solar System as fit-for parameters of a global solution in which she contrasted, in a least-square way, the observations to their predicted values computed with a complete set of dynamical force models including all the known Newtonian and Einsteinian features of motion. As a consequence, any unmodelled exotic force present in nature is, in principle, entirely accounted for by the obtained apsidal extra-precessions  $\Delta \hat{\omega}$  (of course, since modelling is not perfect, in principle,  $\Delta \hat{\omega}$  include also the mismodelled parts of the standard Newtonian/Einsteinian effects, but it turns out that including them in the following computation does not alter our conclusions). See Table 1 for the inner planets and Table 2 for the gaseous giant ones. In regard to them, we must note that modern data sets cover at least one full orbital revolution only for Jupiter, Saturn, and, barely, Uranus; this is why no secular extra precessions of perihelia for Neptune and Pluto are today available.

In order to make a direct comparison with them, we will now analytically work out the secular effects induced by the extra acceleration of (6) on the pericentre of a test particle. To this aim, we will treat (6) as a small perturbation of the Newtonian monopole. The Gauss equation for the variation of  $\hat{\omega}$  under the action of an entirely radial perturbing acceleration  $A_r$  is

$$\frac{d\hat{\omega}}{dt} = -\frac{\sqrt{1-e^2}}{nae} A_r \cos f, \quad (7)$$

in which  $a$  is the semimajor axis,  $e$  is the eccentricity,  $n = \sqrt{GM/a^3}$  is the (unperturbed) Keplerian mean motion related to the orbital period  $P$  by  $n = 2\pi/P$ , and  $f$  is the true anomaly. After being evaluated onto the unperturbed

Keplerian ellipse, (6) must be inserted into (7); then, the average over one orbital period  $P$  must be performed. It is useful to use the eccentric anomaly  $E$  by means of the relations

$$\begin{aligned} r &= a(1 - e \cos E), \\ dt &= \frac{(1 - e \cos E)}{n} dE, \\ \cos f &= \frac{\cos E - e}{1 - e \cos E}, \\ \sin f &= \frac{\sin E \sqrt{1 - e^2}}{1 - e \cos E}. \end{aligned} \quad (8)$$

On using

$$\frac{1}{2\pi} \int_0^{2\pi} \left( \frac{\cos E - e}{1 - e \cos E} \right) dE = \frac{-1 + \sqrt{1 - e^2}}{e}, \quad (9)$$

it is possible to obtain

$$\langle \hat{\omega} \rangle_{\text{in}} = -\alpha \sqrt{\frac{GM(1 - e^2)}{a}} \left( \frac{-1 + \sqrt{1 - e^2}}{e^2} \right). \quad (10)$$

Note that (10) is an exact result with respect to  $e$ . Equation (10) agrees with the precession obtainable divided by  $P$ , the adimensional perihelion shift per orbit  $\Delta \theta_p$  (log) worked out by Adkins and McDonnell [28] within a different perturbative framework; note that for Adkins and McDonnell [28], the constant  $\alpha$  has the dimensions of  $\text{ML}^2\text{T}^{-2}$  (indeed,  $V(r)$  in [28, (47)] is an additional potential energy), so that that the substitution  $\alpha/m \rightarrow -GM\alpha$  must be performed in [28, (50)] to retrieve (10).

It may be interesting to note that for the potential of (4), the rates for the semimajor axis and the eccentricity turn out to be zero; it is not so for the mean anomaly  $\mathcal{M}$ , but no observational determinations exist for its extra rate.

*2.1. Comparison with Data.* According to the general outline of Section 1, we may now consider a pair of planets A and B, take the ratio of their estimated extra rates of perihelia, and compare it to the prediction of (10) in order to see if they are equal within the errors:

$$\Psi_{AB} = \left| \frac{\Delta \hat{\omega}_A}{\Delta \hat{\omega}_B} - \sqrt{\frac{a_B(1 - e_A^2)}{a_A(1 - e_B^2)}} \left( \frac{e_B}{e_A} \right)^2 \left( \frac{-1 + \sqrt{1 - e_A^2}}{-1 + \sqrt{1 - e_B^2}} \right) \right|. \quad (11)$$

If the modification of the gravitational potential (4), not modelled by Pitjeva in estimating  $\Delta \hat{\omega}$ , accounts for what is unmodelled in the perihelia precessions, that is, just for  $\Delta \hat{\omega}$ , then (11) must be compatible with zero, within the errors. It must be noted that our approach is able to directly test the hypothesis that the proposed  $1/r$  exotic force is not zero, irrespective of the magnitude of  $\alpha$  because our prediction for the ratio of the extra rates of perihelia, that is,

$$\frac{\mathcal{P}_A}{\mathcal{P}_B} = \sqrt{\frac{a_B(1 - e_A^2)}{a_A(1 - e_B^2)}} \left( \frac{e_B}{e_A} \right)^2 \left( \frac{-1 + \sqrt{1 - e_A^2}}{-1 + \sqrt{1 - e_B^2}} \right), \quad (12)$$

is just independent of  $\alpha$  itself and is a specific function of the planets' orbital parameters  $a$  and  $e$ . Of course, the ratio of the perihelion rates cannot be used, by definition, to test the zero hypothesis which, instead, can be checked by considering the apsidal precessions separately; indeed, in this case the prediction for the ratio of the perihelion precessions would be, by definition, 0/0. Note that, being  $\Delta\dot{\omega}$  observational quantities, their ratio  $\Pi = \Delta\dot{\omega}_A/\Delta\dot{\omega}_B$  is a well-defined quantity, with an associated uncertainty  $\delta\Pi$  which will be evaluated below.

From (11) and Table 3, it is possible to obtain

$$\begin{aligned}
\Psi_{\text{MarMer}} &= 0.5 \pm 0.2, \\
\Psi_{\text{MerJup}} &= 4.2 \pm 4.1, \\
\Psi_{\text{EarJup}} &= 2.3 \pm 0.2, \\
\Psi_{\text{MarJup}} &= 1.8 \pm 0.2, \\
\Psi_{\text{MerSat}} &= 4.91 \pm 0.02, \\
\Psi_{\text{EarSat}} &= 3.090 \pm 0.001, \\
\Psi_{\text{MarSat}} &= 2.4984 \pm 0.0008, \\
\Psi_{\text{JupSat}} &= 1.36 \pm 0.06, \\
\Psi_{\text{MerUra}} &= 6.9 \pm 0.1, \\
\Psi_{\text{EarUra}} &= 4.383 \pm 0.009, \\
\Psi_{\text{MarUra}} &= 3.543 \pm 0.005, \\
\Psi_{\text{JupUra}} &= 1.9 \pm 0.3.
\end{aligned} \tag{13}$$

The exact eccentricity-dependent factor

$$F(e_A, e_B) = \sqrt{\frac{1 - e_A^2}{1 - e_B^2}} \left(\frac{e_B}{e_A}\right)^2 \left(\frac{-1 + \sqrt{1 - e_A^2}}{-1 + \sqrt{1 - e_B^2}}\right) \tag{14}$$

of (11) is always close to unity, so that its impact on the results of (13) is negligible. The errors in  $\Psi_{AB}$  due to  $\delta a$  and  $\delta\Delta\dot{\omega}$  have been conservatively computed as

$$\begin{aligned}
\delta\Psi_{AB} &\leq \left| \frac{\Delta\dot{\omega}^A}{\Delta\dot{\omega}^B} \right| \left( \frac{\delta\Delta\dot{\omega}^A}{|\Delta\dot{\omega}^A|} + \frac{\delta\Delta\dot{\omega}^B}{|\Delta\dot{\omega}^B|} \right) \\
&+ \frac{1}{2} \left(\frac{a^B}{a^A}\right)^{3/2} \left(\frac{\delta a^A}{a^A} + \frac{\delta a^B}{a^B}\right) F(e_A; e_B).
\end{aligned} \tag{15}$$

We did not optimistically summed the biased terms in a root-sum-square fashion because of the existing correlations among the estimated extra precessions, although they are low with a maximum of about 20% between Mercury and the Earth (L. Iorio thanks E. V. Pitjeva for such an information). It turns out that by rescaling the formal errors in the semimajor axes released by Pitjeva [15] by a factor 10, 100, or more, the results of (13) do not change because the major source of uncertainty is given by far by the errors in the perihelion precessions. In regard to them, let us note that the tightest constraints come from the pairs involving Saturn and Uranus (but not for the pairs A = Uranus/Saturn, B = Saturn/Uranus yielding values of  $\Psi_{AB}$  compatible with zero) for which Pitjeva [15] used only optical data, contrary to

Jupiter (this is why a rescaling of 10 of the formal error in its perihelion extra precession should be adequate). Now, even by further rescaling by a factor 10 the uncertainties, that is, rescaling by a factor 100 their formal, statistical errors, the results do not change (apart from A = Jupiter B = Uranus). In the case of the pairs A = Earth, B = Uranus and A = Mars, B = Uranus even if the real error in the correction to the perihelion precession of Uranus was 1000 times larger than the estimated formal one, the answer of (13) would still remain negative. For A = Mars and B = Saturn, a rescaling of even a factor 10 000 of the formal error in the perihelion extra rate of Saturn would be possible without changing the situation. In the case A = Earth, B = Saturn, a rescaling of 1000 for the Saturn's perihelion extra precession would not alter the result.

As a conclusion, the logarithmic correction to the Newtonian potential of (4) is ruled out by the present-day observational determinations of the Solar System's planetary motions.

### 3. The Power-Law Corrections in the $f(R)$ Extended Theories of Gravity

In the recent years, there has been a lot of interest in the so-called  $f(R)$  theories of gravity [29]. In them, the gravitational Lagrangian depends on an arbitrary analytic function  $f$  of the Ricci scalar curvature  $R$  (see [30] and references therein). These theories are also referred to as "extended theories of gravity," since they naturally generalize, on a geometric ground, GR, in the sense that when  $f(R) = R$  the action reduces to the usual Einstein-Hilbert one, and Einstein's theory is obtained. It has been showed [30] that these theories provide an alternative approach to solve, without the need of dark energy, some puzzles connected to the current cosmological observations and, furthermore, they can explain the dynamics of the rotation curves of the galaxies, without requiring dark matter. Indeed, for instance Capozziello et al. [31], starting from  $f(R) = f_0 R^k$ , obtained a power-law correction to the Newtonian gravitational potential of the form

$$V_\beta = -\frac{GM}{r} \left(\frac{r}{r_c}\right)^\beta, \tag{16}$$

(where  $\beta$  is related to the exponent  $k$  of the Ricci scalar  $R$ ), and they applied (16) to a sample of 15 low surface brightness (LSB) galaxies with combined HI and H $\alpha$  measurements of the rotation curve extending in the putative dark matter dominated region. They obtained a very good agreement between the theoretical rotation curves and the data using only stellar disk and interstellar gas when the slope  $k$  of the gravity Lagrangian is set to the value  $k = 3.5$  (giving  $\beta = 0.817$ ) obtained by fitting the SNeIa Hubble diagram with the assumed power-law  $f(R)$  model and no dark matter.

Here, we wish to put on the test (16) in the Solar System with the approach previously examined.

First, let us note that an extra-radial acceleration

$$\mathbf{A}_\beta = \frac{(\beta - 1)GM}{r_c^\beta} r^{\beta-2} \hat{\mathbf{r}} \tag{17}$$

TABLE 3: A B denotes the pair of planets used:  $\Pi = \Delta\dot{\omega}_A/\Delta\dot{\omega}_B$  and  $\mathcal{A} = (a_B/a_A)^{1/2}$ ;  $F(e_A, e_B)$  is given by (14). The perihelion extra rates for the inner planets have been retrieved from [16]; their errors are not the formal statistical ones. The perihelion extra rates for the outer planets come from [27]; their formal errors have been rescaled by a factor 10. The uncertainties in the semimajor axes have been retrieved from [15, Table IV]. They are the formal ones, but, as can be noted, their impact is negligible. While  $\Pi$  is always compatible with zero, this is definitely not the case for  $\mathcal{A}$ . The eccentricity function  $F$  is always close to unity.

| A B             | $\Pi$                | $\mathcal{A}$                     | $F(e_A, e_B)$ |
|-----------------|----------------------|-----------------------------------|---------------|
| Mars Mercury    | $-0.03 \pm 0.2$      | $0.504 \pm \mathcal{O}(10^{-12})$ | 1.008         |
| Mercury Jupiter | $-0.6 \pm 4.1$       | $3.666 \pm \mathcal{O}(10^{-9})$  | 0.989         |
| Earth Jupiter   | $-0.03 \pm 0.25$     | $2.281 \pm \mathcal{O}(10^{-10})$ | 1.000         |
| Mars Jupiter    | $0.02 \pm 0.17$      | $1.847 \pm \mathcal{O}(10^{-10})$ | 0.998         |
| Mercury Saturn  | $0.004 \pm 0.017$    | $4.963 \pm \mathcal{O}(10^{-9})$  | 0.989         |
| Earth Saturn    | $0.0002 \pm 0.0011$  | $3.088 \pm \mathcal{O}(10^{-9})$  | 1.000         |
| Mars Saturn     | $-0.0001 \pm 0.0009$ | $2.501 \pm \mathcal{O}(10^{-9})$  | 0.998         |
| Jupiter Saturn  | $-0.006 \pm 0.060$   | $1.353 \pm \mathcal{O}(10^{-9})$  | 1.000         |
| Mercury Uranus  | $-0.006 \pm 0.152$   | $7.041 \pm \mathcal{O}(10^{-8})$  | 0.989         |
| Earth Uranus    | $-0.0003 \pm 0.0087$ | $4.380 \pm \mathcal{O}(10^{-8})$  | 1.000         |
| Mars Uranus     | $0.0002 \pm 0.0048$  | $3.459 \pm \mathcal{O}(10^{-8})$  | 0.983         |
| Jupiter Uranus  | $0.01 \pm 0.31$      | $1.920 \pm \mathcal{O}(10^{-8})$  | 0.999         |

can be obtained from (16); let us now work out the secular precession of the pericentre of a test particle induced by (17) in the case  $\beta - 2 < 0$ . By proceeding as in Section 2 we get (see also [28])

$$\langle \dot{\omega} \rangle_\beta = \frac{(\beta - 1)\sqrt{GM}}{2\pi r_c^\beta} a^{\beta-3/2} G(e; \beta), \quad (18)$$

with

$$G(e; \beta) = -\frac{\sqrt{1-e^2}}{e} \int_0^{2\pi} \frac{\cos E - e}{(1 - e \cos E)^{2-\beta}} dE. \quad (19)$$

Since we are interested in taking the ratios of the perihelia, there is no need to exactly compute (19); the eccentricities of the Solar System planets are small and similar for all of them, so that we will reasonably assume that  $G(e_{A;\beta})/G(e_B; \beta) \approx 1$ . Note that  $\beta = 0$  would yield a vanishing apsidal precession because  $G(e; 0) = 0$  (the case  $\beta = 1$  is not relevant because it would yield a constant additional potential and no extra force). In fact, the case  $\beta = 0$  is compatible with all the estimated extra rates of Tables 1 and 2;  $\beta = 0$ , within the errors, is also the outcome of different tests performed in the Solar System by Zakharov et al. [32].

**3.1. Comparison with Data.** According to Capozziello et al. [31], a value of  $\beta = 0.817$  in (16) gives very good agreement between the theoretical rotation curves and the data, without need of dark matter. Again, if we consider a pair of planets A and B and take the ratio of their estimated extra rates of perihelia, by taking  $\beta = 0.817$ , we may define

$$\Xi_{AB} = \left| \frac{\Delta\dot{\omega}_A}{\Delta\dot{\omega}_B} - \left( \frac{a_B}{a_A} \right)^{0.683} \right|. \quad (20)$$

Of course, such kind of test could not be applied to the  $\beta = 0$  case since, in this case, we would have the meaningless prediction  $\mathcal{P}_A/\mathcal{P}_B = 0/0$ . If the modification

of the gravitational potential of (16) exists and is accounted for by the estimated corrections  $\Delta\dot{\omega}$  to the standard Newton-Einstein perihelion precessions, then the quantities  $\Xi_{AB}$  must be compatible with zero, within the errors. Instead, what we obtain from Tables 1, 2, 4, and (18) is:

$$\begin{aligned} \Xi_{\text{EarMer}} &= 0.5 \pm 0.2, \\ \Xi_{\text{MarMer}} &= 0.4 \pm 0.2, \\ \Xi_{\text{MerJup}} &= 6.5 \pm 4.2, \\ \Xi_{\text{EarJup}} &= 3.1 \pm 0.2, \\ \Xi_{\text{MarJup}} &= 2.3 \pm 0.2, \\ \Xi_{\text{MerSat}} &= 8.92 \pm 0.02, \\ \Xi_{\text{EarSat}} &= 4.666 \pm 0.001, \\ \Xi_{\text{MarSat}} &= 3.4997 \pm 0.0008, \\ \Xi_{\text{JupSat}} &= 1.52 \pm 0.06, \\ \Xi_{\text{MerUra}} &= 14.4 \pm 0.1, \\ \Xi_{\text{EarUra}} &= 7.523 \pm 0.008, \\ \Xi_{\text{MarUra}} &= 5.642 \pm 0.005, \\ \Xi_{\text{JupUra}} &= 2.4 \pm 0.3. \end{aligned} \quad (21)$$

It is remarkable to note that even if the formal error in the Saturn's apsidal extra precession was rescaled by a factor 10 000 instead of 10, as done in this paper, the pairs A = Earth B = Saturn and A = Mars B = Saturn would still rule out (18). A rescaling by 1000 of the Uranus' perihelion extra precession would still be fatal, as shown by the pairs A = Earth B = Uranus and A = Mars B = Uranus.

Thus, also the power-law correction to the Newtonian potential of (16) with  $\beta = 0.817$  is ruled out. Criticisms to  $R^k$  models of modified gravity were raised on different grounds by Nojiri and Odintsov [33] who proposed more realistic models in [34]. Moreover, it is well established now

TABLE 4: A B denotes the pair of planets used,  $\Pi = \Delta\dot{\omega}_A/\Delta\dot{\omega}_B$ , and  $\mathcal{B} = (a_B/a_A)^{0.683}$  (corresponding to  $\beta = 0.817$ ). The perihelion extra rates for the inner planets have been retrieved from [16]; their errors are not the formal statistical ones. The perihelion extra rates for the outer planets come from [27]; their formal errors have been rescaled by a factor 10. The uncertainties in the semimajor axes have been retrieved from [15, Table IV]; they are the formal ones, but, as can be noted, their impact is negligible. While  $\Pi$  is always compatible with zero, this is definitely not the case for  $\mathcal{B}$ .

| A B             | $\Pi$                | $\mathcal{B}$                     |
|-----------------|----------------------|-----------------------------------|
| Earth Mercury   | $0.05 \pm 0.19$      | $0.523 \pm \mathcal{O}(10^{-13})$ |
| Mars Mercury    | $-0.03 \pm 0.2$      | $0.392 \pm \mathcal{O}(10^{-13})$ |
| Mercury Jupiter | $-0.6 \pm 4.1$       | $5.898 \pm \mathcal{O}(10^{-9})$  |
| Earth Jupiter   | $-0.03 \pm 0.25$     | $3.084 \pm \mathcal{O}(10^{-9})$  |
| Mars Jupiter    | $0.02 \pm 0.17$      | $2.313 \pm \mathcal{O}(10^{-10})$ |
| Mercury Saturn  | $0.004 \pm 0.017$    | $8.921 \pm \mathcal{O}(10^{-8})$  |
| Earth Saturn    | $0.0002 \pm 0.0011$  | $4.665 \pm \mathcal{O}(10^{-9})$  |
| Mars Saturn     | $-0.0001 \pm 0.0009$ | $3.499 \pm \mathcal{O}(10^{-9})$  |
| Jupiter Saturn  | $-0.006 \pm 0.060$   | $1.512 \pm \mathcal{O}(10^{-9})$  |
| Mercury Uranus  | $-0.006 \pm 0.152$   | $14.383 \pm \mathcal{O}(10^{-8})$ |
| Earth Uranus    | $-0.0003 \pm 0.0087$ | $7.522 \pm \mathcal{O}(10^{-8})$  |
| Mars Uranus     | $0.0002 \pm 0.0048$  | $5.642 \pm \mathcal{O}(10^{-8})$  |
| Jupiter Uranus  | $0.01 \pm 0.31$      | $2.438 \pm \mathcal{O}(10^{-8})$  |

that DM shows also particle-like properties. In this respect, the proposal of  $R^k$  gravity as DM (thanks to a change of the Newton potential) is not considered as a realistic one now. A more realistic DM candidate from  $f(R)$  gravity was suggested by Nojiri and Odintsov [35]. They show that not only a correction to the Newton potential appears, but also composite graviton degree of freedom shows particle-like behavior, as requested by DM data.

#### 4. Comments and Conclusions

In this paper, we have studied the secular precession of the pericentre of a test particle in motion around a central mass  $M$  whose Newtonian gravitational potential exhibits a correction which has a logarithmic and power-law behavior. In order to put on the test the hypothesis that such extra forces are not zero, we devised a suitable test by taking into account the ratios (and not the extra rates of the perihelia of each planet at a time separately, or a linear combination of them, since their uncertainties would fatally prevent obtaining any useful constraints) of the corrections to the secular precessions of the longitudes of perihelia estimated by E. V. Pitjeva for several pairs of planets in the Solar System. The results obtained, resumed by (13) and (21), show that modifications of the Newtonian potentials like those examined in this paper are not compatible with the currently available apsidal extra precessions of the Solar System planets. Moreover, the hypothesis that the examined exotic force terms are zero, which cannot be tested by definition with our approach, is compatible with each perihelion extra rate separately, in agreement with our results. It must be noted that to give the hypothesized modifications of the Newtonian law, the benefit of the doubt, and given that the formal errors in  $a$  and  $\Delta\dot{\omega}$  of the outer planets are probably underestimates of the true uncertainties, we multiplied them by a factor ten or even more, as suggested to one of us (L.

Iorio) by some leading experts in the ephemerides generation field like E. M. Standish, but the answers we obtained were still negative. Another thing that should be pointed out is that, in principle, in assessing  $\Psi_{AB}$  and  $\Xi_{AB}$  one should have used  $\Delta\dot{\omega}^* = \Delta\dot{\omega} - \delta\dot{\omega}_{\text{canonical}}$ , where  $\delta\dot{\omega}_{\text{canonical}}$  represents the mismodelled part of the modelled standard Newton/Einstein precessions. However, neglecting them did not affect our conclusions, as can be easily noted by looking at the  $\Pi$  and  $\mathcal{A}/\mathcal{B}$  columns in Tables 3 and 4. Indeed, the residual precessions due to the imperfect knowledge of, for example, the solar quadrupole mass moment [16]  $\delta J_2/J_2 \approx 10\%$  are of the order of  $\leq 10^{-3}''\text{cy}^{-1}$  [36], and even smaller are the mismodelled precessions due to other potential sources of errors like the asteroid ring or the Kuiper Belt objects [37].

However, caution is in order because, at present, no other teams of astronomers have estimated their own corrections to the Newtonian/Einsteinian planetary perihelion rates, as it would be highly desirable. If and when we have, say, two independent determinations of the anomalous perihelion rate of a given planet  $x = x_{\text{best}} \pm \delta x$  and  $y = y_{\text{best}} \pm \delta y$ , we will see if they are compatible with each other and take their difference  $|x_{\text{best}} - y_{\text{best}}|$  as representative of the real uncertainty affecting the apsidal extra precession of that planet. Moreover, it would be interesting to see if for different sets of estimated corrections to the perihelion rates the figures for  $\Pi$  in Tables 3 and 4 change by an extent sufficient to alter the conclusions of (13) and (21).

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