Research Article

Some Traveling Wave Solutions of a Hyperbolic Model for Chemotaxis in 1-D

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The traveling wave solution of a hyperbolic model for chemotaxis in one space dimension is studied in this paper. By using some transformations of dependent variables and independent variables, we apply the tanh method and improved tanh method to the model, from which some traveling wave solutions in explicit form are presented.

1. Introduction

Chemotaxis is a widespread phenomenon in biological systems describing the response of individuals to an external chemical together with its gradient. It has attracted significant interest due to its critical role in a wide range of biological phenomena. Chemotaxis was studied mathematically beginning with the early papers of Patlak [1] and Keller and Segel [2, 3]. Since then the mathematical literature on the modeling of chemotaxis has grown rapidly. The parabolic Keller-Segel model of chemotaxis [2] has proved a cornerstone for much of this work. For a review of the recent literature, see Hillen and Painter [4]. However, the parabolic Keller-Segel model allows arbitrarily large speed of the population, it is physically unrealistic for small time. Motivated by the fact, Hillen and Stevens [5] proposed a hyperbolic model for chemotaxis in 1-D, which deal with finite speed. The model is

$$\begin{aligned} u_t^+ + (\gamma(S, S_t, S_x)u^+)_x &= -\mu^+(S, S_t, S_x)u^+ + \mu^-(S, S_t, S_x)u^-, \\ u_t^- - (\gamma(S, S_t, S_x)u^-)_x &= \mu^+(S, S_t, S_x)u^+ - \mu^-(S, S_t, S_x)u^-, \\ \tau S_t &= D_0 S_{xx} + f(S, u^+ + u^-), \quad \tau \geq 0, \end{aligned} \quad (1.1)$$

where $u^\pm(x, t)$ denotes the particle densities of right (+)/left (−) moving particles, $\gamma(S, S_t, S_x)$ denotes the particle speed, and $\mu^\pm(S, S_t, S_x)$ are turning rates (rates of change of direction from + to − and vice versa). $S(x, t)$ denotes the concentration of the chemical external signal. The production and degradation of the external chemical signal is modeled by the reaction term f .

For the special case with $D = 0$, constant speed γ and $\mu^\pm$ depending only on S and S_x ,

$$\begin{aligned} u_t^+ + \gamma u_x^+ &= -\mu^+(S, S_x)u^+ + \mu^-(S, S_x)u^-, \\ u_t^- - \gamma u_x^- &= \mu^+(S, S_x)u^+ - \mu^-(S, S_x)u^-, \\ S_t &= f(S, u^+ + u^-). \end{aligned} \quad (1.2)$$

Segel in [9] first used it to analyze a very specific scenario. Later Rivero et al. [6] and Ford et al. [7, 8] used it to describe experimental data. In the works [5–9], the issues of local and global in time existence of solutions were considered theoretically and numerically. Hillen and Levine [10] studied finite time blowup of the system (1.2) with

$$f = S(u^+ + u^-), \quad (1.3)$$

and three specific forms of turning rates, one of which is

$$\mu^\pm(S, S_x) = \frac{\gamma}{2D_1} \left(\gamma \mp \frac{\alpha}{S} S_x \right). \quad (1.4)$$

Keller and Segel in 1971 [11] considered an experimental setup that results in visible bands of bacteria traveling up a capillary tube filled with a mixture of oxygen and nutrient rich substrate. It raised the question of existence of a traveling wave solution to the Keller-Segel model of chemotaxis. Liu in 2008 [12] constructed some explicit solutions of the system (1.2) with the choice of f and $\mu^\pm$ as in (1.3) and (1.4).

In this paper, we try to find some exact traveling wave solutions of the system

$$\begin{aligned} u_t^+ + \gamma u_x^+ &= -\mu^+(S, S_x)u^+ + \mu^-(S, S_x)u^-, \\ u_t^- - \gamma u_x^- &= \mu^+(S, S_x)u^+ - \mu^-(S, S_x)u^-, \\ S_t &= D_0 S_{xx} + f(S, u^+ + u^-), \end{aligned} \quad (1.5)$$

where $D_0 \geq 0, \gamma > 0$ are constants, $f(S, u^+ + u^-)$ and $\mu^\pm(S, S_x)$ are defined above (1.3) and (1.4). It is clear that system (1.5) is a special case of the system (1.1) and a more generalized case of (1.2).

The system (1.5) can be transformed into an equivalent system for the total particle density, $u = u^+ + u^-$, and particle flux, $v = u^+ - u^-$. The resulting system for (u, v) is (3.1) below. Using tanh method and improved tanh method, we search for traveling wave solutions of the system (3.1). When $D_0 \neq 0$, three different kinds of traveling wave solutions are found. To the best of our knowledge, those traveling wave solutions are new and have not appeared in literature. When $D_0 = 0$, the solutions obtained here are different from those known ones in [12].

2. The Two Methods

The tanh method and improved tanh method are powerful techniques to search for traveling wave solutions arising from one dimensional nonlinear wave and evolution equations. Let us first review the main features of two methods. They will be used in this paper. For more details, see [13–17] and reference therein.

For both methods, we first use the wave variable $\xi = k(x - pt) + \xi_0$ to carry a PDE in two independent variables

$$H(u, u_t, u_x, u_{xx}, \dots) = 0 \quad (2.1)$$

into an ODE

$$Q(u, u', u'', \dots) = 0, \quad (2.2)$$

where k and p are the wave number and the wave speed, respectively.

2.1. The tanh Method

In the standard tanh method, the tanh is used as a new variable and the solution of (2.2) is expressed as a finite series of tanh

$$u(\xi) = \sum_{i=0}^M a_i T^i, \quad T = \tanh \xi, \quad (2.3)$$

where M is a positive integer that will be determined. Substituting (2.3) into the reduced ODE (2.2) results in an algebraic equation in powers of T . To determine the parameter M , we usually balance the linear terms of highest order in the resulting equation with the highest order nonlinear terms. With M determined, we collect all coefficients of powers of T in the resulting equation where these coefficients have to vanish. This will give a system of algebraic equations involving the parameters k, p , and a_i . Having determined these parameters, we obtain an analytic solution $u(x, t)$ in a closed form.

The tanh method is a computerizable method, in which generating an algebraic system and solving it are two key procedures and laborious to do by hand. But they can be implemented on a computer with the help of computer algebra software such as Mathematica.

2.2. The Improved tanh Method

The crucial point of the improved tanh method is to replace the tanh ξ in (2.3) by the solution of Riccati equation

$$F' = A + BF + CF^2, \quad C \neq 0, \quad (2.4)$$

where $' := d/d\xi$, and A, B, C are some constants.

Supposing that F is a solution of (2.4), it is easy to verify

(1) if $B^2 - 4AC > 0$, (2.4) possesses a solution

$$F(\xi) = -\frac{B}{2C} - \frac{\sqrt{B^2 - 4AC}}{2C} \tanh\left(\frac{\sqrt{B^2 - 4AC}}{2}\xi\right); \quad (2.5)$$

(2) if $B^2 - 4AC = 0$, (2.4) possesses a solution

$$F(\xi) = -\frac{B}{2C} - \frac{1}{C\xi}; \quad (2.6)$$

(3) if $B^2 - 4AC < 0$, (2.4) possesses a solution

$$F(\xi) = -\frac{B}{2C} + \frac{\sqrt{4AC - B^2}}{2C} \tan\left(\frac{\sqrt{4AC - B^2}}{2}\xi\right). \quad (2.7)$$

Based on (2.5), the solution of (2.2) can be expressed as in (2.3), and based on (2.6) and (2.7), the solution of (2.2) can be expressed as

$$u(\xi) = \sum_{i=0}^M a_i \frac{1}{\xi^i}, \quad (2.8)$$

$$u(\xi) = \sum_{i=0}^M a_i \tan^i \xi, \quad (2.9)$$

where parameters M, k, p , and a_i are to be determined.

3. Traveling Wave Solutions

A transformation of the dependent variables is necessary, before the just-described methods can be applied to the hyperbolic model (1.5). As in [10], we rewrite system (1.5) as a system for $u = u^+ + u^-$ and $v = u^+ - u^-$:

$$\begin{aligned} u_t + \gamma v_x &= 0, \\ v_t + \gamma u_x &= \frac{\gamma}{D_1} \cdot \frac{\alpha S_x}{S} u - \frac{\gamma^2}{D_1} v, \\ S_t &= D_0 S_{xx} + Su. \end{aligned} \quad (3.1)$$

Eliminating v in system (3.1), it is clear that (3.1) is of the form

$$\begin{aligned} -\frac{1}{\gamma} u_{tt} + \gamma u_{xx} &= \frac{\alpha \gamma}{D_1} \left(\frac{u S_x}{S} \right)_x + \frac{\gamma}{D_1} u_t, \\ S_t &= D_0 S_{xx} + Su. \end{aligned} \quad (3.2)$$

Taking into account the transformation

$$u = u(\xi), \quad S = e^{\varphi(\xi)}, \quad \xi = k(x - pt) + \xi_0, \quad (3.3)$$

where $k > 0, p > 0$, and ξ_0 is an arbitrary constant, the (3.2) can be put into the format

$$k \left(\gamma - \frac{p^2}{\gamma} \right) u'' = \frac{\alpha \gamma k}{D_1} (u' \varphi' + u \varphi'') - \frac{\gamma p}{D_1} u', \quad (3.4)$$

$$-kp\varphi' = D_0 k^2 (\varphi'^2 + \varphi'') + u, \quad (3.5)$$

where $p^2 \neq \gamma^2$.

3.1. Using tanh Method

Let

$$u(\xi) = \sum_{i=0}^M a_i \tanh^i \xi. \quad (3.6)$$

It is noted that

$$\begin{aligned} \int \tanh x dx &= \ln|\cosh x| + C, \\ \int \tanh^2 x dx &= x - \tanh x + C, \\ \int \tanh^k x dx &= \frac{-\tanh^{k-1} x}{k-1} + \int \tanh^{k-2} x dx, \end{aligned} \quad (3.7)$$

as a result, we guess the φ has the form

$$\varphi(\xi) = b_{-2}\xi + b_{-1} \ln|\cosh \xi| + \sum_{i=0}^{M-2} b_i \tanh^i \xi. \quad (3.8)$$

Substituting $u(\xi)$ and $\varphi(\xi)$ into (3.4)-(3.5), we obtain $M = 2$, and the solutions of system (3.4)-(3.5) is of the form

$$u = a_0 + a_1 \tanh \xi + a_2 \tanh^2 \xi, \quad \varphi = b_{-2}\xi + b_{-1} \ln|\cosh \xi|. \quad (3.9)$$

With the help of the symbolic software Mathematica, substituting (3.9) into (3.4) and (3.5) and setting the coefficients of $\tanh^i \xi$ to be zero, we obtain

$$\begin{aligned}
 2kD_1(\gamma^2 - p^2)a_2 - \alpha\gamma^2k(a_1b_{-2} + a_0b_{-1}) + \gamma^2pa_1 &= 0, \\
 -kD_1(\gamma^2 - p^2)a_1 - \alpha\gamma^2k(a_2b_{-2} + a_1b_{-1}) + \gamma^2pa_2 &= 0, \\
 -3k(2D_1(\gamma^2 - p^2) + \alpha\gamma^2b_{-1})a_2 &= 0, \\
 kpb_{-2} + D_0k^2(b_{-2}^2 + b_{-1}) + a_0 &= 0, \\
 kpb_{-1} + 2D_0k^2b_{-2}b_{-1} + a_1 &= 0, \\
 D_0k^2(b_{-1}^2 - b_{-1}) + a_2 &= 0.
 \end{aligned} \tag{3.10}$$

If $D_0 \neq 0$ and p satisfies

$$\begin{aligned}
 \Delta &= (\gamma^2 - p^2)^2 D_1^2 (4D_0^2 + 4\alpha D_0 + 5\alpha^2) \\
 &\quad + 2(\gamma^2 - p^2) D_1 \alpha^3 \gamma^2 - \alpha^2 \gamma^4 D_0 (\alpha + D_0) \\
 &> 0,
 \end{aligned} \tag{3.11}$$

solving the algebraic equations (3.10) with the aid of Mathematica, we arrive at

$$\begin{aligned}
 k &= \frac{\gamma^2 p \sqrt{\Delta}}{2D_0 D_1 [(4D_1(\gamma^2 - p^2) + \alpha\gamma^2)(\gamma^2 - p^2)]}, \\
 b_{-1} &= \frac{-2D_1(\gamma^2 - p^2)}{\alpha\gamma^2}, \\
 b_{-2} &= \frac{1}{\alpha k D_0} \cdot \frac{2pD_0 D_1(\gamma^2 - p^2) + pD_0 \alpha \gamma^2 - pD_1 \alpha(\gamma^2 - p^2)}{4D_1(\gamma^2 - p^2) + \alpha\gamma^2}, \\
 a_0 &= \frac{2k^2 D_0 D_1(\gamma^2 - p^2)}{\alpha\gamma^2} \\
 &\quad - \frac{p(p\alpha\gamma^2 D_0 - p\alpha D_1(\gamma^2 - p^2) + 2pD_0 D_1(\gamma^2 - p^2))}{\alpha D_0(\alpha\gamma^2 + 4D_1(\gamma^2 - p^2))} \\
 &\quad - \frac{D_0(p\alpha\gamma^2 D_0 - p\alpha D_1(\gamma^2 - p^2) + 2pD_0 D_1(\gamma^2 - p^2))^2}{\alpha^2 D_0^2(\alpha\gamma^2 + 4D_1(\gamma^2 - p^2))^2},
 \end{aligned}$$

$$\begin{aligned}
a_1 &= 2kD_1(\gamma^2 - p^2) \\
&\times \frac{p\alpha^2\gamma^2 + 2p\alpha\gamma^2D_0 + 2p\alpha D_1(\gamma^2 - p^2) + 4pD_0D_1(\gamma^2 - p^2)}{\alpha^2\gamma^2(\alpha\gamma^2 + 4D_1(\gamma^2 - p^2))}, \\
a_2 &= \frac{-2k^2D_0D_1(\gamma^2 - p^2)(\alpha\gamma^2 + 2D_1(\gamma^2 - p^2))}{(\alpha\gamma^2)^2},
\end{aligned} \tag{3.12}$$

which leads to the solutions of system (3.1)

$$\begin{aligned}
u(x, t) &= a_0 + a_1 \tanh(k(x - pt) + \xi_0) + a_2 \tanh^2(k(x - pt) + \xi_0), \\
S(x, t) &= |\cosh(k(x - pt) + \xi_0)|^{b_{-1}} \cdot \exp(b_{-2}(k(x - pt) + \xi_0)), \\
v(x, t) &= \frac{p}{\gamma} u(x, t).
\end{aligned} \tag{3.13}$$

If $D_0 = 0$, we found $a_2 = 0$, and

$$\begin{aligned}
b_{-1} &= -\frac{D_1(\gamma^2 - p^2)}{\alpha\gamma^2}, & b_{-2} &= \frac{p}{2\alpha k}, \\
a_0 &= -\frac{p^2}{2\alpha}, & a_1 &= \frac{kpD_1(\gamma^2 - p^2)}{\alpha\gamma^2}.
\end{aligned} \tag{3.14}$$

Consequently, we obtain the following solutions of system (3.1)

$$\begin{aligned}
u(x, t) &= -\frac{p^2}{2a} + \frac{D_1kp(\gamma^2 - p^2)}{\alpha\gamma^2} \tanh(k(x - pt) + \xi_0), \\
S(x, t) &= (\cosh(k(x - pt) + \xi_0))^{-D_1(\gamma^2 - p^2)/\alpha\gamma^2} \cdot \exp\left(\frac{p}{2ak}(k(x - pt) + \xi_0)\right), \\
v(x, t) &= -\frac{p^3}{2a\gamma} + \frac{D_1kp^2(\gamma^2 - p^2)}{\alpha\gamma^3} \tanh(k(x - pt) + \xi_0),
\end{aligned} \tag{3.15}$$

which is different from the solutions obtained in [12].

3.2. Using Improved tanh Method

Case 1. We seek solutions in the form of (2.8) $u(\xi) = \sum_{i=0}^M a_i / \xi^i$. Motivated by the expression of (3.5), we turn to the assumption $\varphi(\xi) = b_{-2}\xi + b_{-1} \ln|\xi| + \sum_{j=0}^{M-2} b_j / \xi^j$ and also imply that $M = 2$. Therefore, we have

$$u(\xi) = a_0 + a_1 \frac{1}{\xi} + a_2 \frac{1}{\xi^2}, \quad \varphi(\xi) = b_{-2}\xi + b_{-1} \ln|\xi|. \quad (3.16)$$

By substituting (3.16) into (3.4) and (3.5), and setting the coefficients of $1/\xi^j$ ($j = 0, 1, 2$) to be zero, we set

$$\begin{aligned} \alpha\gamma k(a_1 b_{-2} + a_0 b_{-1}) - \gamma p a_1 &= 0, \\ 2kD_1(\gamma^2 - p^2)a_1 + 2\alpha\gamma^2 k(a_1 b_{-1} + a_2 b_{-2}) - 2\gamma^2 p a_2 &= 0, \\ 6kD_1(\gamma^2 - p^2)a_2 + 3\alpha\gamma^2 k a_2 b_{-1} &= 0, \\ kpb_{-2} + D_0 k^2 b_{-2}^2 + a_0 &= 0, \\ kpb_{-1} + 2D_0 k^2 b_{-2} b_{-1} + a_1 &= 0, \\ D_0 k^2 (b_{-1}^2 - b_{-1}) + a_2 &= 0. \end{aligned} \quad (3.17)$$

If $D_0 \neq 0$ and

$$\Delta = 1 + \frac{\alpha^3 \pm \alpha(2D_0 + \alpha)\sqrt{D_0^2 + \alpha D_0 + \alpha^2}}{5\alpha^2 D_1 + 4D_0^2 D_1 + 4\alpha D_0 D_1} > 0, \quad (3.18)$$

solving algebraic equations (3.17) gives

$$\begin{aligned} b_{-1} &= \frac{-2D_1(\gamma^2 - p^2)}{\alpha\gamma^2}, \\ b_{-2} &= \frac{1}{\alpha k D_0} \cdot \frac{2pD_0 D_1(\gamma^2 - p^2) + pD_0 \alpha\gamma^2 - pD_1 \alpha(\gamma^2 - p^2)}{4D_1(\gamma^2 - p^2) + \alpha\gamma^2}, \\ a_0 &= -\frac{p^2((2D_0 D_1 - D_1 \alpha)(\gamma^2 - p^2) + \alpha\gamma^2 D_0)}{\alpha^2 D_0 (4D_1(\gamma^2 - p^2) + \alpha\gamma^2)^2} \\ &\quad \times \left((3\alpha D_1 + 2D_0 D_1)(\gamma^2 - p^2) + \alpha^2 \gamma^2 + \alpha\gamma^2 D_0 \right), \\ a_1 &= \frac{2kpD_1(\gamma^2 - p^2)}{\alpha^2 \gamma^2 (\alpha\gamma^2 + 4D_1(\gamma^2 - p^2))} \\ &\quad \times \left(\alpha^2 \gamma^2 + 2\alpha\gamma^2 D_0 + 2\alpha D_1(\gamma^2 - p^2) + 4D_0 D_1(\gamma^2 - p^2) \right), \end{aligned}$$

$$\begin{aligned}
 a_2 &= -\frac{2k^2 D_0 D_1 (\gamma^2 - p^2)}{(\alpha \gamma^2)^2} \left(\alpha \gamma^2 + 2D_1 (\gamma^2 - p^2) \right), \\
 p &= \gamma \sqrt{\Delta},
 \end{aligned} \tag{3.19}$$

where Δ is specified by (3.18).

This in turn gives the solutions of system (3.1)

$$\begin{aligned}
 u(x, t) &= a_0 + \frac{a_1}{k(x - pt) + \xi_0} + \frac{a_2}{(k(x - pt) + \xi_0)^2}, \\
 S(x, t) &= |k(x - pt) + \xi_0|^{b-1} \cdot \exp(b_{-2}(k(x - pt) + \xi_0)), \\
 v(x, t) &= \frac{p}{\gamma} \left(a_0 + \frac{a_1}{k(x - pt) + \xi_0} + \frac{a_2}{(k(x - pt) + \xi_0)^2} \right),
 \end{aligned} \tag{3.20}$$

where k is an arbitrary constant and a_i ($i = 0, 1, 2$), b_{-j} ($j = 1, 2$), p are given in (3.19).

Case 2. We seek solutions in the form of (2.9). Let $u(\xi) = \sum_{i=0}^M a_i \tan^i \xi$ and $\varphi(\xi) = b_{-2} \xi + b_{-1} \ln |\cos \xi| + \sum_{j=0}^{M-2} b_j \tan^j \xi$. Substituting $u(\xi), \varphi(\xi)$ into (3.4) and (3.5), balancing the linear terms of highest order with the highest order nonlinear terms, directly results in $M = 2$ and

$$u = a_0 + a_1 \tan \xi + a_2 \tan^2 \xi, \quad \varphi = b_{-2} \xi + b_{-1} \ln |\cos \xi|. \tag{3.21}$$

To progress further substituting (3.21) into (3.4) and (3.5) and setting the coefficients of $\tan^i \xi$ ($i = 0, 1, 2$) in the resulting equations to be zero, we obtain a set of algebraic equations

$$\begin{aligned}
 2kD_1(\gamma^2 - p^2)a_2 - \alpha\gamma^2k(a_1b_{-2} - a_0b_{-1}) + \gamma^2pa_1 &= 0, \\
 kD_1(\gamma^2 - p^2)a_1 - \alpha\gamma^2k(-a_1b_{-1} + a_2b_{-2}) + \gamma^2pa_2 &= 0, \\
 6a_2kD_1(\gamma^2 - p^2) + 3\alpha\gamma^2ka_2b_{-1} &= 0, \\
 -kpb_{-2} - D_0k^2(b_{-2}^2 - b_{-1}) &= a_0, \\
 kpb_{-1} + 2D_0k^2b_{-1}b_{-2} &= a_1, \\
 -D_0k^2(b_{-1}^2 - b_{-1}) &= a_2.
 \end{aligned} \tag{3.22}$$

If $D_0 \neq 0$ and p satisfies

$$\begin{aligned}\Delta = & -p^4(5D_1^2\alpha^2 + 4D_0D_1^2\alpha + 4D_0^2D_1^2) \\ & + p^2\gamma^2(2D_1\alpha^3 + 10\alpha^2D_1^2 + 8\alpha D_0D_1^2 + 8D_0^2D_1^2) \\ & + \gamma^4(\alpha^3D_0 + \alpha^2D_0^2 - 2\alpha^3D_1 - 5\alpha^2D_1^2 - 4\alpha D_0D_1^2 - 4D_0^2D_1^2) \\ > & 0,\end{aligned}\tag{3.23}$$

we know

$$b_{-1} = -\frac{2D_1(\gamma^2 - p^2)}{\alpha\gamma^2},\tag{3.24}$$

$$b_{-2} = \frac{p}{\alpha k D_0} \cdot \frac{2D_0D_1(\gamma^2 - p^2) + \alpha\gamma^2D_0 - D_1\alpha(\gamma^2 - p^2)}{4D_1(\gamma^2 - p^2) + \alpha\gamma^2},\tag{3.25}$$

$$\begin{aligned}a_0 = & -\frac{p^2}{\alpha D_0} \cdot \frac{2D_0D_1(\gamma^2 - p^2) + \alpha\gamma^2D_0 - \alpha D_1(\gamma^2 - p^2)}{4D_1(\gamma^2 - p^2) + \alpha\gamma^2} \\ & - \frac{p^2}{\alpha^2 D_0} \cdot \left(\frac{2D_0D_1(\gamma^2 - p^2) + \alpha\gamma^2D_0 - \alpha D_1(\gamma^2 - p^2)}{4D_1(\gamma^2 - p^2) + \alpha\gamma^2} \right)^2 \\ & - \frac{2k^2D_0D_1(\gamma^2 - p^2)}{\alpha\gamma^2},\end{aligned}\tag{3.26}$$

$$a_1 = -\frac{2kpD_1(\gamma^2 - p^2)(\alpha + 2D_0)(\alpha\gamma^2 + 2D_1(\gamma^2 - p^2))}{\alpha^2\gamma^2(\alpha\gamma^2 + 4D_1(\gamma^2 - p^2))},\tag{3.27}$$

$$a_2 = -\frac{2k^2D_0D_1(\gamma^2 - p^2)(\alpha\gamma^2 + 2D_1(\gamma^2 - p^2))}{(\alpha\gamma^2)^2},\tag{3.28}$$

$$k = \frac{p\gamma^2}{2} \sqrt{\frac{\Delta}{D_0^2D_1^2(\gamma^2 - p^2)^2(4D_1(\gamma^2 - p^2) + \alpha\gamma^2)^2}},\tag{3.29}$$

where Δ is given by (3.23). To this end, we therefore derive the solutions of system (3.1) as follows:

$$\begin{aligned}u(x, t) &= a_0 + a_1 \tan(k(x - pt) + \xi_0) + a_2 \tan^2(k(x - pt) + \xi_0), \\ S(x, t) &= |\cos(k(x - pt) + \xi_0)|^{b_{-1}} e^{b_{-2}(k(x - pt) + \xi_0)}, \\ v(x, t) &= \frac{(a_0 + a_1 \tan(k(x - pt) + \xi_0) + a_2 \tan^2(k(x - pt) + \xi_0))p}{\gamma},\end{aligned}\tag{3.30}$$

where b_{-j} ($j = 1, 2$) and a_i ($i = 0, 1, 2$) and k are given by (3.24)–(3.29), respectively.

4. Conclusion

In the present paper, we have considered the exact traveling wave solutions of a one-space dimensional hyperbolic model for chemotaxis (1.5). Using a transformation of dependent variables and eliminating the dependent variable v in the resultant system (3.1), the system (1.5) is reduced to a couple system of two equations (3.2). Applying the transformation (3.3) to unite the independent variables x and t , the (3.2) is convert to a coupled ordinary differential system. Then, the tanh method and improved tanh method are used, with the help of symbolic software MATHEMATICA, to search for exact traveling wave solutions of system (3.1). Finally, three kinds of traveling wave solutions including soliton, rational, and triangular solutions are found, all of which are different from those known ones in the literature.

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