Research Article

Dual-Domain Transform for Travelling Wave in FRFT Domain

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Received 11 March 2011; Accepted 7 April 2011

Academic Editor: M. Ayub

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In wireless communication, the travelling electromagnetic (EM) wave nature of the signal is not reported comprehensively in the literature. To date, the majority of reported work analyzed the nature of the signal propagating through the channel in time and frequency domains only, whereas the travelling EM wave which is actually propagating through the channel, has both time and spatial dependencies. Very little work has been reported in which this aspect of EM wave is being analyzed, but the time dependency of frequency and spatial dependency of propagation constant associated with the EM wave have not been included, which will affect the performance of the channel in practical conditions. In this paper, all these aspects of EM wave and channel are considered to define a new method (DDT-FRFT) to analyze the travelling EM wave for wireless communication.

1. Introduction

The fractional Fourier transform (FRFT) is an integral transform defined in time-frequency plane, that is, it can be considered as a generalization to the Fourier transform (FT) with an order (or power) parameter a . The a th-order fractional Fourier transform operator is the a th power of the ordinary Fourier transform operator. The FRFT has already found many applications in the areas of signal processing and optics [1–9]. While originally formulated in an optical context, the same result equally applies to electromagnetic waves satisfying the linear wave equation. One of the central results of diffraction theory is that the far-field diffraction pattern is the Fourier transform of the diffracting object. It is possible to generalize this result by showing that the field patterns at closer distances are the fractional Fourier transforms of the diffracting object [10–12]. As the wave field propagates, its distribution evolves through fractional transforms of increasing orders. So the application of FRFT in wave propagation needs to be investigated more.

Later on, the travelling wave has been analyzed with the help of dual-domain transform (DDT-FT) [13], by transforming the travelling wave expression twice, first from time domain to frequency domain and second in spatial coordinate domain to propagation constant domain, by considering transform method as Fourier transform in both cases. The results of this transform are applied to estimate the transfer function of a wireless communication channel with multiple phase shifts, and, has been established that the channel acts as a band pass filter in both the frequency domain and the propagation constant domain.

But the time dependency of frequency components and spatial variable dependency of propagation constant have not been considered in [13], namely, the case when the electromagnetic wave propagating in a medium has different permittivity for different portions of the medium (e.g., in satellite communication, the wave has to travel from earth station to satellite through different layers of earth's atmosphere and space) and also suffers Doppler shift in frequency due to motion of transmitter and/or receiver. This enforces a constraint on the analysis of electromagnetic wave that analysis should be carried out in time-frequency plane and spatial coordinate-propagation constant plane for time and spatial dependencies respectively, and the solution is FRFT. Hence, one another type of dual-domain transform is needed to be defined, which is based on the FRFT.

Some other works were reported in literature on synthetic aperture radar which has applied FRFT for estimating some of the parameters associated with the received signal. In Miaohong et al.'s work [14], the mechanism of radar target micro-Doppler effect has been applied to analyze the passive positioning system, and there it is demonstrated that the micro-Doppler frequency shifts on the received signal are relevant to both terminal motion and multipath propagation characteristics. And micro-Doppler parameters like terminal velocity and acceleration were estimated by FRFT. Yi et al. [15] gave a method for imaging and locating multiple ground moving targets which is based on the first-order Keystone transform and fractional Fourier transform. With this method, the focused image of moving targets can be obtained, and estimates of moving parameters such as radial velocity, azimuth speed, and radial acceleration can be determined. Similarly, Gao and Su [16] analyzed inverse synthetic aperture radar (ISAR) imaging by using FRFT. Here, the angular velocity can be calculated by knowing the chirp rate of chirp signal. In all these works, the effect of motion of transmitter/receiver on frequency is considered only while the EM wave nature of transmitted signal is not considered, and also the effect of the variation in the permittivity and permeability values for different portion of the overall media is neglected. However, in this paper, a new dual-domain transform based on the FRFT (DDT-FRFT) has been proposed, and with the help of some simulation results, the utility of this transform method in the above-mentioned case has been shown.

The paper is organized as follows. In Section 2, a brief introduction of FRFT is given along with the dual-domain transform defined for FT. The derivation of the proposed dual-domain transform based on FRFT along with its application in channel modeling is included in Section 3. It has been shown in Section 4 that the simulation results are in conformity with the proposed theory. The conclusive remarks are made in Section 5.

2. FRFT and DDT-FT

For an electromagnetic wave, the electric field intensity expression for transverse electromagnetic (TEM) mode, which is the function of both time and spatial domain variable, is given as

$$f(x, t) = \cos(kx - \omega t). \quad (2.1)$$

The dual-domain transform based on Fourier transform maps the signal defined in time and spatial “ x ” domain to frequency and propagation constant “ k ” domain, respectively. DDT-FT of the function $f(x, t)$ is defined as [13]

$$F(k, \omega) = \iint_{-\infty}^{\infty} f(x, t) e^{j(kx - \omega t)} dx dt. \quad (2.2)$$

In DDT-FT, propagation constant should be independent of the spatial variable, and frequency should be independent of time variable. This constraint is violated when trans receiver is moving and channel changes its characteristics with distance. In this scenario, FRFT can play an important role. FRFT was introduced wayback in 1920s but remained largely unknown until the work of Namias in 1980 [17]. The FRFT can be viewed as the chirp-basis expansion directly from its definition, but essentially it can be interpreted as a rotation in the time-frequency plane, that is, the unified time-frequency transform, that is, FRFT is a generalization of Fourier transform (FT). The FRFT, represented by “ $\mathcal{J}$ ”, of a signal $x(t)$ with angle parameter “ α ”, represented by $X_\alpha(u)$, is defined for the entire time-frequency plane as [7]

$$\mathcal{J}[x(t)] = X_\alpha(u) = \int_{-\infty}^{\infty} x(t) K_\alpha(t, u) dt, \quad (2.3)$$

where $K_\alpha(t, u)$ is the kernel of FRFT and is given as

$$K_\alpha(t, u) = \begin{cases} \sqrt{\frac{1 - j \cot \alpha}{2\pi}} e^{j/2 \{(t^2 + u^2) \cot \alpha - 2 t u \csc \alpha\}}, & \text{if } \alpha \neq n\pi, \\ \delta(t - u), & \text{if } \alpha = 2n\pi, \\ \delta(t + u), & \text{if } \alpha = (2n + 1)\pi. \end{cases} \quad (2.4)$$

Many properties of this transform have been documented in the recent past including multiplication, mixed product, differentiation, integration, shift, similarity, and modulation in [7, 8], the shifting property will be utilized in formulating the channel response by DDT-FRFT. The shifting property of FRFT is given as [9]

$$\mathcal{J}[x(t - \tau)] = X_\alpha(u - \tau \cos \alpha) e^{j(\tau^2/2 \sin \alpha \cos \alpha - u\tau \sin \alpha)}. \quad (2.5)$$

3. DDT-FRFT and Its Application in Channel Modeling

The dual-domain transform, represented as “ $\mathfrak{D}_3$ ”, based on FRFT, is a technique which maps the signal defined in time and spatial “ x ” domain to anywhere in time (spatial coordinate)-frequency (propagation constant) plane. The proposed DDT-FRFT of the signal $f(x, t)$ is defined as

$$\mathfrak{D}_3[f(x, t)] = F_\alpha(\beta, u) = \iint_{-\infty}^{\infty} f(x, t) K_\alpha(t, u) K_\alpha(x, \beta) dx dt. \quad (3.1)$$

Based on the definition of DDT-FRFT, the shifting property of this transform is defined as

$$\mathfrak{D}_\gamma [f(x - \delta, t - \tau)] = e^{j \sin \alpha ((\tau^2/2) \cos \alpha - u\tau)} e^{j \sin \alpha ((\delta^2/2) \cos \alpha - \beta\delta)} F_\alpha(\beta - \delta \cos \alpha, u - \tau \cos \alpha). \quad (3.2)$$

This is modified as

$$\mathfrak{D}_\gamma [f(x - \delta, t - \tau)] = e^{j\phi_\tau} e^{j\phi_\delta} F_\alpha(\beta - \delta \cos \alpha, u - \tau \cos \alpha), \quad (3.3)$$

where

$$\begin{aligned} \phi_\tau &= \sin \alpha \left(\frac{\tau^2}{2} \cos \alpha - u\tau \right), \\ \phi_\delta &= \sin \alpha \left(\frac{\delta^2}{2} \cos \alpha - \beta\delta \right). \end{aligned} \quad (3.4)$$

Assume that

$$\phi_n = \phi_\tau + \phi_\delta. \quad (3.5)$$

The output of channel having “ N ” multipath, each has an associated delay “ τ ” and associated path “ δ ”, will be given as

$$\frac{O}{P} = \sum_{n=0}^{N-1} A_n e^{j\phi_n} F_\alpha(\beta - \delta_n \cos \alpha, u - \tau_n \cos \alpha), \quad (3.6)$$

where “ A ” is the amplitude of phase-shifted component. From this expression, it is viewable that the channel imposes the following parameters on the signal propagating through it:

- (i) introduces a gain “ A ”,
- (ii) a time delay τ and path delay δ when transformation is not carried,
- (iii) a phase shift “ ϕ_n ” in FRFT domain, which becomes equal to the phase shift in FT domain when angle parameter $\alpha = \pi/2$,
- (iv) a delay $(\delta_n \cos \alpha, \tau_n \cos \alpha)$ in FRFT domain, which becomes equal to zero at $\alpha = \pi/2$.

Since FRFT converts into FT at angle parameter $\alpha = \pi/2$, so the output of channel is derived for this value of α and is given as

$$\frac{O}{P} = \sum_{n=0}^{N-1} A_n e^{j \phi_{n, \alpha=\pi/2}} F(k, \omega). \quad (3.7)$$

This expression resembles the output expression given for DDT-FT in [7]. The effects introduced by the channel to the propagating signal is actually:

- (i) introduces a gain term “ A ”,
- (ii) introduces a phase shift term in Fourier domain.

Here, the output signal which encounters multiple phase shifts in a wireless communication channel can be represented by

$$A_0 e^{j\phi_0} F(k, \omega) + A_1 e^{j\phi_1} F(k, \omega) + A_2 e^{j\phi_2} F(k, \omega) + \dots, \quad (3.8)$$

where ϕ_n is the various phase shifts in FT domain. So the transfer function (TF) of the channel will be calculated as

$$\text{TF} = \frac{\text{Channel o/p}}{\text{Channel i/p}} = \frac{A_0 e^{j\phi_0} F(k, \omega) + A_1 e^{j\phi_1} F(k, \omega) + \dots}{F(k, \omega)}. \quad (3.9)$$

By expanding each exponential term as a power series in (3.9) and rearranging the expression

$$\begin{aligned} \text{TF} &= (A_0 + A_1 + \dots) + j(A_0 \phi_0 + A_1 \phi_1 + \dots) \\ &\quad + \frac{j^2}{2} (A_0 \phi_0^2 + A_1 \phi_1^2 + \dots) + \dots, \end{aligned} \quad (3.10)$$

and comparing it with the Fourier transform of a rectangular pulse in time domain [9], it can be easily predicted that the TF of channel is the transform of pulse-like characteristic in time domain and pulse-like characteristic in spatial domain.

As the FRFT of a rectangular function is calculated from $\alpha = 0$ to $\alpha = \pi/2$, the rectangular function changes its shape from rectangular to sinc function. Similarly, the channel response to a signal is analyzed in FRFT domain from $\alpha = 0$ to $\alpha = \pi/2$, and it is clear from the discussion above that at $\alpha = 0$, channel imposes time and path delay to the signal, and at $\alpha = \pi/2$, channel imposes phase shift term in frequency and propagation constant domain, that is, filtering action will be performed in these domains, and for $0 < \alpha < \pi/2$, channel will introduce not only phase term but also a delay term in that domain.

4. Utility of DDT-FRFT by Simulation

For establishing the utility of DDT-FRFT in the environment where frequency components of electromagnetic wave become time variant, and propagation constant associated with wave depends on spatial coordinate, a case has been studied with the help of simulation. Here, for simplicity, only the linear variation of propagation constant with spatial variable is assumed. In this example, the propagation constant “ k ” and instantaneous frequency “ ω ”, as shown in (2.1), having respective dependencies are given as

$$\begin{aligned} \omega &= 2\pi(f_0 + a t), \\ k &= 2\pi(\mu_0 + b x). \end{aligned} \quad (4.1)$$

Figure 1: The magnitude of DDT-FT (DDT-FRFT at $\alpha = \pi/2$) of EM wave.

Figure 2: The magnitude of DDT-FRFT of EM wave at $\alpha = 0.2$.

And for these dependencies of “ ω ” and “ k ” on time and spatial variable, the travelling electromagnetic wave has the expression

$$f(x, t) = \cos(2\pi(\mu_0 + bx)x - 2\pi(f_0 + at)t). \quad (4.2)$$

The values of f_0 , a , μ_0 , and b are chosen as 2, 10, 1.5, and 20, respectively. And the DDT-FRFT has been observed for a duration double of the time period of EM wave in time domain and double of the spatial wavelength in the spatial domain. First, the DDT-FRFT of travelling wave expression is evaluated for $\alpha = \pi/2$, as it is the same as DDT-FT (because DDT-FRFT is converted to DDT-FT at $\alpha = \pi/2$), as shown in Figure 1. Next, the DDT-FRFT is simulated for $\alpha = 0.2$, as shown in Figure 2. The results are shown in Figures 1 and 2

5. Conclusion

In this paper, a new dual-domain transform based on fractional Fourier transform (DDT-FRFT) has been proposed. The proposed technique has the application where EM wave

encounters a Doppler shift due to the motion of transmitter and/or receiver along with having different phase velocity for the different portion of the medium in which the wave is propagating. Simulation results shown above are in the conformity of this statement, because at angle parameter " α " associated with FRFT having value of " $\pi/2$ ", that is, the case of Fourier transform, the transformed quantity does not have a clear peak (Figure 1), while for $\alpha = 0.2$, a clear peak is observable in the magnitude response of DDT-FRFT of travelling wave (Figure 2). It shows that in noisy conditions, due to a clear peak, the analysis of travelling wave with DDT-FRFT corresponding to $\alpha = 0.2$ gives better results than DDT-FT.

Also the application of DDT-FRFT is discussed for the channel modeling. And it has been established that a multipath wireless channel exhibits pulse-like characteristic not only in time domain but also in spatial domain, that is, for $\alpha = 0$. Similarly, the channel acts as a band pass filter in both the frequency domain and propagation constant domain, that is, for $\alpha = \pi/2$. And for the values in between these two limits, the channel introduces a delay in that domain along with the filtering action.

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