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Research Article

Second Law Analysis of Mixed Convection in a Laminar, Non-Newtonian Fluid Flow through a Vertical Channel

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The fully developed mixed convection of non-Newtonian laminar flow through a vertical channel is investigated. The boundary conditions of uniform and unequal temperature prescribed at the channel walls are considered. The velocity and temperature fields are obtained by analytically solving the momentum and energy balance equations. The velocity and temperature distributions are used to calculate the entropy generation number (N_S), the irreversibility ratio (Φ), and the Bejan number (Be) for several values of the viscous dissipation parameter ($\mathrm{Br}\Omega^{-1}$), the viscosity index (n), and the appropriate dimensionless coordinates. The results show us the regions of high entropy generation.

1. Introduction

The study of second law analysis of a laminar non-Newtonian, power-law fluid flowing between two heated plates has many significant applications in thermal engineering and industries. The heat transfer of non-Newtonian fluids in ducts is a subject which has received much attention in the last decades. The interest in this field is due, for instance, to many industrial applications which involve polymeric materials. Starting from petroleum industry to various heat exchanger systems, this type of geometry can be observed. Meanwhile, the improvement in thermal systems as well as energy utilization during the convection in any fluid is one of the fundamental problems of the engineering processes, since improved thermal systems will provide better material processing, energy conservation, and environmental effects. One of the methods used for predicting the performance of the engineering processes is the second law analysis. The second law of thermodynamics is applied to investigate the irreversibility in terms of the entropy generation rate. Since the entropy generation is a measure of the destruction of the available work of the system, the determination of conditions which cause the entropy generation and then improve upon it is important in upgrading the system performances. This method was introduced by Bejan [1, 2] and followed by many other investigators.

The most important results on internal flow convection of power law fluids have been outlined in several papers by Cho and Hartnett [3], who considered circular geometry, and Hartnett and Kostic [4], who considered rectangular geometry.

Jones and Ingham [5] and Ingham and Jones [6] studied the entrance region of combined forced and free flow in a vertical parallel-plate channel. The authors employ a finite difference method based on fully implicit scheme. This numerical method allows a direct determination of the stream function, velocity, and temperature distributions. Symmetric boundary conditions are considered. Uniform wall temperature or linearly varying wall temperatures boundary conditions are prescribed.

The analysis of entropy generation rate in a circular duct with imposed heat flux at the wall and its extension to determine the optimum Reynolds number as a function of the Prandtl number were presented by Bejan [1, 2]. Sahin [7] performed the second law analysis on a viscous fluid in a circular duct with isothermal boundary conditions. In another paper, Sahin [7] presented the effect of variable viscosities on the entropy generation rate for the heated circular duct. A comparative study of the entropy generation rate inside ducts of different shapes and the determination of the optimum duct shape subjected to the isothermal boundary condition were performed by Sahin [8]. Mahmud and Fraser [9, 10] applied the second law analysis to forced convective heat transfer problems and to a non-Newtonian fluid flow through channel made of two parallel plates.

Entropy generation is closely associated with thermodynamic irreversibility, which is encountered in all practical heat transfer processes. Different sources are responsible for entropy generation such as the heat transfer in the presence of temperature difference and the viscous dissipation. Fluid flow inside a channel made of two parallel plates is one of the topics attracting great interest in thermal engineering as they appear in many industrial applications.

2. Mathematical Formulation and Analysis

Consider two parallel heated plates with a fully developed non-Newtonian, power-law, flowing fluid between them. Figure 1 shows the flow model.

2.1. Velocity and Temperature Distribution

Let us consider the laminar steady flow of a power-law fluid in a vertical channel. Both channel walls are assumed to be isothermal, one with temperature T_1 and the other with temperature T_2, ($T_2 > T_1$). The flow model and coordinate system are shown in Figure 1. In this figure, L is the half-width and g is the gravitational acceleration. The buoyancy effect is taken into account according to the Boussinesq approximation. The latter is accomplished by employing the equation of state

$$\rho = \rho_0 \left[1 - \beta(T - T_0)\right], \tag{2.1}$$

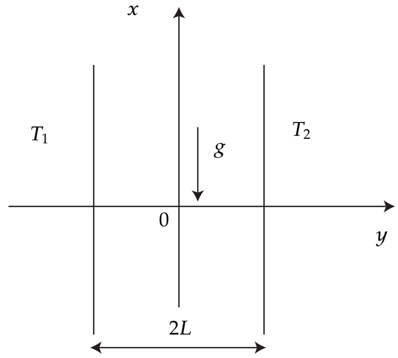

Figure 1: The local flow model and coordinate system.

where T is temperature, ρ is the mass density, β is the thermal expansion coefficient, ρ_0 is the mass density at $T = T_0$, and T_0 is the mean temperature in a channel section, that is,

$$T_0 = \frac{1}{2L} \int_{-L}^{L} T\, dY. \tag{2.2}$$

Let us denote by U and V the X-component and the Y-component of the velocity field, respectively. The mass balance implies that the velocity field is solenoidal, while the conditions of fully developed flow imply that

$$\frac{\partial U}{\partial X} = 0. \tag{2.3}$$

Therefore one can conclude that U depends only on Y and that V is zero. On account of the Ostwald-de Waele constitutive equation, the components

$$\tau_{XY} = \tau_{YX} = \tau = \eta \left| \frac{dU}{dY} \right|^{(1-m)/m} \frac{dU}{dY}, \tag{2.4}$$

where η is the consistency factor and m is the inverse of the power-law index. The case $m < 1$ corresponds to dilatant fluid behavior, while the case $m > 1$ occurs for pseudo plastic fluids.

Let us assume that the thermophysical properties of the fluid ρ_0, β, η and the thermal conductivity k are independent of temperature. Then, (2.4) implies that τ depends only on Y. The momentum balance along Y-direction yields $\partial P/\partial X = 0$, where $P = p + \rho_0 g\, X$ is the difference between the pressure p and the hydrostatic pressure. The momentum balance along the X-direction can be written as

$$g\beta\rho_0(T - T_0) - \frac{dP}{dX} + \frac{d\tau}{dY} = 0. \tag{2.5}$$

By deriving both sides of (2.5) with respect to X, one obtains

$$\frac{\partial T}{\partial X} - \frac{dT_0}{dX} - \frac{1}{g\beta\rho_0}\frac{d^2P}{dX^2} = 0. \tag{2.6}$$

By integrating both sides of (2.6) with respect to Y in the interval $-L < Y < L$ and by employing (2.2), one is led to the following conclusion:

$$\frac{d^2P}{dX^2} = 0, \qquad \frac{\partial T}{\partial X} = \frac{dT_0}{dX}. \tag{2.7}$$

As a consequence of (2.7), one infers that dP/dX is a constant.

Since the channel walls are isothermal and since (2.7) implies that $\partial T/\partial X$ does not depend on Y, one can deduce that $\partial T/\partial X$ is zero; that is, T depends only on Y. Therefore the energy balance equation yields

$$\frac{d^2T}{dY^2} = 0. \tag{2.8}$$

The analysis presented by Barletta [11] has been used to compute the entropy generation terms, and therefore a summary of his analysis is shown below.

Equation (2.4) can be easily inverted to obtain dU/dY, namely,

$$\frac{dU}{dY} = \frac{\tau}{\eta}\left(\frac{|\tau|}{\eta}\right)^{m-1}. \tag{2.9}$$

On account of no-slip boundary condition for velocity field, (2.9) allows one to obtain the expression

$$U(Y) = \frac{1}{\eta^m}\int_{-L}^{Y} \tau(Y')\left|\tau(Y')\right|^{m-1} dY'. \tag{2.10}$$

Let us choose a reference velocity as follows:

$$U_0 = -\left(\frac{2}{\eta}\right)^m D^{m+1}\left|\frac{dP}{dX}\right|^{m-1}\frac{dP}{dX}, \tag{2.11}$$

where $D = 4L$ is the hydraulic diameter. If one defines the dimensionless quantities,

$$\mathrm{Re} = \frac{\rho_0 U_0 |U_0|^{(m-1)/m} D^{1/m}}{\eta}, \qquad \mathrm{Gr} = \frac{g\beta\rho_0^2 (T_2 - T_1)|U_0|^{2(m-1)/m} D^{(m+2)/m}}{\eta^2}, \qquad \omega = \frac{T_0 - T_1}{T_2 - T_1},$$
$$y = \frac{Y}{D}, \qquad u = \frac{U}{U_0}, \qquad \theta = \frac{T - T_0}{T_2 - T_1}, \qquad \sigma = \frac{2\,\mathrm{Re}\,\tau}{\rho_0 U_0^2}, \qquad \Lambda = \frac{\mathrm{Gr}}{\mathrm{Re}}. \tag{2.12}$$

Equations (2.5), (2.8), and (2.9) can be written in the dimensionless form:

$$\frac{d\sigma}{dy} = -2\Lambda\theta - 1, \tag{2.13}$$

$$\frac{d^2\theta}{dy^2} = 0, \tag{2.14}$$

$$\frac{du}{dy} = \frac{\sigma}{2}\left|\frac{\sigma}{2}\right|^{m-1}. \tag{2.15}$$

The boundary conditions for the dimensionless fields can be expressed as

$$u\left(\frac{-1}{4}\right) = u\left(\frac{1}{4}\right) = 0, \qquad \begin{cases} \theta\left(\frac{-1}{4}\right) = 1 - \omega, \\ \theta\left(\frac{1}{4}\right) = -\omega. \end{cases} \tag{2.16}$$

A further constrain on θ is induced by (2.2), namely,

$$\int_{-1/4}^{1/4} \theta(y)dy = 0. \tag{2.17}$$

Equations (2.2)–(2.17) determine uniquely the functions $u(y)$, $\theta(y)$, $\sigma(y)$ and the parameter ω, provided that the inverse of the power-law index and the parameter

$$\lambda = \frac{\mathrm{Gr}}{\mathrm{Re}} = \frac{g\beta D(T_2 - T_1)}{U_0^2}\,\mathrm{Re} \tag{2.18}$$

are prescribed. On account of (2.13), the integration of both sides of (2.15) with respect to y in the interval $[-1/4,\ 1/4]$ yields the following constrain on $\sigma(y)$:

$$\int_{-1/4}^{1/4} \sigma(y)\left|\sigma(y)\right|^{m-1}dy = 0. \tag{2.19}$$

Equation (2.14) implies that $\theta(y)$ is a linear function of y. Then, the boundary conditions expressed by (2.27) and the additional constraint given by (2.17) yield

$$\theta(y) = 2y, \qquad \omega = \frac{1}{2}. \tag{2.20}$$

On account of (2.1) and (2.12), $\sigma(y)$ can be expressed as

$$\sigma(y) = -2\Lambda y^2 - y + C(m, \Lambda), \tag{2.21}$$

where $C(m,\lambda)$ is an integration constant which can be determined by employing the constraint given by (2.19). Therefore, $C(m,\lambda)$ is the solution of the equation

$$\int\left(2\Lambda y^2+y-C\right)\left|2\Lambda y^2+y-C\right|^{m-1}dy=0. \tag{2.22}$$

On account of (2.15), (2.16), and (2.21), the dimensionless velocity can be evaluated as

$$u(y)=-\frac{1}{2^m}\int_{-1/4}^{y}\left[2\Lambda y^2+y-C(m,\Lambda)\right]\left|2\Lambda y^2+y-C(m,\Lambda)\right|^{m-1}dy. \tag{2.23}$$

On account of (2.15) and (2.21),

$$\frac{du}{dy}=-\frac{1}{2^m}\left\{2\Lambda y^2+y-C(m,\Lambda)\right\}\left|2\Lambda y^2+y-C(m,\Lambda)\right|^{m-1}. \tag{2.24}$$

On account of (2.20),

$$\frac{\partial\theta}{\partial y}=2. \tag{2.25}$$

Since $\partial T/\partial X=0$, the entropy generation rate equation becomes

$$S_g'''=K\left(\frac{\partial T}{\partial Y}\right)^2\frac{1}{T_0^2}+\frac{\mu}{T_0}\left(\frac{\partial U}{\partial Y}\right)^{n+1}. \tag{2.26}$$

In the previous equation, the superscript indicates per unit volume. Writing the entropy generation in nondimensional form by defining the entropy number N_s,

$$N_S=\left(\frac{\partial\theta}{\partial y}\right)^2+\frac{\mu T_0D^{1-n}u_0^{n+1}}{K(T_2-T_1)^2}\left(\frac{\partial u}{\partial y}\right)^{n+1}. \tag{2.27}$$

Using (2.24), (2.25), and (2.27),

$$\begin{aligned}N_s&=4+\frac{\mu T_0D^{1-n}u_0^{n+1}}{K(T_2-T_1)^2}\left[\frac{-1}{2^m}\left\{2\Lambda y^2+y-C(m,\Lambda)\right\}\left|2\Lambda y^2+y-C(m,\Lambda)\right|^{m-1}\right]^{n+1},\\ N_s&=4+\frac{\mathrm{Br}}{\Omega}\left[\frac{-1}{2^m}\left\{2\Lambda y^2+y-C(m,\Lambda)\right\}\left|2\Lambda y^2+y-C(m,\Lambda)\right|^{m-1}\right]^{n+1},\end{aligned} \tag{2.28}$$

where

$$\mathrm{Br}\Omega^{-1}=\frac{\mu T_0D^{1-n}u_0^{n+1}}{K(T_2-T_1)^2}. \tag{2.29}$$

is a dimensionless number.

2.2. Irreversibility Ratio

In convection problems, both fluid friction and the heat transfer contribute to the rate of entropy generation. In order to assess which one among the fluid friction and heat transfer dominates, a criterion known as irreversibility ratio is defined by the following equation.

Irreversibility ratio Φ is the ratio of the entropy generation due to the fluid friction to the total entropy generation due to heat transfer.

Φ is irreversibility due to fluid friction/irreversibility due to heat transfer

$$\Phi = \frac{\left(\mu T_0 D^{1-n} u_0^{n+1}/K(T_2 - T_1)^2\right)(\partial u/\partial y)^{n+1}}{(\partial \theta/\partial y)^2},$$
$$\Phi = \frac{\mathrm{Br}\Omega^{-1}(\partial u/\partial y)^{n+1}}{4}. \tag{2.30}$$

2.3. The Bejan Number

Bejan number is the ratio of heat transfer irreversibility to the total irreversibility due to heat transfer and fluid friction.

Be is irreversibility due to heat transfer/total irreversibility

$$\mathrm{Be} = \frac{4}{\left\{(\partial \theta/\partial y)^2 + \mathrm{Br}\Omega^{-1}(\partial u/\partial y)^{n+1}\right\}}. \tag{2.31}$$

3. Results and Discussion

The previous mathematical analysis is valid for the second law analysis of a laminar non-Newtonian, power-law fluid flowing between two parallel heated plates. The velocity and temperature distributions are used to calculate the entropy generation number, irreversibility ratio, and the Bejan number for the case of a laminar, non-Newtonian, power-law fluid. These are presented graphically for various values of the viscosity index (n), the viscous dissipation parameter ($\mathrm{Br}\Omega^{-1}$), and the dimensionless axial distance (y).

3.1. Viscous Dissipation Parameter ($Br\Omega^{-1}$)

The viscous dissipation parameter ($\mathrm{Br}\Omega^{-1}$) is defined as the product of the Brinkman number and the inverse of dimensionless temperature difference.
We have the following:

(i) the Brinkman number (Br) = $\mu u_0^{1+n} D^{1-n}/k\ \Delta T$,

(ii) Dimensionless temperature difference $\Omega = \Delta T/T_0$,

(iii) Viscous dissipation parameter ($\mathrm{Br}\Omega^{-1}$) = $\mu u_0^{1+n} D^{1-n}\ T_0/k\Delta T^2$,

The viscous dissipation parameter is an important dimensionless number for the irreversibility analysis. It determines the relative importance of the viscous effects for the entropy generation.

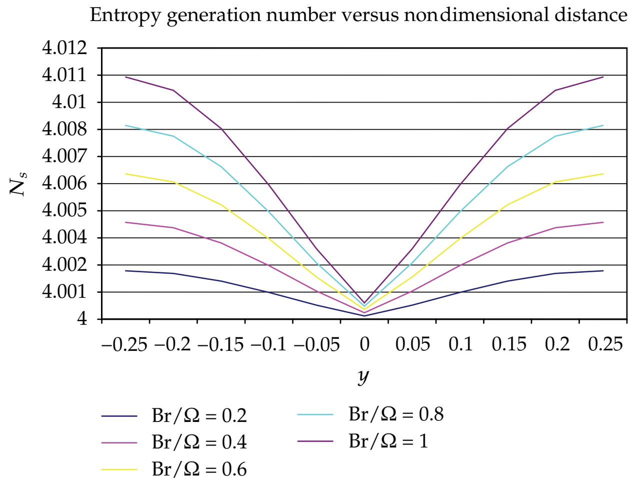

Figure 2: $\lambda = 1,\ n = 1.25$.

3.2. Entropy Generation Number

The spatial distribution of the entropy generation number is plotted in Figures 2, 3, and 4

$$N_S = \left(\frac{\partial\theta}{\partial y}\right)^2 + \mathrm{Br}\Omega^{-1}\left(\frac{\partial u}{\partial y}\right)^{n+1}, \tag{3.1}$$

where N_S = entropy generation number, N_F is $(\mathrm{Br}\Omega^{-1})(\partial u/\partial y)^{n+1}$ = entropy generation due to the fluid friction, and $N_H = (\partial\theta/\partial y)^2$ is entropy generation due to heat transfer. Therefore,

$$N_S = N_H + N_F. \tag{3.2}$$

We note that the entropy generation rate is highest at the isothermal walls and gradually decreases as we move towards the center of the channel. This is because the rate of change of velocity $(\partial u/\partial y)$ is highest at the channel walls and decreases as we move towards the center.

The entropy generation number increases as the viscous dissipation parameter $(\mathrm{Br}\Omega^{-1})$ increases because an increased viscous dissipation parameter increases the entropy generation due to fluid friction.

The entropy generation number increases as the viscosity index (n) increases because the more the viscousity the fluid, the higher the entropy generation rate due to fluid friction which eventually increases the entropy generation number.

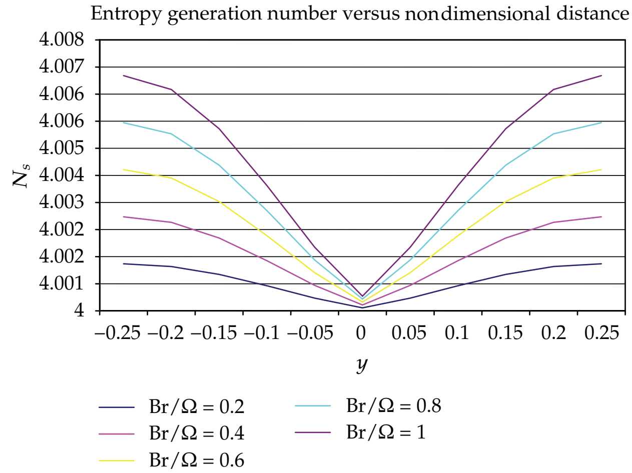

Figure 3: $\lambda = 1$, $n = 1.0$.

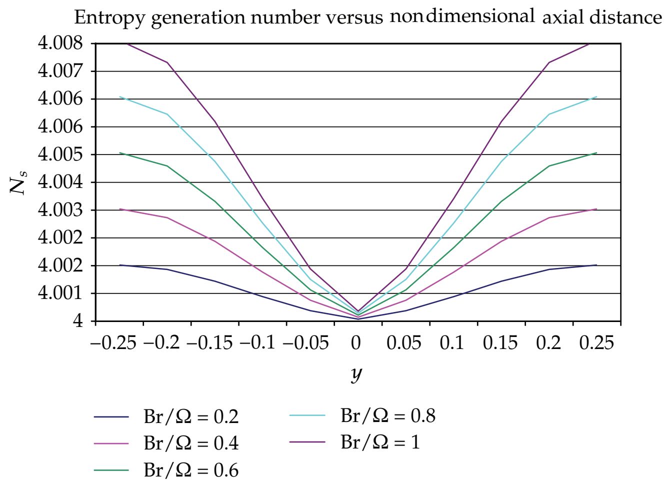

Figure 4: $\lambda = 1$, $n = 0.5$.

3.3. Irreversibility Ratio

In Figures: 5, 6, and 7 irreversibility ratio is plotted as a function of the transverse distance for different values of the viscous dissipation parameter ($\mathrm{Br}\Omega^{-1}$) and viscosity index (n)

$$\Phi = \frac{N_F}{N_H},$$
$$\Phi = \frac{\mathrm{Br}\Omega^{-1}(\partial u/\partial y)^{n+1}}{(\partial \theta/\partial y)^2}. \tag{3.3}$$

The irreversibility ratio is highest at the channel walls and decreases as we move towards the center.

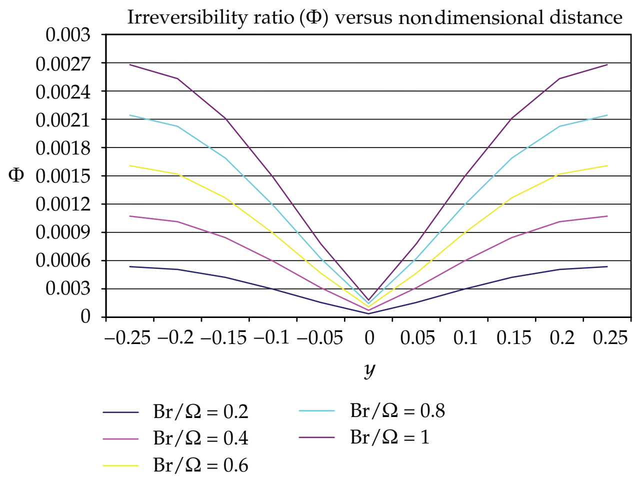

Figure 5: $\lambda = 1,\ n = 1.25$.

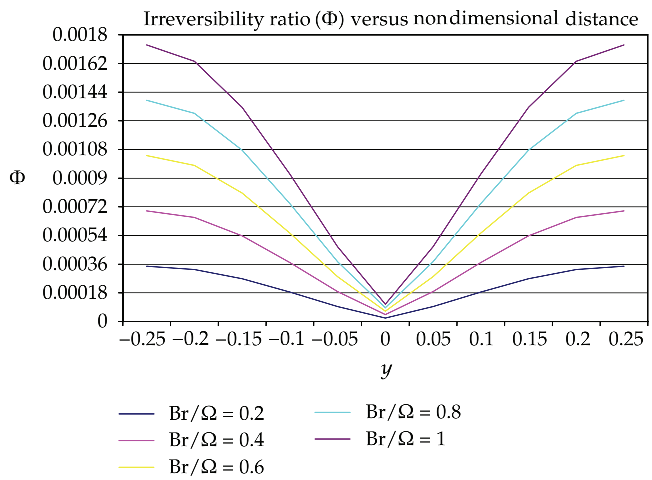

Figure 6: $\lambda = 1,\ n = 1.0$.

Irreversibility ratio increases as the viscous dissipation parameter ($\mathrm{Br}\Omega^{-1}$) and the viscosity index (n) increase.

3.4. The Bejan Number (Be)

The Bejan number is the ratio of irreversibility due to heat transfer to the total irreversibility due to heat transfer and fluid friction

$$\begin{aligned}\text{Bejan number} &= \frac{(N_H)}{(N_H + N_F)} \\ &= \frac{1}{(1\ +\ \Phi)}.\end{aligned} \tag{3.4}$$

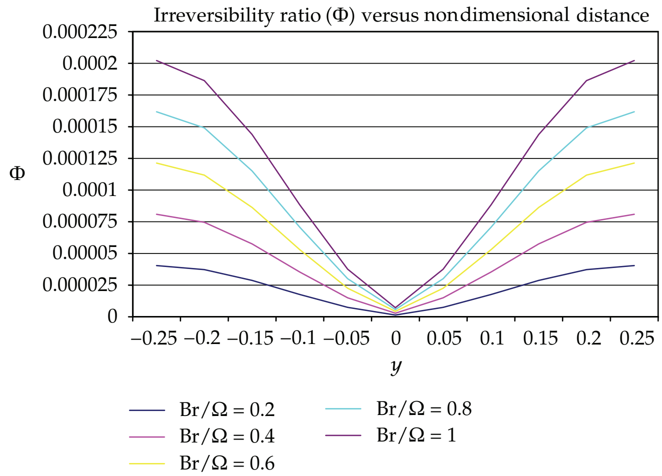

Figure 7: $\lambda = 1,\ n = 0.5$.

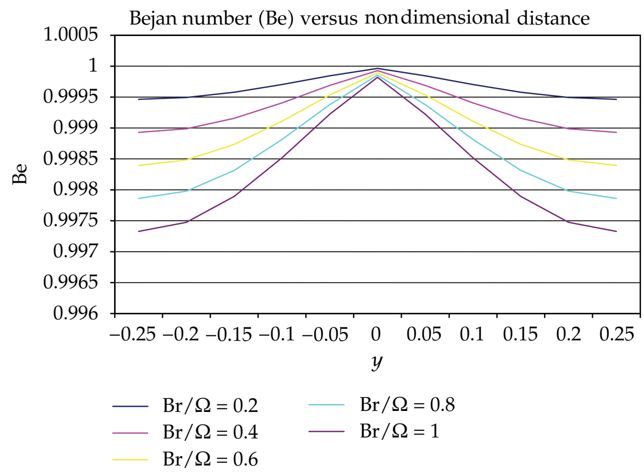

Figure 8: $\lambda = 1,\ n = 1.25$.

In Figures 8, 9, and 10, the Bejan number profiles are shown as functions of the transverse distance for different values of the viscous dissipation parameter ($\mathrm{Br}\Omega^{-1}$), viscosity index (n), and constant (λ).

The Bejan number is highest at the center of the channel and decreases as we move towards the channel walls on either direction.

The Bejan number decreases as the viscous dissipation parameter ($\mathrm{Br}\Omega^{-1}$) and the viscosity index (n) increase.

4. Concluding Remarks

This paper presents the application of the second law of thermodynamics to mixed convection in a laminar, non-Newtonian, power-law fluid flowing between two parallel isothermal

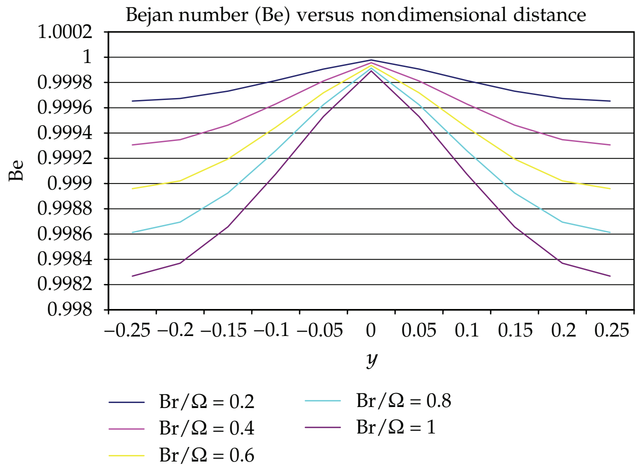

Figure 9: $\lambda = 1,\ n = 1.0$.

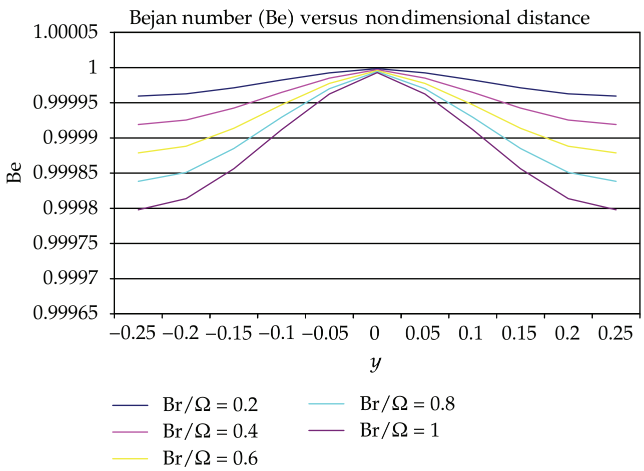

Figure 10: $\lambda = 1,\ n = 0.5$.

vertical plates. The velocity and the temperature profiles are obtained analytically and used to compute the entropy generation number, irreversibility ratio, and the Bejan number for several values of the viscous dissipation parameter ($\mathrm{Br}\Omega^{-1}$), the viscosity index (n), and the dimensionless axial distance (y). The numerical results show that the nondimensional entropy number is least at the center of the channel and increases in the transverse direction on either side owing to an increased velocity gradient near the walls. The entropy generation number increases with increase in the viscosity index and increase in the viscous dissipation parameter ($\mathrm{Br}\Omega^{-1}$).

Irreversibility ratio is highest at the channel walls and decreases as we move towards the center. Irreversibility ratio increases as the viscous dissipation parameter ($\mathrm{Br}\Omega^{-1}$) and the viscosity index (n) increase.

The Bejan number is least at the channel walls and increases in the transverse direction as we move towards the center. The numerical results show that the Bejan number decreases as the viscous dissipation parameter ($\mathrm{Br}\Omega^{-1}$) and the viscosity index (n) increase.

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