Hindawi Publishing Corporation
Physics Research International
Volume 2011, Article ID 387351, 18 pages
doi:10.1155/2011/387351

Review Article

Relativistic Thermodynamics: A Modern 4-Vector Approach

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Received 5 April 2011; Accepted 8 July 2011

Academic Editor: Ashok Chatterjee

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Using the Minkowski relativistic 4-vector formalism, based on Einstein's equation, and the relativistic thermodynamics asynchronous formulation (Grøn (1973)), the isothermal compression of an ideal gas is analyzed, considering an electromagnetic origin for forces applied to it. This treatment is similar to the description previously developed by Van Kampen (van Kampen (1969)) and Hamity (Hamity (1969)). In this relativistic framework Mechanics and Thermodynamics merge in the first law of relativistic thermodynamics expressed, using 4-vector notation, such as $\Delta U^\mu = W^\mu + Q^\mu$, in Lorentz covariant formulation, which, with the covariant formalism for electromagnetic forces, constitutes a complete Lorentz covariant formulation for classical physics.

1. Introduction

During the 1960s and 1970s many physicists devoted considerable effort to finding the most adequate relativistic formulation of thermodynamics [1]. Yuen's 1970 paper [2] presents the state of the art on relativistic thermodynamics at this time. After the work by Van Kampen [3] and Hamity [4] introducing 4 vectors in thermodynamics and the clearly stated asynchronous formulation by Gamba [5], Cavalleri and Salgarelli [6], and Grøn [7] a consensual relativistic thermodynamics formalism should have been achieved. However, no agreement on the correct Relativistic thermodynamics was reached [8] ([9], pp. 303–305). Until recently, papers on this topic have been published [10], mainly on relativistic transformation of temperature [11, 12].

Let Z be a composite body, which moves, in a reference frame S, under the action of k external (conservative and nonconservative) forces $\mathbf{F}_k = (F_{xk}, F_{yk}, F_{zk})$, simultaneously applied during time interval $\mathrm{d}t$, with resultant force $\mathcal{F} = (\sum_k \mathbf{F}_k) = (\mathcal{F}_x, \mathcal{F}_y, \mathcal{F}_z)$, impulse $\mathcal{I} = \mathcal{F}\,\mathrm{d}t$, with nonzero work $\delta W_{\mathrm{ext}} \neq 0$ (only conservative forces perform work) and that experiences a certain thermodynamic process, with internal energy variation $\mathrm{d}U \neq 0$ and heat $\delta Q \neq 0$. In classical physics, the complete description of this process, expressed in Galilean covariant form, is given by [13]: (i) a vectorial equation (linear momentum-impulse equation) $\mathrm{d}\mathbf{p} = \mathcal{I}$:

$$\begin{Bmatrix} \mathrm{d}p_x \\ \mathrm{d}p_y \\ \mathrm{d}p_z \end{Bmatrix} = \begin{Bmatrix} \sum_k F_{xk}\mathrm{d}t \\ \sum_k F_{yk}\mathrm{d}t \\ \sum_k F_{zk}\mathrm{d}t \end{Bmatrix} \tag{1}$$

and (ii) a scalar equation (first law of thermodynamics or energy equation) [14]:

$$\mathrm{d}K_{\mathrm{cm}} + \mathrm{d}U = \delta W_{\mathrm{ext}} + \delta Q. \tag{2}$$

From (1) the following equation can be obtained:

$$\mathrm{d}K_{\mathrm{cm}} = \mathcal{F} \cdot \mathrm{d}\mathbf{x}_{\mathrm{cm}}, \tag{3}$$

or the center of mass equation [15], where $\mathrm{d}\mathbf{x}_{\mathrm{cm}}$ is the displacement of the center of mass (cm) of Z and $\mathrm{d}K_{\mathrm{cm}}$ is its kinetic energy variation throughout the process.

For an observer in frame S_A in standard configuration with respect to frame S, with velocity $\mathbf{V} = (V, 0, 0)$ (Appendix A), it has the corresponding equations

$$\begin{gathered} \mathrm{d}K_{\mathrm{cm}A} = \mathcal{F} \cdot \mathrm{d}\mathbf{x}_{\mathrm{cm}A}, \\ \mathrm{d}K_{\mathrm{cm}A} + \mathrm{d}U = \delta W_{\mathrm{ext}A} + \delta Q, \end{gathered} \tag{4}$$

where the corresponding magnitudes are measured in S_A, the mass, force, interval of time, impulse $\mathcal{I}$, linear momentum variation $\mathrm{d}\mathbf{p}$, internal energy $\mathrm{d}U$, and heat δQ are Galilean invariants, and magnitude velocity $v_A = v - V$, displacement $\mathrm{d}\mathbf{x}_{\mathrm{cm}} = \mathrm{d}\mathbf{x}_{\mathrm{cm}A} - V\mathrm{d}t$, kinetic energy $\mathrm{d}K_{\mathrm{cm}A} = \mathrm{d}K_{\mathrm{cm}} - V\mathrm{d}\mathbf{p}$, and work $\delta W_{\mathrm{extA}} = \delta W_{\mathrm{ext}} - V\mathcal{I}$, have their specific Galilean transformation [16].

It is interesting to note that when forces applied to Z have an electromagnetic origin, with some force $\mathbf{F}_k$ obtained from "Lorentz force" equation $\mathbf{F}_k = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ (q electric charge, $\mathbf{E}$ electric field, and $\mathbf{B}$ magnetic field), the whole formalism is neither covariant under Galilean transformations (Lorentz force is not Galilean covariant [17]) nor covariant under Lorentz transformations (the previous thermodynamics formalism is not Lorentz covariant), in contradiction with Einstein's principle of inertia.

After these considerations about Galilean relativistic thermodynamics, not compatible with electromagnetic interactions, it seems necessary to obtain a formalism for the first law of thermodynamics expressed according to the principles of the special theory of relativity, that is, Lorentzian relativistic thermodynamics, compatible with electromagnetic interactions. As a result of this, it will be possible to obtain a Lorentz covariant formalism for exercises in classical physics that include concepts of mechanics, thermodynamics, and electromagnetism.

A modern view of a relativistic thermodynamics theory requires a clear definition of (i) the tensorial objects which characterize the equilibrium state of the system and of (ii) any tensorial object that characterizes the interaction of the system with its mechanical (work reservoir ([18], Chap. 3)) and thermal (heat reservoir ([18], pp. 89-90)) surroundings, with a prescription of the apparatus which measures it. The observables will depend, in general, on the physical system and on the observer (Appendix A), but the principle of relativity ensures that all inertial observers obtain equivalent descriptions of the same process. So, any relativistic formalism developed to describe a physical process must be according to this principle, that is, it must be Lorentz covariant. This is the course chosen in this paper, in which we solve an exercise on the isothermal (nonquasistatic) compression of an ideal gas in the reference frame S_0 in which the system is at rest and in a frame S_A, in standard configuration to S_0 (Appendix A), using the Minkowski 4-vectors—related through Lorentz transformations [19]—and a Lorentz covariant form for the first law of thermodynamics.

The paper is arranged as follows. In Section 2 the formalism, based on the principle of the inertia of energy (Einstein's equation) and on the asynchronous formulation, is developed. After that, in Section 2.3 the principle of similitude is enunciated, expressing the conditions under which the same equations can be used for an elementary particle and for a composite system. Section 3 presents the 4-vector energy function U^μ for different systems. The asynchronous formulation of 4-vector work W^μ is obtained in Section 4. In Section 5 thermal radiation 4-vector (heat) Q^μ, based on photons, is introduced. In Section 6 the mathematical formulation of the relativistic thermodynamics first law is presented in Lorentz covariant form. In Section 7 the isothermal compression, by two pistons, of an ideal gas is solved by using the previously developed formalism in both frames S_0, zero momentum frame, and S_A, in standard configuration respect S_0. Forces on pistons are described using an electromagnetic interaction, in its relativistic Lorentz covariant form. Finally, Section 8 proposes some conclusions regarding the possibility of solving exercises in classical physics in a complete Lorentz covariant form. Although we assume that the reader is familiar with the Minkowski 4-vector formalism, in Appendix A a brief review on 4-vectors and Lorentz transformation algebra is provided introducing the "metric tensor" $g_{\nu\mu}$ and the "Lorentz transformation" $\mathcal{L}^\mu_\nu(V)$ used in the paper [20].

2. Relativistic Thermodynamics Formalism

Relativistic thermodynamics formalism is developed in two steps: (i) Einstein's equation $E_0 = mc^2$, expressed as the principle of the inertia of energy, which allows us to obtain energy function U and the 4-vector energy function U^μ for a given system; (ii) the asynchronous formulation, that will allow us to obtain the work W performed by forces acting on a system and the 4-vector work W^μ. As a consequence, the principle of similitude can be formulated, according to which, and under very general circumstances, a composite system behaves as a whole in its interactions with its surroundings and equations for an elementary particle can be used with a composite, deformable system.

2.1. Inertia of Energy

2.1. Inertia of Energy. It could be considered, in a broad sense, that the main goal of relativistic thermodynamics is to reach a unified description on point dynamics and extended-body dynamics [13].

In order to ensure that an extended body behaves like a "single particle" interacting with its surroundings—work reservoirs or thermal bath—and so that it is physically meaningful to use Lorentz transformations, it is necessary that all forms of energy that make up the body contribute in the same way to its inertia [21]. These forms of energy must include those related with the mass of its constituent elementary particles, binding—nuclear, chemical, and so forth—energies (Figure 1), internal kinetic energy (see Section 7.1.1), electrostatic energy [22], and so forth, and energy of thermal radiation in equilibrium with matter inside the system [23] (see Section 5).

Einstein's Equation $E_0 = mc^2$ for an extended body can be interpreted by relating its *inertia*—a body's reluctance to undergo a change in velocity [24]—with *energy function* [25]—energy content of the physical system or internal energy [26].

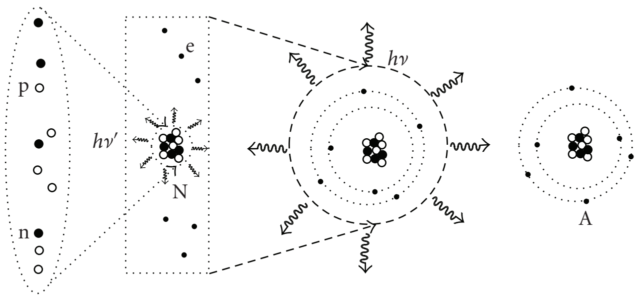

Figure 1: An atom (A)—self-contained structure—is obtained from a nucleus (N), previously assembled from protons (p) and neutrons (n), and an ensemble of electrons (e). Nucleus inertia $\mathcal{M}_{\mathrm{N}} = U_{\mathrm{N}}c^{-2}$ decreases respect the inertia of its elementary particle components $\mathcal{U}_{\mathrm{N}} = 6m_p + 4m_{\mathrm{n}}$, due to the energy $U_{\mathrm{N}} - \mathcal{U}_{\mathrm{N}} = -\widetilde{U}_{\mathrm{N}} = -8h\nu'$ released in its formation. Atom inertia $\mathcal{M}_{\mathrm{A}} = U_{\mathrm{A}}c^{-2}$ decreases with respect to the inertia of its component nucleus and electrons $\mathcal{U}_{\mathrm{A}} = 6m_{\mathrm{e}} + \mathcal{M}_{\mathrm{N}}$ due to the energy $U_{\mathrm{A}} - \mathcal{U}_{\mathrm{A}} = -\widetilde{U}_{\mathrm{A}} = -8h\nu$ released in its formation.

Principle of the Inertia of Energy. for an extended body in complete equilibrium, any kind of energy inside the system, relativistically expressed in reference frame S_0 in which the system as a whole is at rest, contributes to the *energy function* U of the system [27]. Considering that all forms of energy are convertible between them [28] the *inertia* $\mathcal{M}$ of a system [29] in equilibrium is ([30], p. 163)

$$\mathcal{M} = Uc^{-2} . \tag{5}$$

According to Einstein [31, 32] the inertia of a body changes with its content of energy [33] (Section 3).

It is possible to define the inertia $\mathcal{M}$ of a body (we prefer the term *inertia*, instead of mass [34], to avoid confusions when the system includes photons (Section 5)) as [35]: the inertia $\mathcal{M}$ of a composite body equals the sum of its elementary particles mass (protons, neutrons, and electrons) m_0:

$$m_0 = \sum_j m_p + \sum_k m_n + \sum_l m_e, \tag{6}$$

with energy $\mathcal{U} = m_0c^2$, minus the minimum energy $\widetilde{U}$, divided by c^2, necessary to separate its elementary particles so that they are far apart (Section 7.1.1):

$$\mathcal{M} = \left(\mathcal{U} - \widetilde{U}\right)c^{-2} = m_0 - \widetilde{U}c^{-2} , \tag{7}$$

with $U = \mathcal{U} - \widetilde{U}$.

2.2. Asynchronous Formulation. For an extended, deformable body a relativistic theory cannot be directly formulated in an arbitrary inertial frame. It must be based on known prerelativistic descriptions. On the one hand, it seems necessary to maintain the classical concept that the resultant force on the body must be zero (zero total impulse) when the motion remains uniform and to assure that when no torque is applied to the system in a certain reference frame, no torque is applied to it in another frame [36]. On the other hand, in classical mechanics forces on an extended system are applied simultaneously. This simultaneity occurs in all inertial frames. In thermodynamics, heat is a kind of interchanged energy with (assumed implicitly) zero linear momentum.

According to Cavalleri and Salgarelli, when forces on an extended, composite, body are applied, in order to develop a coherent formalism for relativistic thermodynamics, a privileged observer must exist, in reference frame S_0, that performs experiments on the body that remains at rest (Figure 2) [6].

According to Gamba [5]:

> "in the *Asynchronous Formulation*, observers in frames S_0 and S_{A} refer to the *same* experiment (the experiment performed in the privileged frame S_0) and obtain its own physical magnitudes, expressed as 4-vectors. In this formulation both descriptions of the experiment are connected by *true* Lorentz transformations [19]."

The observer in S_0 takes an ideal surface, at rest, which delimits the system considered and measures energy, work or heat, interchanged through the surface during time interval Δt. An observer in S_A obtains the same magnitudes by true Lorentz transformations, from the events considered by an observer in S_0. The observer in S_A does not perform a similar experiment to observer in S_0 (synchronous formulation [37]), it just translates the experiment performed in S_0 to its own physical magnitudes. Owing to the relativity of simultaneity, forces applied simultaneously in S_0 will not be simultaneous in S_A (asynchronous processes).

The existence of frame S_0 guarantees the correspondence between the relativistic and the classical descriptions; an equivalence necessary in the low velocity limit.

In the asynchronous formulation, given the quantity $A^\mu = B^\mu + C^\mu$, where B^μ is defined for the event $x_1^\mu = \{x_1, y_1, z_1, ct_1\}$ and C^μ is defined for the event $x_2^\mu = \{x_2, y_2, z_2, ct_2\}$, with $x_1^\mu \neq x_2^\mu$, but with $t_2 = t_1$,

$$A^\mu\left(x_1^\mu[t_1], x_2^\mu[t_1]\right) = B^\mu\left(x_1^\mu[t_1]\right) + C^\mu\left(x_2^\mu[t_1]\right), \tag{8}$$

then quantity $A^\mu(x_1^\mu[t_1], x_2^\mu[t_1])$ in S_0 is the same as $A_{\mathrm{A}}^\mu(x_{1\mathrm{A}}^\mu[t_{1\mathrm{A}}], x_{2\mathrm{A}}^\mu[t_{2\mathrm{A}}])$ in S_A when all subindex A quantities are obtained from the corresponding quantities in S_0 through Lorentz transformations. Relativity gives rules to relate measurements made by observers in frame S_0 to measurements made by observers in frame S_A only if this definition is adopted [5].

In the asynchronous formulation the 4-vector energy function U^μ is a time-like 4-vector in S_0, with zero linear momentum components [38] (see Section 3):

$$U^\mu = \{0, 0, 0, U\}. \tag{9}$$

In frame S_A, $U_{\mathrm{A}}^\mu = \{cp_{\mathrm{A}}, 0, 0, E_{\mathrm{A}}\}$ transforms under Lorentz transformation as $U_{\mathrm{A}}^\mu = \mathcal{L}_\nu^\mu(V)U^\nu$.

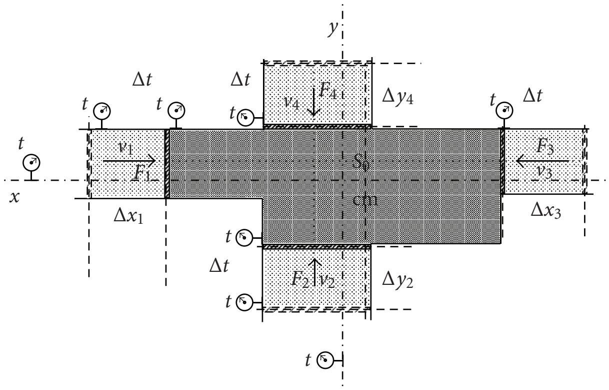

FIGURE 2: Compression process in frame S_0. A set of external forces F_k ($k = 1, 2, 3, 4$) are applied on an extended, deformable system during the same time interval Δt, as measured in frame S_0, with zero total impulse and zero torque. The kth force has associated the displacement $\Delta \mathbf{r}_k$ ($\mathbf{r} = (x, y)$) and 3-vector velocity $\mathbf{v}_k = \Delta \mathbf{r}_k / \Delta t$. The center of mass (cm) of the system does not move during the process.

For work due to external forces applied simultaneously in frame S_0 with total zero impulse, the 4-vector work W^μ is a timelike 4-vector [1] (see Section 4):

$$W^\mu = \{0, 0, 0, W\}. \tag{10}$$

As a generalization of this asynchronous formulation, in frame S_0 every flux of energy through the frontier of the system as thermal radiation (heat) is exchanged with zero total impulse. Thus, heat is exchanged with zero linear momentum in frame S_0 with a 4-vector Q^μ related to thermal radiation exchange given by [3] (see Section 5)

$$Q^\mu = \{0, 0, 0, Q\}. \tag{11}$$

In frame S_A the same Lorentz transformation $\mathcal{L}^\mu_\nu(V)$ is common to U^μ_A, W^μ_A, with $W^\mu_A = \mathcal{L}^\mu_\nu(V) W^\mu$ and to the 4-vector Q^μ_A, that transforms as the energy (timelike) part of a (time-like) 4-vector, $Q^\mu_A = \mathcal{L}^\mu_\nu(V) Q^\mu$.

2.3. Principle of Similitude. The asynchronous formulation and the principle of the inertia of energy guarantee that the system can be described as a "single particle" [39] characterized by its energy function U or its inertia $\mathcal{M}$. These considerations permit us to enunciate [13] the following.

Principle of Similitude. The mathematical expression for a physical law is the same when referred to an elementary particle, with tabulated mass m, or when referred to a composite body, well characterized by its energy function U, and inertia $\mathcal{M} = Uc^{-2}$.

In the asynchronous formulation there is no difference between Lorentz transformations for an elementary particle and Lorentz transformations for an extended body, provided that the system is in equilibrium, that is, energy function U of the body is well defined. In frames like S_A, in which the system has velocity V, differences between point dynamics and extended-body dynamics are due to the relativity of simultaneity [6], that is, forces applied simultaneously in S_0 but at different points of the body will not be simultaneous in S_A.

The principle of similitude has the following meaning. Physics equations, such as the Lorentz force equation $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$, Newton's second law of classical mechanics $\mathbf{F} = m\mathbf{a}$, or relativistic equations, such as $E^2 = m^2c^4 + c^2\mathbf{p}^2$ or $\mathbf{p} = (E/c^2)\mathbf{v}$, are correct when they are applied to an elementary particle, with mass m and charge q, because every magnitude is well defined, for example, total energy $E = \gamma(v)mc^2$, linear momentum $\mathbf{p} = \gamma(v)m\mathbf{v}$, and so forth, as well as the electric field $\mathbf{E}$, the magnetic field $\mathbf{B}$, and so forth, and forces applied are local forces, all of them applied at the same point. Similarly, a 4-vector, like $C^\mu = A^\mu + B^\mu$ or $C^\mu = c^{-1} q \mathcal{E}^\mu_\nu v^\nu$, transforms between frames S_0 and S_A in standard configuration, by using the Lorentz transformation, $\mathcal{L}^\mu_\nu(V)$, with $C^\mu_A = \mathcal{L}^\mu_\nu(V) C^\nu$, and so forth, and where C^μ_A, and so forth, is in S_A the same 4-vector C^μ in S_0, because all of them are locally defined.

When one wants to apply these equations to a process described on a composite, deformable body (e.g., a Ni atomic nucleus, a gas enclosed in a cylinder-piston system, a macroscopic chunk of Fe, etc.) and one wants to use the Lorentz transformation between reference inertial frames to transform 4-vectors, it is necessary to have previously ensured that the body behaves as a whole and that the principle of inertia of energy is satisfied. Because on an extended body different forces are applied at different points, it is necessary to ensure previously that there exists a reference frame S_0 in which the center of mass does not move during the process. This goal is achieved when external forces are applied according to the asynchronous formulation and when the interval of time during which forces are applied on the mobile parts of the system is greater than the relaxation time of the system.

Consider a gas enclosed in a cylinder-piston system. If the force on piston is applied in such a way that the velocity of the piston v_k is greater than the velocity of the sound in the gas v_s, with a characteristic gas relaxation time t_C given by $t_C \approx L/v_s$, where L is a characteristic linear dimension of the system, then the system does not behave as a whole during time intervals $\Delta t < t_C$ because there are parts of it that do not feel the perturbation and so do not contribute to the inertia of the system. In this case the description of the process cannot be made according to the relativistic formalism to be developed here, the principle of similitude is not applicable and another formalism must be used to describe the process [40].

When a process on a composite, extended body is carried out in such a way that the principle of inertia of energy is satisfied, the same set of equations valid for elementary particles can be used on the body.

Consider a macroscopic body with well-defined energy function U. In general, this energy function is temperature dependent $U \equiv U(T)$ (see Section 7.1.1) (U dependence on volume will not be considered volume [41]) and also its inertia $\mathcal{M}(T) = U(T)c^{-2}$, according to the principle of inertia of energy ([9], p. 289). When moving with velocity V

(one-dimensional) in frame S_A its linear momentum p_A and total energy E_A are given by

$$p_A = \gamma(V)\mathcal{M}(T)V,$$
$$E_A = \gamma(V)U(T), \quad (12)$$
$$p_A = \frac{E_A}{c^2 V}.$$

As previously noted, these results can be obtained from $U_A^\mu = \mathcal{L}_\nu^\mu(V)U^\nu$. These equations constitute the generalization for an extended body of equations $p_A = \gamma(V)mV$, $E_A = \gamma(V)mc^2$, and $p_A = (E_A/c^2)V$ for an elementary particle of mass m and velocity V. The total energy E_A of the body can be expressed as

$$E_A^2 = [U(T)]^2 + c^2 p_A^2 = [\mathcal{M}(T)]^2 c^4 + c^2 p_A^2 . \quad (13)$$

Equation (13) is the generalization, in a thermodynamics context, of the equation $E^2 = m^2c^4 + c^2p^2$ for an elementary particle.

For an elementary particle mass m and velocity V, the kinetic energy K is $K = [\gamma(V) - 1]mc^2$. The kinetic energy K_A for an extended body, defined as $K_A = E_A - U(T)$, is

$$K_A = \frac{c^2 p_A^2}{E_A + U(T)} = [\gamma(V) - 1]U(T)$$
$$= [\gamma(V) - 1]\mathcal{M}(T)c^2. \quad (14)$$

The kinetic energy of the body in frame S_0, in which its linear momentum is null, is zero.

3. Four-Vector Energy Function

Energy function U of a composite body is obtained from the energy function of its components (Section 2.1).

(1) Universal constants (c, h (Planck), k_B(Boltzmann), G, ϵ_0, etc.) are relativistic invariants having the same value for all inertial observers in relative motion.

(2) For an elementary particle—proton, neutron, and electron—the inertia equals its tabulated mass—m_p. m_n, m_e, respectively.

(3) For a nucleus, ${}^A_Z\mathrm{N}$, with Z protons and $(A - Z)$ neutrons, its inertia $\mathcal{M}_N$ equals the sum of the inertia of the elementary particles—with all elementary particles at infinite separation as initial arrangement—minus its binding energy (strong interaction) [42] $\widetilde{U}_N$ divided by c^2 (Figure 1):

$$U_N = \left[Zm_p + (A - Z)m_n\right]c^2 - \left|\widetilde{U}_N\right|,$$
$$\mathcal{M}_N = U_N c^{-2}. \quad (15)$$

(4) For an atom, the inertia $\mathcal{M}_A$ equals the sum of the inertia of its nucleus and electrons minus released energy $\widetilde{U}_A$ (electromagnetic interaction) [43] divided by c^2 (Figure 1):

$$U_A = U_N + n_p m_e c^2 - \left|\widetilde{U}_A\right|,$$
$$\mathcal{M}_A = U_A c^{-2}. \quad (16)$$

For instance, energy function u for a ^{4}He atom (2 protons, 2 neutrons, and 2 electrons) [21] is given [31] by

$$u = u_0 - \left(\left|\widetilde{U}_N\right| + \left|\widetilde{U}_A\right|\right),$$
$$u_0 = 2\left(m_p + m_e\right)c^2 + 2m_n c^2. \quad (17)$$

(5) For a molecule, formed by k atoms, the inertia $\mathcal{M}_M$ is the sum of the inertia of its individual atoms minus the energy released when chemical bonds are formed [44] divided by c^2:

$$U_M = \sum_k U_{Ak} - \left|\widetilde{U}_M\right|, \qquad \mathcal{M}_M = U_M c^{-2}. \quad (18)$$

Energy function U_C of a composite, self-contained (stable) system is less than the sum of the energy function of its k constituents [45] $\mathcal{U} = \sum_k U_k$, $U_C < U$.

(6) For a system of free noninteracting components [46] like a gas of He atoms, the inertia equals the sum of the total energy of components $U = \sum_k (k_k + u_k)$—kinetic energy and energy function of the kth component, respectively—divided by c^2 (see Section 7.1.1).

(7) For thermal radiation (photons in a cavity with energy density proportional to fourth power of absolute temperature) filling a cavity [47] its total energy U_p contributes to the total inertia of the system [48]. The thermal radiation emitted by a body can be described as radiation in a cavity [49] (see Section 5).

As previously noted, in the zero-momentum frame S_0 of a composite system with energy function $U \equiv U(T)$ the 4-vector that denotes the state of the system is given by $U^\mu = \{0, 0, 0, U(T)\}$. For an observer in frame S_A, 4-vector energy function $U_A^\mu = \{cp_A, 0, 0, E_A\}$ is $U_A^\mu = \mathcal{L}_\nu^\mu(V)U^\nu$, and one obtains

$$U_A^\mu = \{-c\gamma(V)\mathcal{M}(T)V, 0, 0, \gamma(V)U(T)\},$$
$$\mathcal{M}(T) = U(T)c^{-2},$$
$$p_A = -\gamma(V)\mathcal{M}(T)V, \quad (19)$$
$$E_A = \gamma(V)U(T),$$

according to the principle of inertia of energy (Section 7.2.1).

Einstein Equation. for a completely isolated system that performs any kind of internal process, for example, annihilation or creation of particles, disintegration, inelastic collisions, and so forth, the inertia does not change along the process [13], according to the Principle of Inertia of Energy, with

$$\Delta U^{\mu} = 0. \tag{20}$$

4. Four-Vector Work

In order to obtain a complete characterization of forces applied to a thermodynamical system (i.e., based on a fundamental interaction), we will describe forces as the interaction between an electric charge q_k located on the kth piston and a (static) electric field $\mathbf{E}_k = (E_{xk}, E_{yk}, E_{zk})$. This procedure guarantees a detailed description of forces and of its relativistic transformation between reference frames (Appendix B).

Consider in frame S_0 a set of k forces $\mathbf{F}_k = (F_{xk}, F_{yk}, F_{zk}) = q(E_{xk}, E_{yk}, E_{zk})$, with an electromagnetic origin, simultaneously applied, on different k pistons, on an extended body (Figure 3) during the same interval of time Δt, according to the previously discussed asynchronous formulation. Impulse $\mathbf{I}_k = (I_{xk}, I_{yk}, I_{zk})$ and work W_k for the kth force are given by

$$\begin{aligned} \mathbf{I}_k &= \left(F_{xk}\mathrm{d}t, F_{yk}\mathrm{d}t, F_{zk}\mathrm{d}t\right), \\ W_k &= \mathbf{F}_k \cdot \mathrm{d}\mathbf{r}_k = F_{xk}\mathrm{d}x_k + F_{yk}\mathrm{d}y_k + F_{zk}\mathrm{d}z_k. \end{aligned} \tag{21}$$

The kth field $\mathbf{E}_k$ is represented by the $4\times4\times4$-tensor $E_{k\,\nu}^{\,\mu}$:

$$\varepsilon_{k\,\nu}^{\,\mu} = \begin{Bmatrix} 0 & 0 & 0 & E_{xk} \\ 0 & 0 & 0 & E_{yk} \\ 0 & 0 & 0 & E_{zk} \\ E_{xk} & E_{yk} & E_{zk} & 0 \end{Bmatrix}, \tag{22}$$

with the $4 \times 4 \times 4$-tensor electromagnetic force $F_{k\,\nu}^{\,\mu}$:

$$F_{k\,\nu}^{\,\mu} = q\varepsilon_{k\,\nu}^{\,\mu} = \begin{Bmatrix} 0 & 0 & 0 & F_{xk} \\ 0 & 0 & 0 & F_{yk} \\ 0 & 0 & 0 & F_{zk} \\ F_{xk} & F_{yk} & F_{zk} & 0 \end{Bmatrix}. \tag{23}$$

The kth piston has a 4-vector displacement $\mathrm{d}x_k^{\mu}$ and a 4-vector velocity v_k^{μ}:

$$\begin{gathered} \mathrm{d}x^{\mu} = \{\mathrm{d}x_k, \mathrm{d}y_k, \mathrm{d}z_k, c\mathrm{d}t\}, \\ v_k^{\mu} = \frac{\mathrm{d}x_k^{\mu}}{\mathrm{d}\tau_k} = \gamma(v_k)\left\{v_{xk}, v_{yk}, v_{zk}, c\right\}, \\ \frac{\mathrm{d}t}{\mathrm{d}\tau_k} = \gamma(v_k), \end{gathered} \tag{24}$$

where τ_k is the proper time of kth piston displacement.

For the kth piston, two 4-vectors can be obtained: (i) the 4-vector Minkowski force F_k^{μ} and (ii) the 4-vector work W_k^{μ}.

(1) The 4-vector Minkowski force F_k^{μ} is given by ([20], Chap. 33)

$$\begin{aligned} F_k^{\mu} &= c^{-1}\mathcal{F}_{k\,\nu}^{\,\mu} v_k^{\nu} \\ &= \gamma(v_k)\left\{F_{xk}, F_{yk}, F_{zk}, c^{-1}[\mathbf{F}_k \cdot \mathbf{v}_k]\right\}, \end{aligned} \tag{25}$$

with $\mathbf{F}_k \cdot \mathbf{v}_k = F_{xk}v_{xk} + F_{yk}v_{yk} + F_{zk}v_{zk}$.

(2) The 4-vector work δW_k^{μ} is given by

$$\begin{aligned} \delta W_k^{\mu} &= \mathcal{F}_{k\,\nu}^{\,\mu}\mathrm{d}x_k^{\nu} \\ &= \left\{cF_{xk}\mathrm{d}t, cF_{yk}\mathrm{d}t, cF_{zk}\mathrm{d}t, F_{xk}\mathrm{d}x_k \right. \\ &\qquad \left. + F_{yk}\mathrm{d}y_k + F_{zk}\mathrm{d}z_k\right\} \\ &= \left\{c\mathcal{I}_{xk}, c\mathcal{I}_{yk}, c\mathcal{I}_{zk}, W_k\right\}, \end{aligned} \tag{26}$$

a 4-vector with units of energy. The 4-vector F_k^{μ} can be obtained by deriving δW_k^{μ} in respect to proper time $\mathrm{d}\tau_k$ of the kth piston as

$$\frac{\delta W_k^{\mu}}{\mathrm{d}\tau_k} = \frac{\mathrm{d}t}{\mathrm{d}\tau_k}\frac{\delta W^{\mu}}{\mathrm{d}t} = cF_k^{\mu}. \tag{27}$$

This obtention of the 4-vector F_k^{μ} shows that δW_k^{μ} is a 4-vector itself (Appendix A). For a finite interval of time Δt, with constant force $\mathbf{F}_k$, and 4-vector interval $\Delta x_k^{\mu} = \{\Delta x_k, \Delta y_k, \Delta z_k, c\Delta t\}$, the 4-vector work W_k^{μ} is

$$\begin{aligned} W_k^{\mu} &= \mathcal{F}_{k\,\nu}^{\,\mu}\Delta x_k^{\nu} \\ &= \left\{cF_{xk}\Delta t, cF_{yk}\Delta t, cF_{zk}\Delta t, F_{xk}\Delta x_k + F_{yk}\Delta y_k + F_{zk}\Delta z_k\right\}. \end{aligned} \tag{28}$$

For the set of k forces simultaneously applied to the body at different pistons in frame S_0 during the finite interval of time Δt (Figure 4), the total 4-vector "force-displacement product" (work) W^{μ} is the sum of the 4-vector W_k^{μ}. The 4-vector total work W^{μ} is given by $W^{\mu} = \sum_k W_k^{\mu}$, with condition $\sum_k \mathbf{I}_k = 0$:

$$W^{\mu} = \{0, 0, 0, W\}; \quad W = \sum_k W_k. \tag{29}$$

In frame S_{A}, $W_{\mathrm{A}}^{\mu} = \mathcal{L}_{\nu}^{\mu}(V)W^{\nu} = \{c\mathcal{I}_{x\mathrm{A}}, 0, 0, W_{\mathrm{A}}\}$, with impulse $\mathcal{I}_{x\mathrm{A}}$ and "force-displacement product" W_{A} being

$$\begin{aligned} \mathcal{I}_{x\mathrm{A}} &= -\gamma(V)(Wc^{-2})V, \\ W_{\mathrm{A}} &= \gamma(V)W. \end{aligned} \tag{30}$$

Adiabatic First Law ([18], Section 4.2). A system that changes its energy function owing to forces applied to it, in an adiabatic process, is

$$\Delta U^{\mu} = W^{\mu}. \tag{31}$$

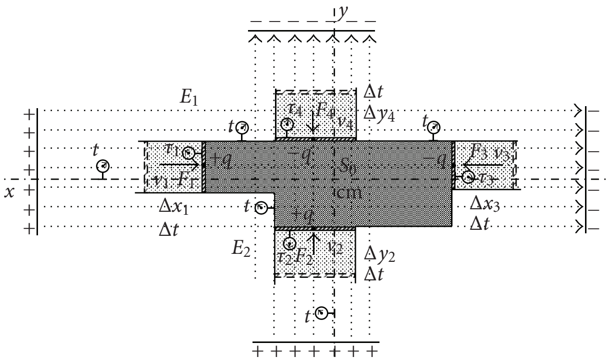

FIGURE 3: Extended system, with k pistons ($k = 1, 2, 3, 4$), to which forces $F_k = qE_k$ are simultaneously applied during time interval Δt as measured in frame S_0 by a set of synchronized clocks at rest (Figure 1). On the kth piston there is a clock that measures its proper time τ_k. The kth piston displaces $d\mathbf{r}_k$ during time interval dt, speed $\mathbf{v}_k = d\mathbf{r}_k/dt$, and proper time interval $d\tau_k$. Forces F_k have an electromagnetic origin: an electric field E_k, produced by a plane-parallel charged capacitor, interacts with a charge q_k fixed to the kth piston.

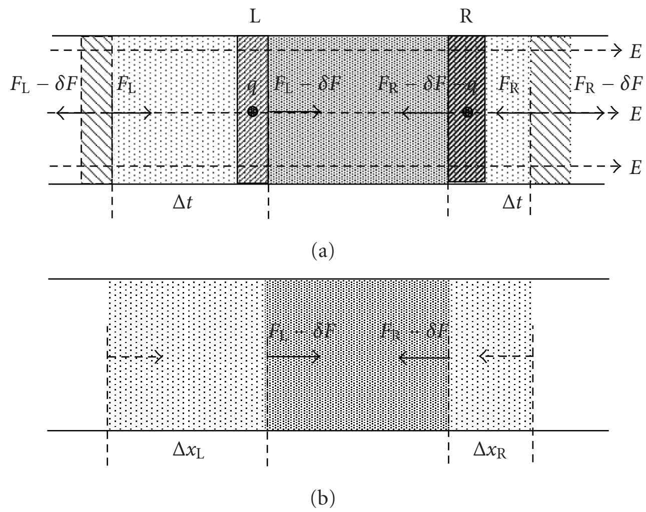

FIGURE 4: (a) A gas contained in a cylinder is compressed under the action of two pistons, L, with electric charge $+q$ fixed to it, and R, with electric charge $-q$ fixed to it. On piston L, a force $F_L = qE$ is exerted by the electric field E and a force $-(F_L - \delta F)$ slightly smaller by the gas. On piston R a force $F_R = -qE$ is exerted by the electric field, an a force $(F_R - \delta F)$ by the gas. (b) Thus the gas is compressed under the action of force F_L applied to a displacement Δx_L and a force F_R applied to a displacement Δx_R. Both forces F_L and F_R are applied simultaneously during time interval Δt. Every piston acts as an intermediate agent between the work reservoir (electric field and battery to which the capacitor is connected) and the thermodynamics system (the gas).

5. Four-Vector Heat

Work is described in thermodynamics as oriented (nonrandom) internal energy transferred between a body and a work reservoir (Figure 4). However, heat is described as random (or nondirected) internal energy transferred between two bodies at different temperatures [50]. Nondirected means "without linear momentum."

According to Rindler, in the special theory of relativity any transfer of energy, being equivalent to a transfer of inertia, necessarily involves momentum [51, p. 91]. This assessment is valid for all forms of radiation and must be valid for heat [52], whatever definition of heat is being used.

The most direct argument on relativistic heat transformations is provided by Arzeliés [1]. Based on the principle of equivalence work-heat, this author assumes that relativistic heat transforms as relativistic work.

The 4-vector heat, Q^μ, is obtained in two steps. First, we obtain the 4-vector for thermal radiation (its frequency distribution fulfills Planck's frequency distribution) enclosed in a cavity with walls at temperature T, and then the thermal radiation exchanged by a body as heat is described as thermal radiation in a cavity.

In a generalization of the asynchronous formulation, we assume that in frame S_0 (zero momentum frame) heat is emitted or absorbed with zero linear momentum ([3] p. 173). With the 4-vector Q^μ given in frame S_0 as $Q^\mu = \{0, 0, 0, Q\}$, in frame S_A, standard configuration, with $Q_A^\mu = \{cp_A, 0, 0, E_A\}$, and $Q_A^\mu = \mathcal{L}^\mu_\nu(V)Q^\nu$, a linear momentum associated to Q, must be ([4] p. 1746)

$$p_A = -\gamma(V)\frac{Q}{c^2}V. \tag{32}$$

The relativistic linear momentum of heat in frame S_A requires a physical interpretation—because of the contrast with no momentum for heat in classical thermodynamics [53]. In order to provide the relativistic interpretation of heat and the description of a thermal bath, we will describe thermal radiation as an ensemble of emitted photons enclosed in a cavity [54].

A cavity with walls at temperature T, measured with a gas thermometer at constant volume, and filled with photons that fit Planck's frequency distribution—that is, thermal photons—constitutes a thermal bath. In frame S_0 in which cavity walls are at rest, the total linear momentum of the photon ensemble is zero. In the monochromatic approximation [55] to Planck's distribution, every photon has the same frequency ν, with $\nu(T) = AT$ (Wien's Law), where A is a constant (Figure 5).

The 4-vector wave ω^μ for a photon ([56], pp. 255–257) of wavelength λ and angular frequency $\omega = 2\pi/\mathcal{T}$, period $\mathcal{T}$, that propagates in a direction given by the wave vector $\mathbf{k}$,

$$\mathbf{k} = \left(\frac{2\pi}{\lambda}\cos\,\theta, \frac{2\pi}{\lambda}\sin\,\theta, 0\right), \tag{33}$$

is ([20], pp. 269-270)

$$\omega^\mu = \left\{c\frac{2\pi}{\lambda}\cos\,\theta, c\frac{2\pi}{\lambda}\sin\,\theta, 0, \frac{2\pi}{\mathcal{T}}\right\}. \tag{34}$$

For a given rth photon, with frequency ν, $\lambda\nu = c$, and moving in direction $\mathbf{k} = (\cos\,\theta_r, \sin\,\theta_r, 0)$, there exists an energy 4-vector ($\hbar = h/2\pi$),

$$q_r^\mu = \hbar\omega_r^\mu = \left\{c\left[\frac{h\nu}{c}\right]\cos\,\theta_r, c\left[\frac{h\nu}{c}\right]\sin\,\theta_r, 0, h\nu\right\}. \tag{35}$$

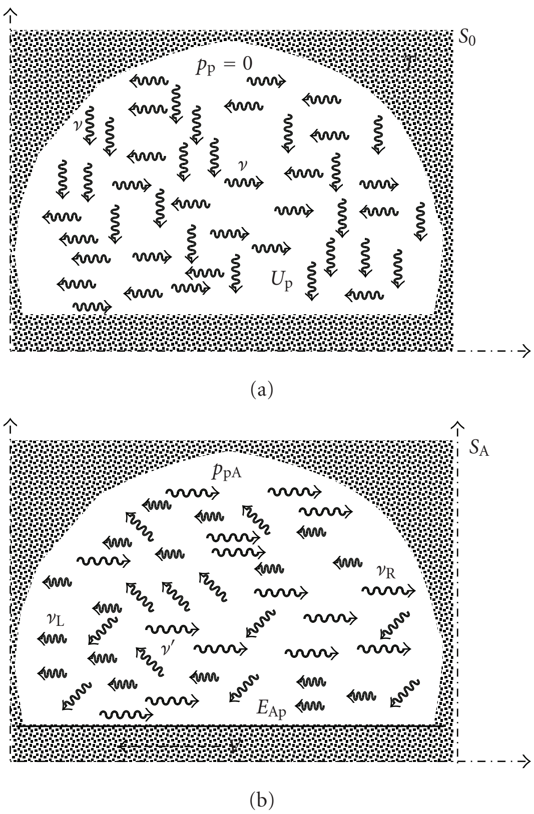

FIGURE 5: (a) Cavity with walls at temperature T filled with thermal radiation (photons) frequency $\nu = \nu(T)$ (monochromatic approximation). In frame S_0 linear momentum for this ensemble of photons is zero $p_p = 0$ and its energy function is $U_p = Nh\nu$. (b) In frame S_A, the same ensemble of photons, with frequencies $\nu_L > \nu$, $\nu_R < \nu$ (relativistic Doppler effect) and ν' (aberration effect), has linear momentum $p_{pA} = \gamma(V)(U_p c^{-1})V$, according to the principle of inertia of energy, and total energy $E_p = \gamma(V)U_p$.

The norm of this 4-vector is $\|q_r^\mu\| = 0$. An individual photon has no inertia.

In frame S_0, total linear momentum for the N photons inside the cavity at temperature T, p_p, and its total energy E_p are given by

$$
\begin{aligned}
p_{xp} &= \sum_r \frac{h\nu}{c} \cos\theta_r = 0, \\
p_{yp} &= \sum_r \frac{h\nu}{c} \sin\theta_r = 0, \\
E_p(T) &= \sum_r h\nu = Nh\nu(T).
\end{aligned}
\tag{36}
$$

In the zero-momentum frame S_0, total energy $E_p(T)$ is the energy function $U_p(T)$ of the system. The 4-vector thermal radiation $\mathcal{Q}^\mu$, $\mathcal{Q}^\mu = \sum_r q_r^\mu$ is

$$
\mathcal{Q}^\mu = \left\{0, 0, 0, U_p(T)\right\}, \qquad U_p(T) = Nh\nu(T). \tag{37}
$$

In frame S_A,

$$
\begin{gathered}
\mathcal{Q}_A^\mu = \mathcal{L}_\nu^\mu(V)\mathcal{Q}^\nu = \left\{-c\gamma(V)\mathcal{M}_p(T)V, 0, 0, \gamma(V)U_p(T)\right\}, \\
\mathcal{M}_p(T) = Nh\nu(T)c^{-2}.
\end{gathered}
\tag{38}
$$

The energy function $U_p(T)$ is the norm of the 4-vector $\|\mathcal{Q}_A^\mu\| = Nh\nu(T)$. This photons ensemble has nonzero inertia [48] $\mathcal{M}_p = U_p c^{-2}$.

Consider for a moment this cavity filled with thermal radiation containing one mole of atoms of a gas also. It is interesting to note that (i) photons of thermal radiation enclosed in a cavity, with Planck's frequency distribution, the atoms of the gas, with its (ii) electrons distributed in electronic orbitals following Boltzmann's energy distribution, and (iii) atoms moving with Maxwell's (or Juttner distribution [57]) kinetic energy distribution, every distribution with the same parameter temperature T, contribute to the energy function and to the inertia [35] of the system. As previously discussed, energy function for an ensemble of atoms and energy function for an ensemble of thermal photons transform between inertial frames in the same way. Thus, thermal equilibrium at temperature T between matter and radiation is a relativistic invariant and every inertial observer will agree on that equilibrium (Figure 6).

After the obtention of a 4-vector $\mathcal{Q}^\mu$ for the contribution to its energy function by thermal radiation inside a cavity, it is necessary to characterize as a 4-vector heat Q^μ the exchanged energy by a body as thermal photons.

First of all, systems thermally interacting with each other cannot be in equilibrium if they are in relative motion [58]. In the Asynchronous Formulation generalization to heat, there exists a privileged frame S_0 in which the system is at rest with respect to the thermal bath and in S_0 thermal radiation (photons) is absorbed or emitted with zero total linear momentum.

The energy absorbed, or emitted, by a body as thermal radiation (heat) throughout a process can be modeled as photons inside a cavity. A thermal system can absorb or emit photons through its frontier except in adiabatic processes. A photon emitted by a body, with frequency ν and direction $\mathbf{u}$, contributes with $-h\nu u/c$ to the linear momentum variation of the body and with $-h\nu$ to the total energy variation of the body that emits it. A photon absorbed by a body, with frequency ν and direction $\mathbf{u}$, contributes with $+h\nu\mathbf{u}/c$ to the linear momentum variation and with $+h\nu$ to total energy increment of the system.

Absorbed or emitted photons can be considered different phases in thermal equilibrium [59]. Thus, there is not "force-displacement product" (work) associated with emission or absorption of thermal radiation (photons).

With the system and thermal bath mutually at rest, the ensemble of emitted photons (when the system is at higher temperature than thermal reservoir) is described as an ensemble of thermal photons in a cavity with zero total linear momentum, and so the ensemble of absorbed photons (when the system is at lower temperature than thermal reservoir).

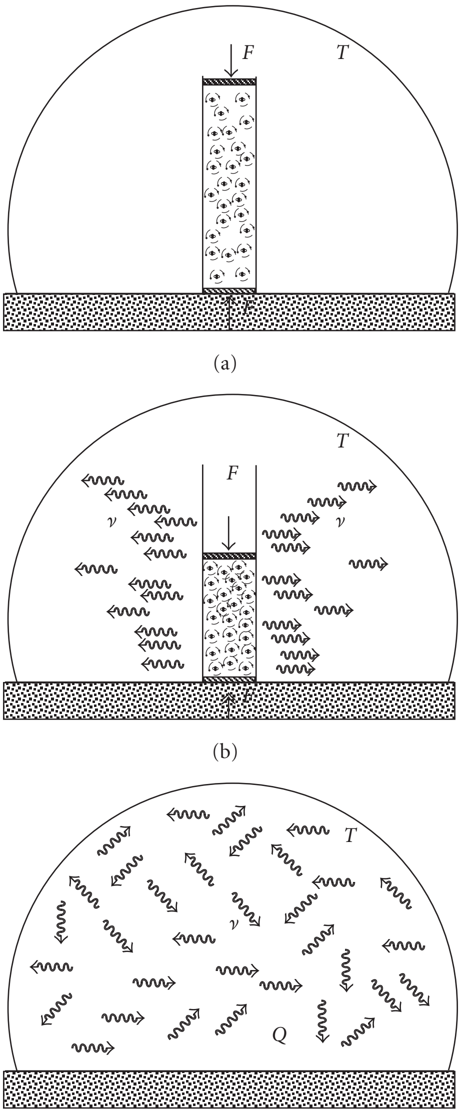

Figure 6: (a) A gas contained in a cylinder is compressed by a force F applied to a piston inside a cavity (thermal resevoir) at temperature T (isothermal process). The centre of mass of the gas remains at rest in its initial and final equilibrium states. (b) During the compression process photons with frequency ν (monochromatic approximation) are emitted with zero linear momentum. (c) Heat Q emitted during the compression is characterized as the energy associated with the thermal radiation made up of all photons emitted contained into the cavity (with minus sign).

In frame S_0, the 4-vector *thermal radiation* (*heat*) Q^μ associated when the body emits (−) or absorbs (+) N photons with frequency $\nu(T)$ is given by

$$Q^\mu = -\mathcal{Q}^\mu = \left\{0, 0, 0, \mp U_p(T)\right\}, \tag{39}$$

with $U_p(T) = \dot{N} h\nu \Delta t$, where $\dot{N} = \mathrm{d}N/\mathrm{d}t$ is the flux of photons (net number of photons exchanged in unit time) and $N = \dot{N}\Delta t$ is the net number of photons exchanged by the body during time interval Δt.

For an observer in frame S_{A}, $Q^\mu_{\mathrm{A}} = \{c p_{\mathrm{A}}, 0, 0, E_{\mathrm{A}}\}$, from $Q^\mu_{\mathrm{A}} = \mathcal{L}^\mu_\nu(V) Q^\nu$ with linear momentum $p_{p\mathrm{A}}$ and total energy $E_{p\mathrm{A}}$:

$$\begin{gathered} p_{pA} = -\gamma(V)\mathcal{M}_p(T)V\ ; \quad \mathcal{M}_p(T) = U_p(T)c^{-2}, \\ E_{pA} = \gamma(V)U_p(T) = \gamma(V)\mathcal{M}_p(T)c^2, \end{gathered} \tag{40}$$

Physical interpretation of linear momentum for heat in frame S_{A} will be obtained from relativistic Doppler and aberration effects applied to photons (see Section 7.2.3). The norm of Q^μ_A is

$$\left\|Q^\mu_{\mathrm{A}}\right\| = \left[E^2_{pA} - c^2 p^2_{pA}\right]^{1/2} = U_p(T) = \mathcal{M}_p(T)c^2, \tag{41}$$

with energy function $U_p(T)$ and inertia $\mathcal{M}_p(T)$ relativistic invariants [60].

If heat is defined as the total energy associated with the emitted (or absorbed) photons *as measured in the observer's frame*, then $Q_A = \gamma(V)Q$, with $Q = U_p(T)$. If heat is defined as the emitted (or absorbed) energy carried by photons in frame S_0 in which the interchange of photons with a thermal surrounding mutually at rest and zero linear momentum happens—as it is defined (implicitly) in classical thermodynamics—then Q is the norm of any 4-vector $Q^\mu = \|Q^\mu_A\|$ and it is a relativistic invariant. In any case, it is the 4-vector that possesses physical meaning, not its components.

Heat. For a system that changes its energy function without forces applied to it, by heating, or cooling, in diathermal contact with a thermal bath, system and bath at mutual rest, is

$$\Delta U^\mu = Q^\mu. \tag{42}$$

6. Relativistic Thermodynamics First Law

According to the generalized asynchronous formulation of relativistic thermodynamics, the description of a certain process on a composite, deformable system, and after the obtention of 4-vectors energy function U^μ, initial U^μ_i and final U^μ_f, work W^μ, and heat Q^μ, is as follows.

Relativistic Thermodynamics First Law. Mathematical: the relationship between variations in energy function of a system after a certain process, during which it interacts with a mechanical reservoir, with forces simultaneously applied to it during that process, and a thermal reservoir, system and reservoir mutually at rest, with thermal radiation interchanged by the system during the process, with every magnitude expressed as a 4-vector, is [4, 61]

$$\Delta U^\mu = U^\mu_f - U^\mu_i = W^\mu + Q^\mu. \tag{43}$$

No matter whether a system is self-contained or free (confined in a container), any energy, momentum, U^μ, W^μ or Q^μ, is always a 4-vector provided that one performs a covariant summation at constant time (simultaneously) in the frame S_0 in which the system is mutually at rest [2] (at least instantaneously) with its mechanical and thermal surroundings.

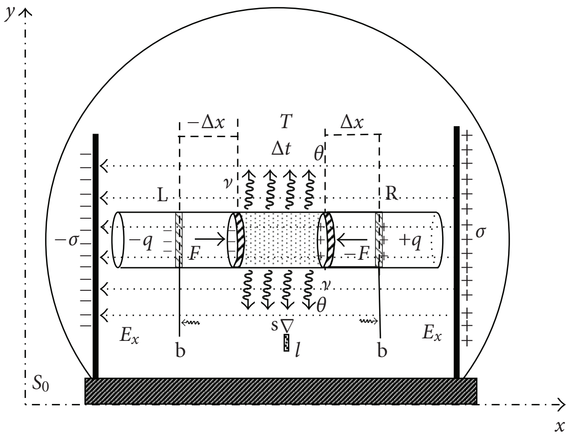

FIGURE 7: Cylinder with gas, diathermal walls, closed by two pistons, R and L, inside a plane-parallel capacitor with charge surface density σ. Laser (l), beam splitter (s), pistons of blocked mechanism (b) are used to assure that forces are applied simultaneously [62]. The gas in cylinder is compressed by forces acting on pistons. Force $F_{\mathrm{L}} = F$ acts on piston L during time interval Δt and displacement $\Delta x_{\mathrm{L}} = \Delta x$. Force $F_{\mathrm{R}} = -F$ acts on piston R during time interval Δt and displacement $\Delta x_{\mathrm{R}} = -\Delta x$. During the compression process, photons are emitted by cylinder walls with frequency ν and zero linear momentum in frame S_0.

In the frame reference S_A, the first law is expressed as

$$\Delta U_A^\mu = U_{fA}^\mu - U_{iA}^\mu = W_A^\mu + Q_A^\mu. \tag{44}$$

Every 4-vector in S_A can be obtained by a Lorentz transformation for the corresponding 4-vector in S_0

$$\mathcal{L}_\nu^\mu(V)\left[U_f^\nu - U_i^\nu = W^\nu + Q^\nu\right] \longrightarrow U_{f\mathrm{A}}^\mu - U_{i\mathrm{A}}^\mu = W_{\mathrm{A}}^\mu + Q_{\mathrm{A}}^\mu. \tag{45}$$

This circumstance guarantees the first law Lorentz covariance.

7. Ideal Gas Isothermal Compression

A horizontal cylinder (Figure 7), with thin metallic walls, section A, length L, containing 1 mol, N_A atoms, of ^{4}He gas, enclosed by two pistons, left (L) and right (R). We assume that helium behaves as an ideal gas, described by thermal equation of state $P\mathcal{V} = N_A k_B T$. The gas is in equilibrium under pressure P_i and at temperature T, volume $\mathcal{V}_i$, $\mathcal{V}_i = RT/P_i$. The limits of the system are the walls of the cylinder, considered diathermal. Pistons are considered adiabatic.

7.1. Compression in Frame S_0. As privileged frame S_0 we take the frame in which cylinder walls, plate parallel capacitor and thermal reservoir walls are at rest. During the compression process, forces on gas are applied simultaneously, during time interval Δt. Thermal radiation is interchanged with zero impulse in S_0 and the gas center of mass remains at the same point, with initial and final zero velocity.

7.1.1. Energy Function in S_0. For simplicity, we assume that the atoms of He inside the cylinder possess only translational energy, that is, all atoms are in its ground electronic state. In general, one can assume that gas velocity distribution is Juttner distribution [57] ([9], pp. 289–293). For simplicity, one assumes that atoms are randomly distributed inside the container and that every atom has the same translational energy, that is, every He atom moves with the same speed $v = v(T)$ (monokinetic approximation [50]), same modulus, but with different vectorial components $\mathbf{v} = (v_x, v_y, v_z)$, $v = |\mathbf{v}|$. In this approximation, $v(T) = aT^{1/2}$. Constant a is obtained by imposing

$$k = [\gamma(v) - 1]u = \frac{3}{2}k_B T, \tag{46}$$

where $k = [\gamma(v) - 1]u$ is the kinetic energy of a He atom and u its energy function (Section 3).

Linear momentum $\mathbf{p}_j = (p_{xj}, p_{yj}, 0)$ (for simplicity we assume $x - y$ as movement of the atoms) and total energy E_j for the jth atom are

$$\begin{gathered} p_{xj} = \gamma(v) m v_x, \\ p_{yj} = \gamma(v) m v_y, \\ e_j = \gamma(v) u, \quad m = uc^{-2}. \end{gathered} \tag{47}$$

Initial total linear momentum $\mathbf{p}_i = (p_{xi}, p_{yi}, 0)$ and initial total energy (energy function) U_i are given by

$$\begin{gathered} p_{\mathrm{x}i} = \sum_j p_{xj} = \gamma(v) m \sum_k v_{xj} = 0, \\ p_{yi} = \sum_j p_{yj} = \gamma(v) m \sum_k v_{yj} = 0, \\ U_i = U(T) = \gamma(v) \sum_j u = N_A \gamma(v) u. \end{gathered} \tag{48}$$

The 4-vector initial energy function U_i^μ is then

$$U_i^\mu = \{0, 0, 0, U_i\}; \qquad U_i = \gamma(v) N_A u. \tag{49}$$

Energy function U_i depends on temperature through temperature dependence on velocity $v = v(T)$. In S_0 ($\mathbf{p}_i = 0$), total system energy ([63], Sec. 8.3) is, by definition, its energy function $U_i = K_i + \mathcal{U}_i$, sum of the kinetic energy of helium atoms $K_i = \sum_k k_k = N_A[\gamma(v) - 1]u$, and $\mathcal{U}_i = \sum_k u = N_A u$.

For an ideal gas in an isothermal process, energy function remains constant, as well as the temperature, and also He atom speed. The 4-vector final energy function U_f^μ is then:

$$U_f^\mu = \left\{0, 0, 0, U_f\right\}, \tag{50}$$

with $U_f = U_i = U(T)$.

7.1.2. Work in S_0. Forces on pistons are described as produced by the interaction of an electric charge with an electromagnetic field (Figure 7). Static electric positive $+q$ and negative $-q$ charges are located on right and left pistons, respectively. The whole device, gas plus pistons, is located inside the homogeneous electric field E produced by a plane-parallel capacitor ([64], Chap. 13) with charge surface density σ.

In frame S_0 the capacitor is at rest. A horizontal electric field E_x is created inside the capacitor. For this (uniform) electric field the potential 4-vector $\Phi^\mu = \{A_x, A_y, A_z, \phi\}$, where $\mathbf{A} = (A_x, A_y, A_z)$ and ϕ are the vector potential and the scalar potential, respectively, is given by the contravariant 4-vector [65]:

$$\Phi^\mu = \{0, 0, 0, -E_x x\}, \quad E_x = \frac{\sigma}{2\epsilon_0}. \tag{51}$$

The electromagnetic 4×4-tensor $\mathcal{E}^{\nu\mu}$—double contravariant—is given by ([51], Section 42)

$$\mathcal{E}^{\nu\mu} = \frac{\partial \Phi^\nu}{\partial x^\mu} - \frac{\partial \Phi^\mu}{\partial x^\nu}. \tag{52}$$

For the horizontal plane-parallel capacitor the 4×4-tensor $\mathcal{E}^{\nu\mu}$ is

$$\mathcal{E}^{\nu\mu} = \begin{Bmatrix} 0 & 0 & 0 & E_x \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -E_x & 0 & 0 & 0 \end{Bmatrix}. \tag{53}$$

The mixed 4×4-tensor $\mathcal{E}^\mu_\nu$ is obtained as

$$\mathcal{E}^\mu_\nu = g_{\nu\xi} \mathcal{E}^{\xi\mu} = \begin{Bmatrix} 0 & 0 & 0 & -E_x \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -E_x & 0 & 0 & 0 \end{Bmatrix}. \tag{54}$$

Initially, pistons are locked by a blocked mechanism (b in Figure 7). A laser (l in Figure 7) and a beam splitter (s in Figure 7) that is located just between the pistons, are used to release simultaneously both pistons. When the laser is turned on the split beams will arrive at the blocked mechanism of both left and right pistons and, at time $t = 0$, are unlocked allowing electric field charges interaction simultaneously on both L and R pistons [62].

When pistons are released, the Minkowski force F^μ_k on kth piston is

$$F^\mu_k = q \mathcal{E}^\mu_\nu v^\mu_k, \quad v^\mu_k = \frac{\mathrm{d}x^\nu_k}{\mathrm{d}\tau_k}, \tag{55}$$

where q is the electric charge on piston, $\mathcal{E}^\mu_\nu$ is the 4×4-tensor electromagnetic field, and v^μ_k and τ_k are the kth piston 4-vector velocity and proper time, respectively [66].

For the left piston (subindex L), displacement $\Delta x_{\mathrm{L}} = \Delta x$ and velocity $v_{\mathrm{L}} = \Delta x_{\mathrm{L}}/\Delta t = v$, 4-vector $\Delta x^\mu_{\mathrm{L}}$ and 4-vector velocity v^μ_{L}, are respectively

$$\Delta x^\mu_{\mathrm{L}} = \{\Delta x, 0, 0, c\Delta t\}, \quad v^\mu_{\mathrm{L}} = \gamma(v)\{v, 0, 0, c\}. \tag{56}$$

For the right piston (R), displacement $\Delta x_{\mathrm{R}} = -\Delta x$ and velocity $v_{\mathrm{R}} = \Delta x_{\mathrm{R}}/\Delta t = -v$, 4-vector $\Delta x^\mu_{\mathrm{R}}$ and 4-vector velocity v^μ_{R}, are respectively

$$\Delta x^\mu_{\mathrm{R}} = \{-\Delta x, 0, 0, c\Delta t\}, \qquad v^\mu_{\mathrm{R}} = \gamma(v)\{-v, 0, 0, c\}. \tag{57}$$

The Minkowski forces due to electromagnetic interaction are:

$$\begin{aligned} F^\mu_{\mathrm{L}} &= -q \mathcal{E}^\mu_\nu v^\nu_{\mathrm{L}} = \gamma(v)\{+cqE_x, 0, 0, qE_x v\}, \\ F^\mu_{\mathrm{R}} &= +q \mathcal{E}^\mu_\nu v^\nu_{\mathrm{R}} = \gamma(v)\{-cqE_x, 0, 0, qE_x v\}. \end{aligned} \tag{58}$$

With forces acting on pistons, $F_{\mathrm{L}} = (qE_x, 0, 0)$ and $F_{\mathrm{R}} = (-qE_x, 0, 0)$, 4-vectors work are, respectively,

$$\begin{aligned} W^\mu_{\mathrm{L}} &= -q \mathcal{E}^\mu_\nu \Delta x^\nu_{\mathrm{L}} = \{+cqE_x \Delta t, 0, 0, qE_x \Delta x\}, \\ W^\mu_{\mathrm{R}} &= +q \mathcal{E}^\mu_\nu \Delta x^\nu_{\mathrm{R}} = \{-cqE_x \Delta t, 0, 0, qE_x \Delta x\}. \end{aligned} \tag{59}$$

The total 4-vector work W^μ is then

$$W^\mu = W^\mu_{\mathrm{L}} + W^\mu_{\mathrm{R}} = \{0, 0, 0, 2qE_x \Delta x\}. \tag{60}$$

7.1.3. Heat in S_0. During (slow) gas compression, $N = \dot{N}\Delta t$ photons with frequency $\nu(T)$ are emitted, with zero total linear momentum. Photons are emitted through the horizontal walls of the cylinder, $N/2$ photons are emitted in direction $\theta_+ = \pi/2$ and $N/2$ photons in direction $\theta_- = -\pi/2$. Total linear momentum for this ensemble of photons $\mathbf{p}_p = (p_{xp}, p_{yp}, 0)$ is

$$\begin{aligned} p_{xp} &= 0, \\ p_{yp} &= \frac{N}{2} h\nu \sin\ \theta_+ + \frac{N}{2} h\nu \sin\ \theta_- = 0. \end{aligned} \tag{61}$$

Total energy of these emitted photons $E = Nh\nu$, with $\mathbf{p}_p = 0$, is its energy function U_p:

$$U_p = Nh\nu. \tag{62}$$

According to the principle of inertia of energy, these N photons have inertia [63] $\mathcal{M}_p = U_p c^{-2}$. The 4-vector for heat Q^μ (thermal radiation emitted from the point of view of the system) is then given by

$$Q^\mu = \{0, 0, 0, -\dot{N}h\nu\Delta t\}. \tag{63}$$

7.1.4. First Law in S_0. From first law $U^\mu_f - U^\mu_i = W^\mu + Q^\mu$, one obtains $0 = 2qE_x\Delta x - \dot{N}h\nu\Delta t$ or

$$\dot{N}h\nu\Delta t = 2qE_x \Delta x, \qquad \dot{N}h\nu = 2qE_x v. \tag{64}$$

The configurational work done on the gas provides the energy emitted as heat.

7.2. Compression in Frame S_A. An observer in frame S_{A} obtains the corresponding 4-vector $U^\mu_{i\mathrm{A}}$, $U^\mu_{f\mathrm{A}}$, W^μ_{A}, and Q^μ_{A} by measuring different magnitudes—displacements Δx_{A}, time intervals Δt_{A}, velocities v_{A}, forces F_{A}, photon frequency ν_{A}, and so forth—in its own frame (Figure 8). With first law in frame S_{A} expressed as $U^\mu_{f\mathrm{A}} - U^\mu_{i\mathrm{A}} = W^\mu_{\mathrm{A}} + Q^\mu_{\mathrm{A}}$, the Lorentz invariance of this equation assures that the same result as in S_0, that is, $\dot{N}h\nu\Delta t = 2qE\Delta x$, is obtained.

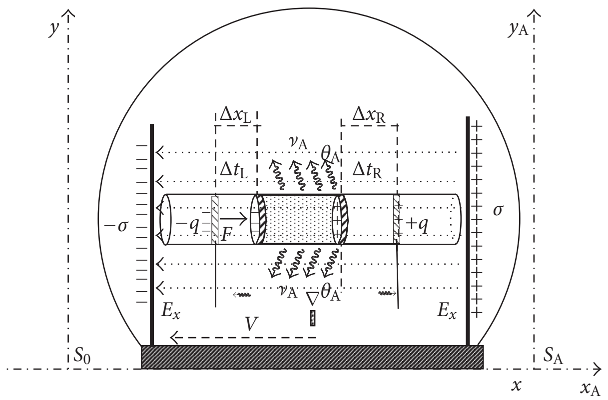

FIGURE 8: Isothermal compression given in Figure 7, described from the point of view of an observer in frame S_A, standard configuration, with velocity V with respect to frame S_0. In frame S_A forces are not applied simultaneously and heat carries linear momentum.

7.2.1. Energy Function in S_A. Let there be one mol of He atoms moving inside the cylinder. In frame S_0, with zero total linear momentum, velocities of atoms are measured *simultaneously*. During the same interval of time Δt, displacement $\Delta \mathbf{r}_j = (\Delta x_j, \Delta y_j, 0)$ for the jth atom is measured and its 3-vector velocity is given by $\mathbf{v}_k = (\Delta \mathbf{r}_j/\Delta t) = (v_x, v_y, 0)$. In order to ensure that in frame S_0 total linear momentum is zero, for every atom j moving with velocity $\mathbf{v}_j = (v_{xj}, v_{yj}, 0)$ there must exist another atom n moving with opposite velocity $\mathbf{v}_n = (v_{xn}, v_{yn}, 0)$, such that $v_{xj} = -v_{xn}$ and $v_{yj} = -v_{yn}$.

The velocity $\mathbf{v}_A = (v_{xsA}, v_{ysA}, 0)$ of the sth atom is given, in terms of its velocity $\mathbf{v} = (v_{xs}, v_{ys}, 0)$ measured in S_0 and velocity $\mathbf{V} = (V, 0, 0)$ of frame S_A with respect to frame S_0, by the equation ([51], Section 12)

$$\mathbf{v}_{sA} = \frac{1}{\gamma(V)[1 - v_{xs}V/c^2\ ]}\left(\gamma(V)[v_{xs} - V], v_{ys}, 0\right), \tag{65}$$

with the useful relations ([67], p. 69)

$$\begin{aligned} \gamma(v_{sA}) &= \gamma(v)\gamma(V)\left[1 - \frac{v_{xs}V}{c^2}\right], \\ \gamma(v_{sA})v_{xsA} &= \gamma(v)\gamma(V)(v_{xs} - V), \\ \gamma(v_{sA})v_{ysA} &= \gamma(v)v_{ys}. \end{aligned} \tag{66}$$

For every pair $j - n$ of opposite atoms, total momentum and total energy are easily obtained using the previous transformations:

$$\begin{aligned} p_{x(j+n)A} &= \gamma\left(v_{jA}\right)(uc^{-2})v_{xjA} + \gamma(v_{rA})(uc^{-2})v_{xnA} \\ &= \gamma(V)[2\gamma(v)uc^{-2}]V, \end{aligned} \tag{67}$$

$$p_{y(j+n)A} = \gamma\left(v_{jA}\right)(uc^{-2})v_{yjA} + \gamma(v_{rA})(uc^{-2})v_{ynA} = 0,$$

and total energy is given by

$$\begin{aligned} E_{jA} &= \gamma\left(v_{jA}\right)u = \gamma(v)\gamma(V)\left[1 - \frac{v_xV}{c^2}\right]u, \\ E_{nA} &= \gamma(v_{nA})u = \gamma(v)\gamma(V)\left[1 + \frac{v_xV}{c^2}\right]u, \\ E_{(j+n)A} &= E_{jA} + E_{nA} = \gamma(V)[2\gamma(v)u]\ . \end{aligned} \tag{68}$$

For the $N/2$ total pairs of opposite atoms, one has

$$\begin{aligned} U^{\mu}_{iA} &= \left\{-c\gamma(V)\left[\frac{\gamma(v)N_Au}{c^2}\right]V, 0, 0, \gamma(V)[\gamma(v)N_Au]\right\} \\ &= \{-c\gamma(V)\mathcal{M}V, 0, 0, \gamma(V)U(T)\}, \quad \mathcal{M} = U(T)c^{-2} \end{aligned} \tag{69}$$

This is the same result obtained from Lorentz transformation [68] on the 4-vector energy function in S_0, given in (49), $U^{\mu}_{iA} = \mathcal{L}^{\mu}_{\nu}U^{\nu}_i$.

A similar description for 4-vector final energy function U^{μ}_{fA}, with $U^{\mu}_{fA} = \mathcal{L}^{\mu}_{\nu}U^{\nu}_f$, is

$$U^{\mu}_{fA}\{-c\gamma(V)\mathcal{M}V, 0, 0, \gamma(V)U(T)\}. \tag{70}$$

7.2.2. Work in S_A. By considering the locked-unlocked piston set, laser beam, splitter, blocked mechanism described previously in frame S_0, it is evident that forces that are simultaneously applied in S_0 are not simultaneous in frame S_A (Figure 8) [1].

To obtain the 4-vector W^{μ}_A in S_A, relativistic transformations of time intervals, spatial displacements, and forces must be used.

In S_A, 4-vector displacements are

$$\begin{aligned} \Delta x^{\mu}_{RA} &= \mathcal{L}^{\mu}_{\nu}(V)\Delta x^{\mu}_R \\ &= \left\{\gamma(V)[-\Delta x - V\Delta t], 0, 0, c\gamma(V)\left[\Delta t + \frac{V}{c^2}\Delta x\right]\right\}, \\ \Delta x^{\mu}_{LA} &= \mathcal{L}^{\mu}_{\nu}(V)\Delta x^{\mu}_L \\ &= \left\{\gamma(V)[+\Delta x - V\Delta t], 0, 0, c\gamma(V)\left[\Delta t - \frac{V}{c^2}\Delta x\right]\right\}. \end{aligned} \tag{71}$$

Spatial displacements Δx_{LA} and Δx_{RA} associated with forces F_{LA} and F_{RA}, respectively, are different in S_A as well as time intervals: $\Delta t_{LA} \neq \Delta t_{RA}$ [1].

It is assumed that 4-vector forces acting on extended bodies are transformed in the same way as 4-vector forces acting on point particles [69, 70]. Force $\mathbf{F}_A = (F_{xA}, F_{yA}, F_{zA})$ measured with respect to S_A is given, in terms of force $\mathbf{F} = (F_x, F_y, F_z)$ measured with respect to S_0 and the velocity $\mathbf{V} = (V, 0, 0)$ of frame S_A with respect to frame S_0, by [71]

$$\begin{aligned} F_{xA} &= \frac{F_x - (V/c^2)\left(F_xv_x + F_yv_y\right)}{1 - v_xV/c^2}, \\ F_{yA} &= \frac{\gamma^{-1}(V)F_y}{1 - v_xV/c^2}. \end{aligned} \tag{72}$$

For horizontal forces ($F_y = F_z = 0$) and $F_x = \pm F$,

$$F_{\mathrm{LA}} = qE_x, \qquad F_{\mathrm{RA}} = -qE_x\ . \tag{73}$$

Total impulse $\mathcal{I}_{\mathrm{A}}$ and work W_{A} in frame S_{A} are given by

$$\begin{aligned}\mathcal{I}_{\mathrm{A}} &= F_{\mathrm{LA}}\Delta t_{\mathrm{LA}} + F_{\mathrm{RA}}\Delta t_{\mathrm{RA}} = -\gamma(V)\left[2qE_x\Delta x c^{-2}\right]V\ ,\\ W_{\mathrm{A}} &= F_{\mathrm{LA}}\Delta x_{\mathrm{LA}} + F_{\mathrm{RA}}\Delta x_{\mathrm{RA}} = \gamma(V)\left[2qE_x\Delta x\right],\end{aligned} \tag{74}$$

with

$$W^{\mu}_{\mathrm{A}} = \{c\mathcal{I}_{\mathrm{A}}, 0, 0, W_{\mathrm{A}}\}. \tag{75}$$

This is the same result for W^{μ}_{A} obtained by using Lorentz transformation on 4-vector work in S_0 given by (60), $W^{\mu}_{\mathrm{A}} = \mathcal{L}^{\mu}_{\nu}(V)W^{\nu}_{\mathrm{A}}$.

The same result is obtained if one considers relativistic transformation of electric and magnetic fields ([72], Section 10.5).

The 4×4-tensor electromagnetic field in S_{A}, ${\mathcal{E}_{\mathrm{A}}}^{\mu}_{\nu}$, can be obtained as (Appendix A) ([20], p. 281)

$${\mathcal{E}_{\mathrm{A}}}^{\mu}_{\nu} = \mathcal{L}^{\mu}_{\nu}(V){\mathcal{E}_{\mathrm{A}}}^{\mu}_{\nu}{\mathcal{L}^{+}}^{\mu}_{\nu}(V) = \begin{Bmatrix} 0 & 0 & 0 & -E_x \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -E_x & 0 & 0 & 0 \end{Bmatrix}. \tag{76}$$

The electromagnetic field does not change in frame S_{A} with respect to the field in S_0 (no magnetic field appears in S_{A}). The 4-vector work W^{μ}_{LA} and W^{μ}_{RA} can by obtained by applying its definition in S_{A}:

$$\begin{aligned}W^{\mu}_{\mathrm{LA}} &= q{\mathcal{E}_{\mathrm{A}}}^{\mu}_{\nu}\Delta x^{\nu}_{\mathrm{LA}} = \left\{cqE_x\gamma(V)\left[\Delta t - \left(\frac{V}{c^2}\right)\Delta x\right], 0, 0, q\gamma(V)\right.\\ &\qquad \left.\times E_x\left[\Delta x - V\Delta t\right]\right\},\\ W^{\mu}_{\mathrm{RA}} &= -q{\mathcal{E}_{\mathrm{A}}}^{\mu}_{\nu}\Delta x^{\nu}_{\mathrm{RA}} = \left\{-cqE_x\left[\Delta t + \left(\frac{V}{c^2}\right)\Delta x\right], 0, 0, -q\right.\\ &\qquad \left.\times E_x\left[-\Delta x - V\Delta t\right]\right\}.\end{aligned} \tag{77}$$

In S_{A} total impulse,

$$\mathcal{I}_{\mathrm{A}} = -c\gamma(V)\left[\frac{2qE\Delta x}{c^2}\right]V, \tag{78}$$

is not zero and total work is

$$W_{\mathrm{A}} = \gamma(V)\left[2qE\Delta x\right]. \tag{79}$$

The 4-vector work W^{μ}_{A} in S_{A} is then

$$\begin{aligned}W^{\mu}_{\mathrm{A}} &= W^{\mu}_{\mathrm{LA}} + W^{\mu}_{\mathrm{RA}}\\ &= \left\{-c\gamma(V)\left[\frac{2qE\Delta x}{c^2}\right]V, 0, 0, \gamma(V)\left[2qE\Delta x\right]\right\},\end{aligned} \tag{80}$$

a result previously obtained.

7.2.3. Heat in S_A. From relativistic Doppler effect, frequency ν_{A} in frame S_{A} for a photon emitted in frame S_0 with frequency ν and direction $(\cos\theta, \sin\theta, 0)$ is given by

$$\nu_{\mathrm{A}} = \gamma(V)\left[1 - \beta(V)\cos\theta\right]\nu. \tag{81}$$

The relativistic aberration effect [73] indicates that photon direction $(\cos\ \theta_{\mathrm{A}}, \sin\ \theta_{\mathrm{A}}, 0)$ as measured in S_{A} is

$$\begin{aligned}\cos\theta_{\mathrm{A}} &= \frac{\cos\theta - \beta(V)}{1 - \beta(V)\cos\theta},\\ \sin\theta_{\mathrm{A}} &= \frac{\gamma^{-1}(V)\ \sin\theta}{1 -\ \beta(V)\cos\theta}.\end{aligned} \tag{82}$$

In frame S_0, for a photon jth emitted with angle $\theta_j = +\pi/2$ there is another photon nth emitted with angle $\theta_n = -\pi/2$ (both with frequency ν, in order to assure zero total linear momentum for emitted photons). In frame S_{A} photons are emitted with frequency $\nu_{\mathrm{A}} = \gamma(V)\nu$, higher than frequency measured in S_0 (transverse Doppler effect). Photons in S_{A} are emitted with angles larger than $\pi/2$ (in absolute value) (Figure 8).

In frame S_{A}, total linear momentum and total energy are easy to obtain for this pair of opposite emitted photons (in S_0) and for the $N/2$ pairs of emitted photons pairs, total linear momentum and energy are

$$\begin{aligned}p_{xp\mathrm{A}} &= -c\gamma(V)\left[Nh\nu c^{-2}\right]V,\\ p_{yp\mathrm{A}} &= 0,\\ E_{p\mathrm{A}} &= \gamma(V)Nh\nu = \gamma(V)\dot{N}h\nu\Delta t.\end{aligned} \tag{83}$$

For the ensemble of emitted photons the 4-vector is

$$Q^{\mu}_{p} = \left\{-c\gamma(V)\mathcal{M}_p V, 0, 0, \gamma(V)Nh\nu\right\}, \quad \mathcal{M}_p = Nh\nu c^{-2}. \tag{84}$$

In frame S_{A}, $N/2$ are emitted with angles $\pm\theta_{\mathrm{A}}$, with zero linear momentum in y direction. These photons carry linear momentum in direction $-x$ and its total linear momentum is $p_{x\mathrm{A}} = -\gamma(V)(h\nu/c^2)V$.

The 4-vector heat emitted by the body $Q^{\mu}_{\mathrm{A}} = -Q^{\mu}_{p}$ is

$$Q^{\mu}_{\mathrm{A}} = \left\{c\left[\gamma(V)\frac{Nh\nu}{c^2}\right]V, 0, 0, -\gamma(V)Nh\nu\right\}. \tag{85}$$

This result was obtained from Lorentz transformation on the 4-vector heat in S_0, given by (63), $Q^{\mu}_{\mathrm{A}} = \mathcal{L}^{\mu}_{\nu}(V)Q^{\nu}$. An ensemble of photons with energy $\dot{N}h\nu\Delta t$ and zero linear momentum, has an inertia $Nh\nu c^{-2}$ associated. The ensemble of N thermal photons does not transform between frames like a photon but like an elementary particle.

Total energy for photons $E_{p\mathrm{A}}$ as measured in S_{A} and total energy for photons U_p in S_0 are related as

$$E_{p\mathrm{A}} = \gamma(V)U_p. \tag{86}$$

The norm $\|Q^{\mu}_{\mathrm{A}}\|$ of the 4-vector heat Q^{μ}_{A} is an invariant, with

$$\left\|Q^{\mu}_{\mathrm{A}}\right\| = U_p = \dot{N}h\nu\Delta t. \tag{87}$$

7.2.4. First Law in S_A. From the first law in S_A, $U^{\mu}_{fA} - U^{\mu}_{iA} = W^{\mu}_{A} + Q^{\mu}_{A}$ and the 4-vectors given in (69),(80),(85), one obtains

$$0 = -c\gamma(V)\left[\frac{2qE\Delta x}{c^2}\right]V + c\gamma(V)\left[\frac{Nh\nu}{c^2}\right]V, \qquad 0 = \gamma(V)[2qE\Delta x] - \gamma(V)[Nh\nu]. \tag{88}$$

These two equations are redundant, and one obtains

$$Nh\nu = 2qE\Delta x. \tag{89}$$

This is a result previously obtained in frame S_0.

In frame S_A, total energy E_A, linear momentum p_A, and energy function U remain constant during the compression process—these magnitudes remain constant in frame S_0 too. In frame S_A the set of forces applied to the gas produces a net impulse —in contrast with the net zero impulse in frame S_0—and the ensemble of photons emitted during the process carries linear momentum—in contrast with the net zero momentum for emitted photons in S_0. Impulse and work provided by external forces on the gas, represented by the 4-vector W^{μ}_{A}, are transmitted to the ensemble of emitted photons, represented by Q^{μ}_{A}. When photons are emitted, the gas gets a (positive) linear momentum due to this emission of thermal radiation that compensates for the (negative) linear momentum provided by forces applied on it, with a result of total zero linear momentum variation. Similarly, energy carried for photons is provided by the work done by the forces. This transformation, $Q^{\mu}_{A} = -W^{\mu}_{A}$, is the relativistic generalization for the complete transformation of work W into heat Q for an isothermal process on an ideal gas, $U = U(T)$, with $\Delta U = 0$. The description of this process in frame S_0 is the usual description in classical thermodynamics, with energy associated with heat but with no linear momentum.

8. Conclusions

A coherent development of modern relativistic thermodynamics requires (i) a guarantee that the system behaves according to the principle of the inertia of energy, that is, forces are applied in such a way that the system behaves as a whole [29] and (ii) that the experiment in frame S_0 is performed in such a way that equations for elementary (point) particles can be applied to the extended thermodynamic system (principle of similitude). When this goal is achieved, in the asynchronous formulation formalism, the Minkowski 4-vector calculus in special relativity can be used for nonlocal (extended bodies) as well as for local (elementary particles) 4-vector quantities.

For the description of the process performed by body Z (Section 1) using the special theory of Relativity with the Minkowski 4-vector formalism, (1)-(2) merge in the first law of relativistic thermodynamics:

$$dU^{\mu} = \delta W^{\mu} + \delta Q^{\mu}. \tag{90}$$

According to the asynchronous formulation, in frame S_0 body Z is instantaneously at rest ([74], p. 41), with $v_i = 0$, and $v_f = v$, $\mathbf{v} = (v_x, v_y, v_z)$, $v = |\mathbf{v}|$,—with equations

$$\begin{Bmatrix} \gamma(v)\mathcal{M}_f v_x \\ \gamma(v)\mathcal{M}_f v_y \\ \gamma(v)\mathcal{M}_f v_z \\ \gamma(v)U_f - U_i \end{Bmatrix} = \begin{Bmatrix} \sum_k F_{xk}dt \\ \sum_k F_{yk}dt \\ \sum_k F_{zk}dt \\ \delta W \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \delta Q \end{Bmatrix}, \tag{91}$$

with $\mathbf{F}_k$ conservative forces simultaneously applied to Z and with

$$\delta W = \sum_k (\mathbf{F}_k \cdot \mathbf{v}_k)dt, \qquad \delta Q = \dot{N}h\nu dt, \qquad \mathcal{M}_i = U_i c^{-2}, \qquad \mathcal{M}_f = U_f c^{-2}. \tag{92}$$

where $\dot{N}$ is the flux of photons between Z and its thermal reservoir. Thus, the coherence of the developed formalism is based on the principle of the inertia of energy, with the Lorentz transformation, that guarantees that any kind of energy, U, W, or Q, that contributes to the temporal (energy) component of a 4-vector in frame S_0, contributes with the inertia $\mathcal{M} = Uc^{-2}$ to the spatial (linear momentum) component of the body in frame S_A.

If every force F_k acting on Z has its origin in the interaction of an electric charge q with an electromagnetic field, with a 4 × 4-tensor $\mathcal{E}_{k\nu}^{\mu}$, the 4-vector Minkowski force, F^{μ}_{k}, is given by

$$F^{\mu}_{k} = \frac{q}{c}\mathcal{E}_{k\nu}^{\mu} v^{\mu}_{k} = \gamma(v_k)\left\{F_{xk}, F_{yk}, F_{zk}, c^{-1}\mathbf{F}_k \cdot \mathbf{v}_k\right\}, \tag{93}$$

and the corresponding 4-vector work (infinitesimal), δW^{μ}_{k}, is

$$\delta W^{\mu}_{k} = q\mathcal{E}_{k\nu}^{\mu} dx^{\mu}_{k} = \left\{cF_{xk}dt, cF_{yk}dt, cF_{zk}dt, \mathbf{F}_k \cdot dx_k\right\}, \tag{94}$$

with

$$cF^{\mu}_{k} = \frac{\delta W^{\mu}_{k}}{d\tau_k} \tag{95}$$

and with

$$\delta W^{\mu} = \sum_k \delta W^{\mu}_{k} = c\sum_k F^{\mu}_{k} d\tau_k, \qquad d\tau_k = \gamma^{-1}(v_k)dt. \tag{96}$$

In conclusion, the formulation of the first law of relativistic thermodynamics using Minkowski 4-vector formalism, introducing 4-vector U^{μ} and 4-vector Q^{μ}, and considering an electromagnetic origin for the 4-vector work W^{μ}, allows us to solve exercises in classical physics, including concepts of mechanics, thermodynamics, and electromagnetism, in a complete Lorentz covariant formalism.

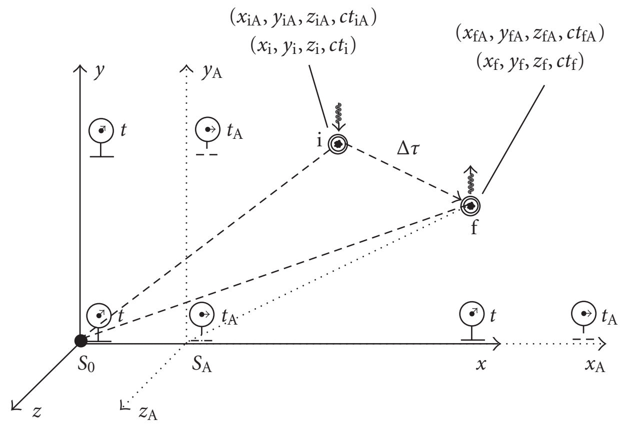

Figure 9: Frames S_0 and S_A in "standard configuration." Axes y and z in both frames are parallel, and frame S_A moves with velocity V along axis x of frame S_0. At time $t = t_A = 0$ origins coincide. Every frame has its own set of synchronized clocks. (i) Initial event, atom photon absorption, (f) final event, atom photon emission. $\Delta\tau$ is the proper time of the atom (measured by a clock that travels with it) between events i and f.

Appendices

A. Minkowski 4-Vector Formalism and the Lorentz Transformation

The special theory of relativity is characterized by the group of Lorentz transformations that describe the way in which two different observers relate their experimental observations to the same process on the same physical system. A quantity is therefore physically meaningful—it is of the same nature to all observers—if it behaves as a 4-vector under Lorentz transformation [19]. This can be cited as the *Minkowski hypothesis*.

Two rigid reference frames S_0 and S_A, with identical units of length and time, are given to be in standard configuration [67] when the S_A origin moves with velocity $\mathbf{V} = (V, 0, 0)$ along the x-axis of S_0, the x_A-axis coincides with the x-axis, while the y- and y_A-axes remain parallel, as do the z- and z_A-axes (parallel movement) and all clocks are set to zero when origins meet (Figure 9).

(1) It is important to note that an "observer" is a huge, extended, information-gathering system. An inertial observer is a coordinate system for spacetime, which makes an observation by recording the location (x, y, z) and time (t) of any event. An "observation" made by the inertial observer is the act of assigning to any event the coordinates x, y, z of the location of its occurrence and the time t read by the clock at (x, y, z) when the event occurred ([74], pp. 3-4) (Figure 9).

(2) An event is described by a contravariant (Greek index, column), and 4-vector x^μ and x^μ_A in observers in S_0 and in S_A, respectively (x, y, z, t) in S_0 and (x_A, y_A, z_A, t_A) in S_A, are expressed as contravariant 4-vectors [23]

$$x^\mu = \begin{Bmatrix} x \\ y \\ z \\ ct \end{Bmatrix}, \qquad x^\mu_A = \begin{Bmatrix} x_A \\ y_A \\ z_A \\ ct_A \end{Bmatrix}. \tag{A.1}$$

(For the sake of typographic simplicity, a contravariant 4-vector will be written as row 4-vector, but maintaining its contravariant index.)

(3) The Lorentz transformation for standard configuration, with constant velocity V, is given by ([65], Chap. 4)

$$\mathcal{L}^\mu_\nu(V) = \begin{Bmatrix} \gamma(V) & 0 & 0 & -\beta(V)\gamma(V) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta(V)\gamma(V) & 0 & 0 & \gamma(V) \end{Bmatrix}, \tag{A.2}$$

where $\beta(V) = V/c$ and $\gamma(V) = [1 - \beta^2(V)]^{-1/2}$ (Lorentz factor).

(4) The inverse Lorentz transformation, $\mathcal{L}^{+\nu}_{\ \mu}(V) = \mathcal{L}^\nu_\mu(-V)$, with $\mathcal{L}^{+\nu}_{\ \mu}(V)\mathcal{L}^\nu_\mu(V) = 1^\mu$, which transforms an S_A 4-vector into a S_0 4-vector, is given by ([20], p. 280)

$$\mathcal{L}^{+\nu}_{\ \mu}(V) = \begin{Bmatrix} \gamma(V) & 0 & 0 & \beta(V)\gamma(V) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta(V)\gamma(V) & 0 & 0 & \gamma(V) \end{Bmatrix}. \tag{A.3}$$

(5) The 4-vectors relative to the same event are related as [75] (the Minkowski hypothesis)

$$x^\mu = \mathcal{L}^\mu_\nu(V) x^\nu_A\ , \tag{A.4}$$

(6) The raising and lowering of suffixes of 4×4-tensors is effected by means of the metric tensor $g_{\nu\mu}$ [51]. When the invariant interval between two events, initial (x_i, y_i, z_i, ct_i) and final (x_f, y_f, z_f, ct_f), with (infinitesimal) displacement 4-vector $\mathrm{d}x^\mu = \{\mathrm{d}x, \mathrm{d}y, \mathrm{d}z, c\mathrm{d}t\}$ ($\mathrm{d}x = x_f - x_i$, etc.) is defined as ([74], p. 9)

$$\mathrm{d}s^2 = c^2(\mathrm{d}t)^2 - \left[(\mathrm{d}x)^2 + (\mathrm{d}y)^2 + (\mathrm{d}z)^2\right], \tag{A.5}$$

which is written in the form

$$\mathrm{d}s^2 = g_{\nu\mu}\mathrm{d}x^\nu \mathrm{d}x^\mu\ , \tag{A.6}$$

then $g_{\nu\mu}$ is given by ([76], pp. 21-22):

$$g_{\nu\mu} = \begin{Bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{Bmatrix}. \tag{A.7}$$

(7) For a contravariant 4-vector A^μ, $A^\mu = \{A_x, A_y, A_z, A_t\}$, with "spatial" components $A = (A_x, A_y, A_z)$ and "temporal" component A_t:

(a) the corresponding covariant (Greek subindex, row) 4-vector is defined as $A_\mu = g_{\mu\nu}A^\nu$, changing the sign of A^μ spatial components, $A_\mu = \{-A_x, -A_y, -A_z, A_t\}$;

(b) given a covariant 4-vector B_μ, $B_\mu = \{B_x, B_y, B_z, B_t\}$, the *inner product* $B_\mu A^\mu$, or *projection* of A^μ along B_μ, is

$$B_\mu A^\mu = B_t A_t - \left(B_x A_x + B_y A_y + B_z A_z\right); \tag{A.8}$$

The inner product of two 4-vectors is a relativistic invariant, that is, $B_\mu A^\mu = B_{A\mu} A_A^\mu$,

(c) its *norm* $\|A^\mu\|$ defined as $\|A^\mu\| = A_\mu A^\mu$ is

$$\|A^\mu\| = \left[A_t^2 - \left(A_x^2 + A_y^2 + A_z^2\right)\right]^{1/2}; \tag{A.9}$$

the norm of a 4-vector is a relativistic invariant, that is, $\|A^\mu\| = \|A_A^\mu\|$.

(d) a linear combination of two 4-vectors is again a 4-vector. For a given 4-vector C^μ, $D^\mu = (aA^\mu + cC^\mu)$, where a and c are constants, D^μ is a 4-vector;

(e) two 4-vectors A^μ and B^μ are said to be equal if, for all j

$$A_j = B_j; \tag{A.10}$$

The property of two 4-vectors being equal is an invariant property. Consequently, a 4-vector equation is an invariant equation. This suggests that the most general manner of writing a physical law into a covariant form would be to formulate it as a 4-vector equation ([77], pp. 69-71)

(8) The proper time $d\tau$ for the 4-vector displacement dx^μ is the time measured by a clock that moves with the system (Figure 9):

$$d\tau = \left\{(dt)^2 - c^{-2}\left[(dx)^2 + (dy)^2 + (dz)^2\right]\right\}^{1/2}, \tag{A.11}$$

being $c d\tau$, the norm of the 4-vector displacement. Thus,

$$\frac{dt}{d\tau} = \gamma(v). \tag{A.12}$$

This equation expresses $d\tau$ as a function of the time dt measured in S_0, frame chosen for the description of events.

(9) The contravariant 4-vector velocity v^μ is defined as [23]

$$v^\mu = \frac{dx^\mu}{d\tau} = \gamma(v)\left\{v_x, v_y, v_z, c\right\}. \tag{A.13}$$

B. Electromagnetic Field

An elementary particle (structureless), with electric charge q, moves with velocity $\mathbf{v} = (v_x, v_y, v_z)$, with 4-vector velocity $v^\mu = \gamma(v)\{v_x, v_y, v_z, c\}$. This particle moves in an electric field $\mathbf{E} = (E_x, E_y, E_z)$ given by the 4×4-tensor—double contravariant—electromagnetic field ([51], p. 126) $\mathcal{E}^{\nu\mu}$:

$$\mathcal{E}^{\nu\mu} = \begin{Bmatrix} 0 & 0 & 0 & -E_x \\ 0 & 0 & 0 & -E_y \\ 0 & 0 & 0 & -E_z \\ E_x & E_y & E_z & 0 \end{Bmatrix}. \tag{B.1}$$

The corresponding mixed 4×4-tensor ([78], pp. 66–68) $\mathcal{E}_\nu^\mu$ is given by $\mathcal{E}_\nu^\mu = g_{\nu\xi}\mathcal{E}^{\xi\mu}$, with:

$$\mathcal{E}_\nu^\mu = \begin{Bmatrix} 0 & 0 & 0 & E_x \\ 0 & 0 & 0 & E_y \\ 0 & 0 & 0 & E_z \\ E_x & E_y & E_z & 0 \end{Bmatrix}. \tag{B.2}$$

The 4×4-tensor electromagnetic force $\mathcal{F}_\nu^\mu$ is defined as $\mathcal{F}_\nu^\mu = q\mathcal{E}_\nu^\mu$:

$$\mathcal{F}_\nu^\mu = \begin{Bmatrix} 0 & 0 & 0 & F_x \\ 0 & 0 & 0 & F_y \\ 0 & 0 & 0 & F_z \\ F_x & F_y & F_z & 0 \end{Bmatrix}, \tag{B.3}$$

with $F_x = qE_x, F_y = qE_y, F_z = qE_z$. The so-called 4-vector Minkowski force ([51] p. 131) F^μ on the particle is given by

$$\begin{aligned} F^\mu &= \frac{1}{c}\mathcal{F}_\nu^\mu v^\nu \\ &= \gamma(v)\left\{qE_x, qE_y, qE_z, \frac{q}{c}\left(E_x v_x + E_y v_y + E_z v_z\right)\right\}. \end{aligned} \tag{B.4}$$

For the electromagnetic field characterized by the 4×4-tensor $\mathcal{E}_\nu^\mu = g_{\nu\xi}\mathcal{E}^{\xi\mu}$ in S_0 frame, the same field is characterized by the 4×4-tensor $\mathcal{E}_{A\nu}^{\ \mu}$ in S_A frame, given by

$$\mathcal{E}_{A\nu}^{\ \mu} = \mathcal{L}_\xi^\mu(V)\mathcal{E}_\chi^\xi \mathcal{L}^{+\chi}_{\ \nu}(V). \tag{B.5}$$

Acknowledgment

The author would like to thank Professor R. Valiente for critical reading of the paper and for help with its final redaction.

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