International Scholarly Research Network
ISRN Mechanical Engineering
Volume 2011, Article ID 505673, 10 pages
doi:10.5402/2011/505673

Research Article

Effect of Temperature-Dependent Variable Viscosity on Magnetohydrodynamic Natural Convection Flow along a Vertical Wavy Surface

Nazma Parveen and Md. Abdul Alim

Department of Mathematics, Bangladesh University of Engineering and Technology, Dhaka 1000, Bangladesh

Correspondence should be addressed to Nazma Parveen, nazma@math.buet.ac.bd

Received 17 January 2011; Accepted 21 February 2011

Academic Editors: C.-Y. Huang, G.-J. Wang, and M. Al-Nimr

The effect of temperature dependent variable viscosity on magnetohydrodynamic (MHD) natural convection flow of viscous incompressible fluid along a uniformly heated vertical wavy surface has been investigated. The governing boundary layer equations are first transformed into a nondimensional form using suitable set of dimensionless variables. The resulting nonlinear system of partial differential equations are mapped into the domain of a vertical flat plate and then solved numerically employing the implicit finite difference method, known as Keller-box scheme. The numerical results of the surface shear stress in terms of skin friction coefficient and the rate of heat transfer in terms of local Nusselt number, the stream lines and the isotherms are shown graphically for a selection of parameters set consisting of viscosity parameter (ε), magnetic parameter (M), and Prandtl number (Pr). Numerical results of the local skin friction coefficient and the rate of heat transfer for different values are also presented in tabular form.

1. Introduction

Laminar natural convection boundary layer flow and heat transfer problem from a vertical wavy surface gets a great deal of attention in various branches of engineering. If the surface is roughened, the flow is disturbed by the surface and this alters the rate of heat transfer. These types of roughened surface are taken into account in several heat transfer collectors, flat plate condensers in refrigerators and heat exchanger. One common example of a heat exchanger is the radiator used in cars, in which the heat generated from engine transferred to air flowing through the radiator. Machine-roughened surface enhanced heat transfer and the interface between concurrent or countercurrent two-phase flow is another example remotely related to this problem. Such an interface is always wavy and momentum transfer across it is by no means similar to that across a smooth, flat surface and neither is the heat transfer.

The study of the flow of electrically conducting fluid in the presence of magnetic field is also important from the technical point of view and such types of problems have received much attention by many researchers. The specific problem selected for study is the flow and heat transfer in an electrically conducting fluid adjacent to the surface. The surface is maintained at a uniform temperature T_w, which may either exceed the ambient temperature T_∞ or may be less than T_∞. When $T_w \geq T_\infty$, an upward flow is established along the surface due to free convection, where as for $T_w \leq T_\infty$, there is a down flow. The interaction of the magnetic field and the moving electric charge carried by the flowing fluid induces a force, which is proportional to the magnitude of the longitudinal velocity and acts in the opposite direction is also very small. Additionally, a magnetic field of strength β_0 acts normal to the surface. Consequently, the influence of the magnetic field on the boundary layer is exerted only through induced forces within the boundary layer itself, with no additional effects arising from the free stream pressure gradient. The problem of magnetohydrodynamic free convection in a strong cross field was investigated by Kuiken [1]. Also the effect of magnetic field on free convection heat transfer has been studied by Sparrow and Cess [2].

The viscosity of the fluid is proportional to a linear function of temperature which was proposed by Charraudeau [3]. The effects of surface waviness on the natural convection boundary layer flow of a Newtonian fluid have studied by Yao [4] and Moulic and Yao [5, 6]. In these papers they used an extended Prantdl's transposition theorem and a finite-difference scheme. Yao proposed a simple transformation to study the natural convection heat transfer for an isothermal vertical sinusoidal surface. This simple coordinate transformation method to change the wavy surface into a flat plate. Transient-free convection flow with temperature dependent viscosity in a fluid saturated porous media has shown by Mehta and Sood [7]. In this paper they found that the flow characteristics substantially change when the effect of temperature dependent viscosity is considered. The effects of temperature dependent viscosity on the free and mixed convection flow from a vertical flat plate in the region near the leading edge have studied by Kafoussius and Williams [8] and Kafoussius and Rees [9]. Rees and Pop [10–12] investigated the natural convection boundary layer induced by vertical and horizontal wavy surface exhibiting small amplitude waves embedded in a porous medium. Hady et al. [13] investigated the mixed convection boundary layer flow on a continuous flat plate with variable viscosity. Alam et al. [14] have also studied the problem of free convection from a wavy vertical surface in presence of a transverse magnetic field. On the other hand, the combined effects of thermal and mass diffusion on the natural convection flow of a viscous incompressible fluid along a vertical wavy surface have been investigated by Hossain and Rees [15]. In this paper the effect of waviness of the surface on the heat and mass flux is investigated in combination with the species concentration for a fluid having Prandtl number equal to 0.7. The natural convection heat and mass transfer near a vertical wavy surface with constant wall temperature and concentration in a porous medium studied by Cheng [16]. Hossain et al. [17] investigated the natural convection flow past a permeable wedge for the fluid having temperature dependent viscosity and thermal conductivity. Free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of magnetic field investigated by Elbashbeshy [18]. Munir et al. [19, 20] investigated natural convection with variable viscosity and thermal conductivity along a vertical wavy cone. The problems of natural convection of fluid with temperature dependent viscosity along a heated vertical wavy surface have studied by Kabir et al. [21]. Molla et al. [22] have studied natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation/absorption. Nasrin and Alim [23] investigated magnetohydrodynamic free convection flow along a vertical flat plate with thermal conductivity and viscosity depending on temperature. Numerical study on a vertical plate with variable viscosity and thermal conductivity has investigated by Palani and Kim [24]. Aldoss et al. [25] investigated MHD mixed convection from a horizontal circular cylinder. Al-Nimr and Alkam [26] have studied magneto-hydrodynamics transient free convection in open-ended vertical annuli. Al-Nimr and Hader [27] also investigated MHD free convection flow in open-ended vertical porous channels. Damseh et al. [28] investigated entropy generation during fluid flow in a channel under the effect of transverse magnetic field. From the above investigations it is found that variation of viscosity with temperature of magnetic field is an interesting macroscopic physical phenomenon in fluid mechanics.

The present study is to incorporate the idea that the effects of temperature dependent viscosity in presence of magnetic field of electrically conducting fluid with free convection boundary layer flow along a vertical wavy surface. However, it is known that this physical property (viscosity) may be change significantly with temperature. To predict accurately the flow behavior, it is necessary to take into account of viscosity. The governing partial differential equations are reduced to locally non-similar partial differential forms by adopting some appropriate transformations. The transformed boundary layer equations are solved numerically using implicit finite difference scheme together with the Keller-box technique [29]. We have focused our attention on the evolution of the surface shear stress in terms of local skin friction coefficient and the rate of heat transfer in terms of local Nusselt number, the stream lines and the isotherms for selected values of parameters consisting of the magnetic parameter M, Prandtl number Pr and the viscosity variation parameter ε.

2. Formulation of the Problem

The steady two-dimensional laminar free convection boundary layer flow of a viscous incompressible and electrically conducting fluid along a vertical wavy surface in presence of uniform transverse magnetic field of strength β_0 with temperature dependent variable viscosity is considered. It is assumed that the wavy surface is electrically insulated and is maintained at a uniform temperature T_w. Far above the wavy plate, the fluid is stationary and is kept at a temperature T_∞, where $T_w > T_\infty$. The boundary layer analysis outlined below allows $\overline{\sigma}(\overline{x})$ being arbitrary, but our detailed numerical work assumed that the surface exhibits sinusoidal deformations. The wavy surface may be described by

$$\overline{y}_w = \overline{\sigma}(\overline{x}) = \alpha \sin\left(\frac{n\pi\overline{x}}{L}\right), \tag{1}$$

where L is the characteristic length associated with the wavy surface.

The geometry of the wavy surface and the two-dimensional Cartesian coordinate system are shown in Figure 1.

The governing equations of such flow under the usual boundary layer and Boussinesq approximation can be written as

$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} = 0,$$

$$\begin{aligned} \overline{u}\frac{\partial \overline{u}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{u}}{\partial \overline{y}} &= -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial \overline{x}} + \frac{1}{\rho}\nabla \cdot (\mu \nabla \overline{u}) \\ &\quad + g\beta(T - T_\infty) - \frac{\sigma_0\beta_0^2}{\rho}\overline{u}, \end{aligned}$$

$$\bar{u}\frac{\partial\bar{v}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{v}}{\partial\bar{y}}=-\frac{1}{\rho}\frac{\partial\bar{p}}{\partial\bar{y}}+\frac{1}{\rho}\nabla\cdot(\mu\nabla\bar{v}),$$
$$\bar{u}\frac{\partial T}{\partial\bar{x}}+\bar{v}\frac{\partial T}{\partial\bar{y}}=\frac{k}{\rho C_p}\nabla^2 T, \tag{2}$$

where $(\bar{x},\bar{y})$ are the dimensional coordinates along and normal to the tangent of the surface and $(\bar{u},\bar{v})$ are the velocity components parallel to $(\bar{x},\bar{y})$, $\nabla^2(=\partial^2/\partial x^2+\partial^2/\partial y^2)$ is the Laplacian operator, g is the acceleration due to gravity, $\bar{p}$ is the dimensional pressure of the fluid, ρ is the density, β_0 is the strength of magnetic field, σ_0 is the electrical conduction, k is the thermal conductivity, β is the coefficient of thermal expansion, $\mu(T)$ is the viscosity of the fluid depending on temperature T of the fluid in the boundary layer region and C_p is the specific heat due to constant pressure.

The boundary conditions relevant to the above problem are

$$\bar{u}=0,\quad \bar{v}=0,\quad T=T_w\quad \text{at } \bar{y}=\bar{y}_w=\bar{\sigma}(\bar{x}),$$
$$\bar{u}=0,\quad T=T_\infty,\quad \bar{p}=p_\infty\quad \text{as } \bar{y}\longrightarrow\infty, \tag{3}$$

where T_w is the surface temperature, T_∞ is the ambient temperature of the fluid and P_∞ is the pressure of fluid outside the boundary layer.

There are very few forms of viscosity variation available in the literature. Among them we have considered that one which is appropriate for liquid introduced by Hossain et al. [17] as follows:

$$\mu=\mu_\infty[1+\varepsilon^*(T-T_\infty)], \tag{4}$$

where μ_∞ is the viscosity of the ambient fluid and $\varepsilon^*=(1/\mu_f)(\partial\mu/\partial T)_f$ is a constant evaluated at the film temperature of the flow $T_f=1/2(T_w+T_\infty)$.

Following Yao [4], we now introduce the following nondimensional variables:

$$x=\frac{\bar{x}}{L},\qquad y=\frac{\bar{y}-\bar{\sigma}}{L}\mathrm{Gr}^{1/4},\qquad p=\frac{L^2}{\rho\nu^2}\mathrm{Gr}^{-1}\bar{p},$$
$$u=\frac{\rho L}{\mu_\infty}\mathrm{Gr}^{-1/2}\bar{u},\qquad v=\frac{\rho L}{\mu_\infty}\mathrm{Gr}^{-1/4}(\bar{v}-\sigma_x\bar{u}),$$
$$\theta=\frac{T-T_\infty}{T_w-T_\infty}, \tag{5}$$
$$\sigma_x=\frac{d\bar{\sigma}}{d\bar{x}}=\frac{d\sigma}{dx},\qquad \mathrm{Gr}=\frac{g\beta(T_w-T_\infty)}{\nu^2}L^3,$$

where θ is the dimensionless temperature function and (u,v) are the dimensionless velocity components parallel to (x,y) and $\nu\ (=\mu/\rho)$ is the kinematic viscosity.

Introducing the above dimensionless dependent and independent variables into (2), the following dimensionless form of the governing equations are obtained after ignoring terms of smaller orders of magnitude in Gr, the Grashof number defined in(5)

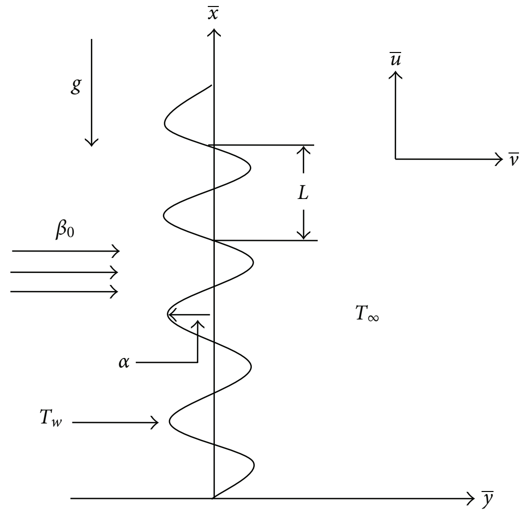


Figure 1: The coordinate system and the physical model.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{6}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\mathrm{Gr}^{1/4}\sigma_x\frac{\partial p}{\partial y}+(1+\sigma_x^2)(1+\varepsilon\theta)\frac{\partial^2 u}{\partial y^2}$$
$$+\varepsilon(1+\sigma_x^2)\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial y}-Mu+\theta, \tag{7}$$

$$\sigma_x\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\mathrm{Gr}^{1/4}\frac{\partial p}{\partial y}+\ \sigma_x(1+\sigma_x^2)(1+\varepsilon\theta)\frac{\partial^2 u}{\partial y^2}$$
$$+\varepsilon\sigma_x(1+\sigma_x^2)\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial y}-\sigma_{xx}u^2, \tag{8}$$

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{1}{\mathrm{Pr}}(1+\sigma_x^2)\frac{\partial^2\theta}{\partial y^2}. \tag{9}$$

In the above equations Pr, ε, and M are, respectively, known as the Prandtl number, dimensionless viscosity variation parameter and dimensionless magnetic parameter, which are defined as

$$\mathrm{Pr}=\frac{C_p\mu_\infty}{k},\qquad \varepsilon=\varepsilon^*(T_w-T_\infty),\qquad M=\frac{\sigma_0\beta_0^2L^2}{\mu\mathrm{Gr}^{1/2}}, \tag{10}$$

It can easily be seen that the convection induced by the wavy surface is described by (6)–(9). We further notice that, (8) indicates that the pressure gradient along the y-direction is $O(\mathrm{Gr}^{-1/4})$, which implies that lowest-order pressure gradient along x-direction can be determined from the inviscid flow solution. For the present problem this pressure gradient $(\partial p/\partial x\ =\ 0)$ is zero. Equation (8) further shows that $\mathrm{Gr}^{1/4}\partial p/\partial y$ is $O(1)$ and is determined by the left-hand side of this equation. Thus, the elimination of $\partial p/\partial y$ from (7) and (8) leads to

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(1+\sigma_x^2\right)(1+\varepsilon\theta)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_x\sigma_{xx}}{1+\sigma_x^2}u^2+\varepsilon\left(1+\sigma_x^2\right)\frac{\partial u}{\partial y}\frac{\partial \theta}{\partial y}-\frac{M}{1+\sigma_x^2}u+\frac{1}{1+\sigma_x^2}\theta. \tag{11}$$

The corresponding boundary conditions for the present problem then turn into

$$\begin{aligned} &u=v=0, \quad \theta=1 \quad \text{at } y=0\\ &u=\theta=0, \quad p=0 \quad \text{as } y\longrightarrow\infty. \end{aligned} \tag{12}$$

Now we introduce the following transformations to reduce the governing equations to a convenient form:

$$\psi=x^{3/4}f(x,\eta), \qquad \eta=yx^{-1/4}, \qquad \theta=\theta(x,\eta), \tag{13}$$

where η is the pseudo similarity variable and ψ is the stream function that satisfies (6) and is defined by

$$u=\frac{\partial\psi}{\partial y}, \qquad v=-\frac{\partial\psi}{\partial x}. \tag{14}$$

Introducing the transformations given in (13) and into (11) and (9), we have

$$\begin{aligned} &\left(1+\sigma_x^2\right)(1+\varepsilon\theta)f'''+\frac{3}{4}ff''-\left(\frac{1}{2}+\frac{x\sigma_x\sigma_{xx}}{1+\sigma_x^2}\right)f'^2\\ &\quad+\frac{1}{1+\sigma_x^2}\theta-\frac{Mx^{1/2}}{1+\sigma_x^2}f'+\varepsilon\left(1+\sigma_x^2\right)\theta' f''\\ &\quad=x\left(f'\frac{\partial f'}{\partial x}-f''\frac{\partial f}{\partial x}\right), \end{aligned} \tag{15}$$

$$\frac{1}{\Pr}\left(1+\sigma_x^2\right)\theta''+\frac{3}{4}f\theta'=x\left(f'\frac{\partial\theta}{\partial x}-\theta'\frac{\partial f}{\partial x}\right). \tag{16}$$

The boundary conditions (12) now take the following form:

$$\begin{aligned} &f(x,o)=f'(x,o)=0, \qquad \theta(x,o)=1,\\ &f'(x,\infty)=0, \qquad \theta(x,\infty)=0. \end{aligned} \tag{17}$$

In the above equations prime denotes the differentiation with respect to η.

3. Local Skin-Friction Coefficient and the Local Rate of Heat Transfer

In practical applications, the physical quantities of principle interest are the shearing stress τ_w and the rate of heat transfer in terms of the skin friction coefficient C_{fx} and Nusselt number Nu_x, respectively, which can be written as

$$C_{fx}=\frac{2\tau_w}{\rho U_\infty{}^2}, \qquad \mathrm{Nu}_x=\frac{q_w x}{k(T_w-T_\infty)}, \tag{18}$$

where

$$\tau_w=\left(\mu\bar{n}\cdot\nabla\bar{u}\right)_{y=0}, \qquad q_w=-k\left(\bar{n}\cdot\nabla T\right)_{y=0}. \tag{19}$$

Using the transformations (13) and (19) into (18), the local skin friction coefficient C_{fx} and the rate of heat transfer in terms of the local Nusselt number Nu_x take the following form:

$$\begin{aligned} &\frac{C_{fx}(\mathrm{Gr}/x)^{1/4}}{2}=(1+\varepsilon)\sqrt{1+\sigma_x^2}\,f''(x,o),\\ &\mathrm{Nu}_x\left(\frac{\mathrm{Gr}}{x}\right)^{-1/4}=-\sqrt{1+\sigma_x^2}\theta'(x,o). \end{aligned} \tag{20}$$

4. Method of Solution

Along with the boundary condition (17), the solution of the parabolic differential equations (15) and (16) will be found by using the implicit finite difference method together with Keller- box scheme [29], which is well documented by Cebeci and Bradshaw [30]. This method has been extensively used recently by Hossain et al. [14, 15, 17, 19–22]. To apply the aforementioned method, (15) and (16) their boundary condition (17) are first converted into the following system of first-order equations. For this purpose we introduce new dependent variables $u(\xi,\eta)$, $v(\xi,\eta)$, $p(\xi,\eta)$, $g(\xi,\eta)$ and $f'=u$, $u'=v$, $g'=p$ so that the transformed momentum and energy equations can be written as

$$\begin{aligned} &P_1Tv'+P_2fv-P_3u^2+P_4g-P_5u+P_6pv=\xi\left(u\frac{\partial u}{\partial\xi}-v\frac{\partial f}{\partial\xi}\right),\\ &\frac{1}{\Pr}P_1p'+P_2fp=\xi\left(u\frac{\partial g}{\partial\xi}-p\frac{\partial f}{\partial\xi}\right), \end{aligned} \tag{21}$$

where $x=\xi$, $\theta=g$ and

$$\begin{aligned} &P_1=\left(1+\sigma_x^2\right), \qquad P_2=\frac{3}{4}, \qquad P_3=\frac{1}{2}+\frac{x\sigma_x\sigma_{xx}}{1+\sigma_x^2},\\ &P_4=\frac{1}{1+\sigma_x^2}, \qquad P_5=\frac{Mx^{1/2}}{1+\sigma_x^2},\\ &P_6=\varepsilon\left(1+\sigma_x^2\right), \qquad T=(1+\varepsilon\theta) \end{aligned} \tag{22}$$

and the boundary conditions (17) are

$$\begin{aligned} &f(\xi,0)=0, \qquad u(\xi,0)=0, \qquad g(\xi,0)=1\\ &u(\xi,\infty)=0, \qquad g(\xi,\infty)=0. \end{aligned} \tag{23}$$

Table 1: Comparison of Skin friction coefficient C_{fx} and the local rate of heat transfer Nu_x against x for the variation of Prandtl number Pr with and without effects of magnetic parameter M and viscosity parameter ε with $\alpha = 0.3$.

Pr	$\varepsilon = 0.0, M = 0.0$		$\varepsilon = 0.0, M = 0.5$		$\varepsilon = 5.0, M = 0.0$		$\varepsilon = 5.0, M = 0.5$	
	C_{fx}	Nu_x	C_{fx}	Nu_x	C_{fx}	Nu_x	C_{fx}	Nu_x
0.7	0.97482	0.34175	0.87097	0.31346	1.69448	0.26456	1.55731	0.24755
1.74	0.84656	0.46767	0.76566	0.43181	1.42192	0.34631	1.32449	0.32597
3.0	0.76895	0.55765	0.70032	0.51737	1.26523	0.40490	1.18768	0.38310
7.0	0.65372	0.72178	0.60129	0.67456	1.04514	0.51175	0.99193	0.48832

The finite difference approximation according to box method then reduced the system of equation into nonlinear system of difference equation. These are as follows:

$$\begin{aligned}
&\frac{1}{2}(P_1T)^n_{j-1/2}\left(\frac{v^n_j - v^n_{j-1}}{h_j}\right) + \frac{1}{2}\,(P_1T)^n_{j-1/2}\left(\frac{v^{n-1}_j - v^{n-1}_{j-1}}{h_j}\right)\\
&\quad + (P_2fp)^{n-1/2}_{j-1/2} - (P_3u^2)^{n-1}_{j-1/2}\\
&\quad + (P_4g)^{n-1}_{j-1/2} - (P_5u)^{n-1}_{j-1/2} + (P_6pv)^{n-1}_{j-1/2}\\
&= \xi^{n-1/2}_{j-1/2}\left(u^{n-1/2}_{j-1/2}\,\frac{u^n_{j-1/2} - u^{n-1}_{j-1/2}}{k_n} - v^{n-1/2}_{j-1/2}\,\frac{f^n_{j-1/2} - f^{n-1}_{j-1/2}}{k_n}\right),\\
&\frac{1}{2\mathrm{Pr}}\left\{(P_1)^n_{j-1/2}\right\}\left(\frac{p^n_j - p^n_{j-1}}{h_j}\right)\\
&\quad + \frac{1}{2\mathrm{Pr}}\left\{(P_1)^{n-1}_{j-1/2}\right\}\left(\frac{p^{n-1}_j - p^{n-1}_{j-1}}{h_j}\right) + (P_2fp)^{n-1/2}_{j-1/2}\\
&= \xi^{n-1/2}_{j-1/2}\left(u^{n-1/2}_{j-1/2}\,\frac{g^n_{j-1/2} - g^{n-1}_{j-1/2}}{k_n} - p^{n-1/2}_{j-1/2}\,\frac{f^n_{j-1/2} - f^{n-1}_{j-1/2}}{k_n}\right).
\end{aligned} \tag{24}$$

The above equations are to be linearized by using Newton's quasilinearization method. Then linear algebraic equations can be written in a block matrix, which forms a coefficient matrix. The whole procedure, namely reduction to first-order followed by central difference approximations, Newton's quasilinearization method and the block Thomas algorithm, is well known as Keller-box method.

5. Results and Discussion

In this paper the effect of temperature dependent variable viscosity on magnetohydrodynamic natural convection flow of viscous incompressible fluid along a uniformly heated vertical wavy surface has been investigated. Although there are four parameters of interest in the present problem, our main aim is to determine the effects of varying ε, the temperature dependent viscosity, the strength of magnetohydrodynamic field and Prandtl number Pr.

Numerical values of local shearing stress and the rate of heat transfer are calculated in terms of the skin friction coefficient C_{fx} and Nusselt number Nu_x from (20) for natural convection flow of a variable viscosity electrically conducting fluid in the presence of magnetic field for a wide range of the axial distance variable x starting from the leading edge. These are shown in tabular form in Table 1 and graphically in Figures 4–7 for different values of the aforementioned parameters ε, M and Pr. From this table it can be concluded that if the value of Pr increases the values of the skin-friction coefficient C_{fx} decreases and the rate of heat transfer in terms of Nusselt number Nu_x increases for any value of ε and M. It is also shown that if the value of M increases the values of C_{fx} decreases while it increases of ε and the rate of heat transfer in terms of Nusselt number Nu_x decreases for any value of ε and M.

Figure 2 illustrate the effects of temperature dependent variable viscosity parameter ε and magnetic parameter M on the development of streamlines which are plotted for the amplitude of the wavy surface $\alpha = 0.3$ and Prandtl number $\mathrm{Pr} = 0.73$. When $\varepsilon = 0$ and $M = 0$, the problem discussed by Yao [4], where the viscosity is independent of temperature and in absence of magnetic parameter as shown in Figure 2(a). In this case $\psi_{\max} = 9.25$. In Figure 2(b) it is found that an increase in the value of M causes the effects of the wavy surface to be attenuated and the boundary layer becomes thinner where $\psi_{\max} = 6.20$. For the case of fluid with constant viscosity ($\varepsilon = 0$) and in presence of magnetic field (i.e., $M > 0$) which was studied by Alam et al. [14]. The magnetic field acting along the horizontal direction retards the fluid velocity. For this there creates a lorentz force by the interaction between the applied magnetic field and flow field. This force acts against the fluid flow and reduce the velocity distribution. On the other hand, when the value of ε increases similar thing happens and the maximum value of ψ, that is, $\psi_{\max}$ is 8.08 which is shown in Figure 2(c). The combined effects of ε and M are shown in Figure 2(d). In this case the maximum value of ψ is 5.23 decreases comparatively in presence of magnetic parameter M with variable viscosity parameter ε.

The influence of the variable viscosity parameter ε and the magnetic parameter M on the isotherms profile for $\alpha = 0.3$ and $\mathrm{Pr} = 0.73$ are shown in Figure 3. As mentioned before, owing to the presence of the variable viscosity parameter ε and the magnetic field M effects ($\varepsilon > 0$ and $M > 0$), the thermal state of the fluid increases, causing the thermal boundary layer to increase. In the down stream region the temperature distribution is negligible in this case. For absence of the variable viscosity parameter ε and the magnetic field M, it can be observed that the opposite phenomenon happens.

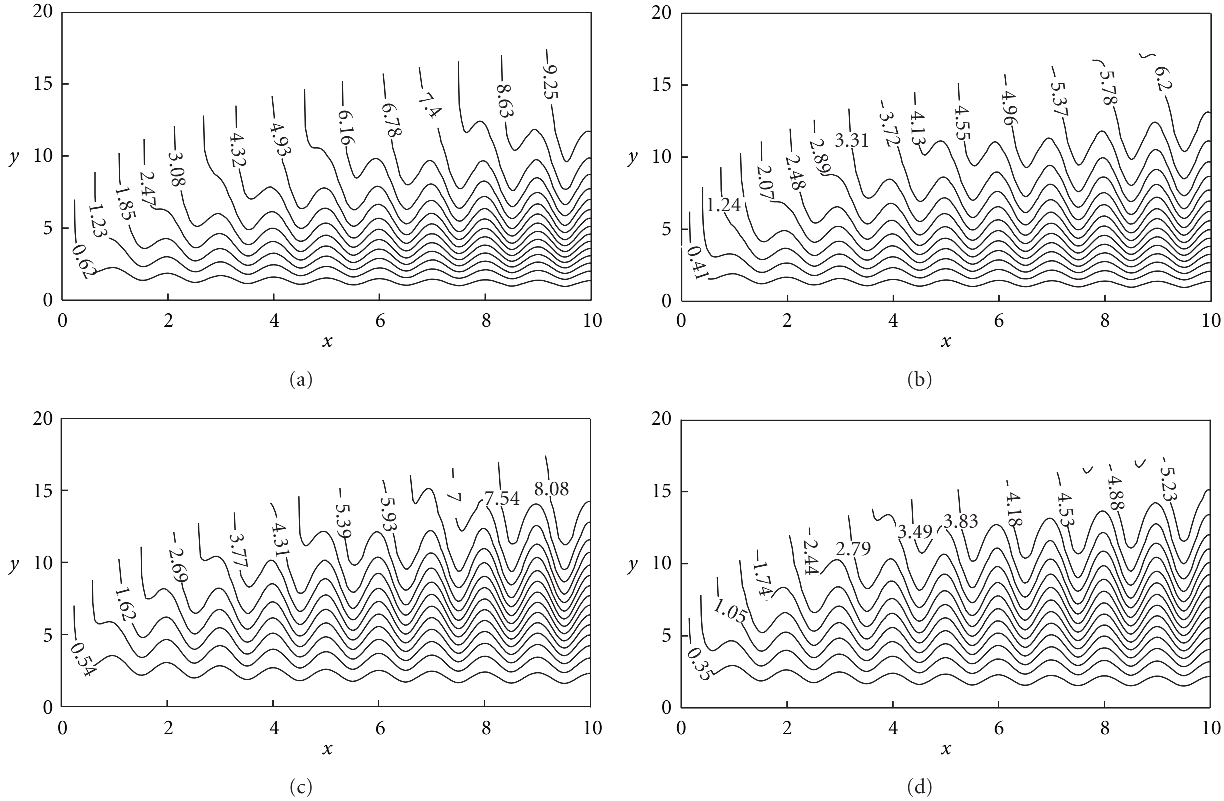


Figure 2: Streamlines for (a) $\varepsilon = 0.0$, $M = 0.0$ (b) $\varepsilon = 0.0$, $M = 0.5$ (c) $\varepsilon = 5.0$, $M = 0.0$ (d) $\varepsilon = 5.0$, $M = 0.5$ while $\mathrm{Pr} = 0.73$, and $\alpha = 0.3$.

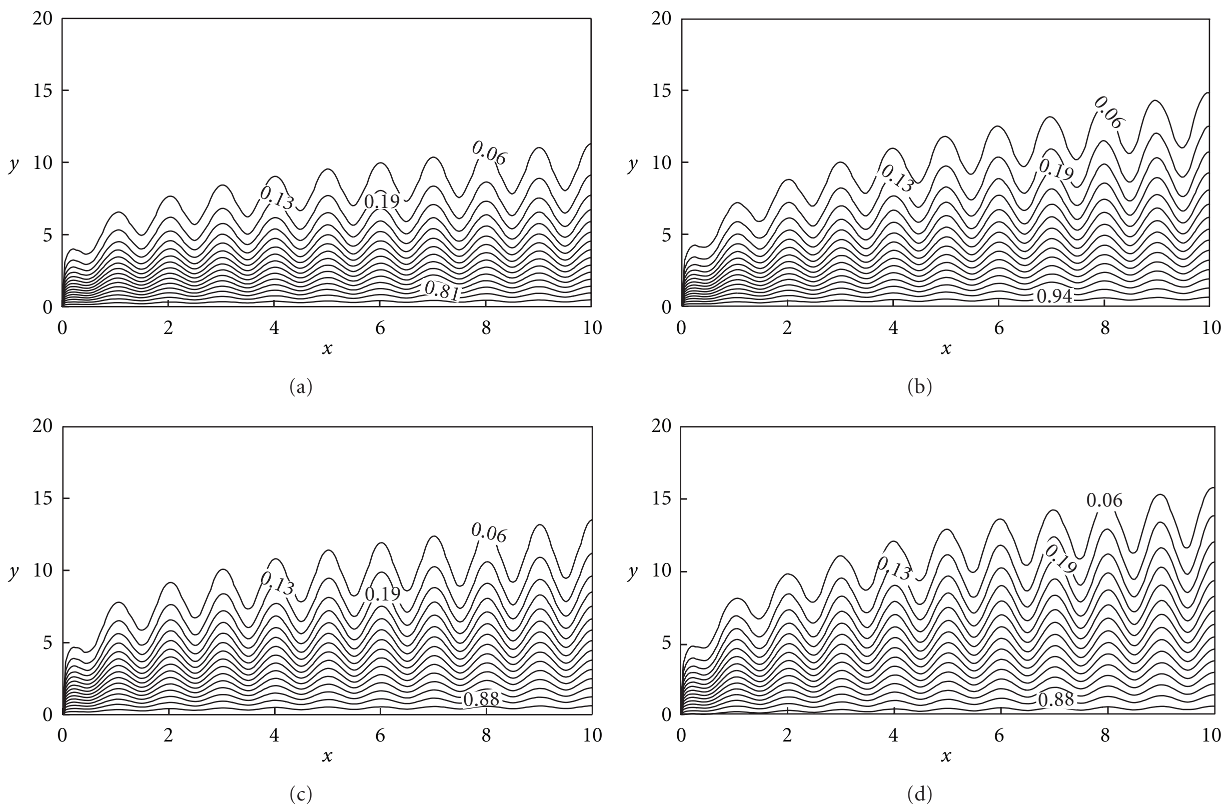


Figure 3: Isotherms for (a) $\varepsilon = 0.0$, $M = 0.0$ (b) $\varepsilon = 0.0$, $M = 0.5$ (c) $\varepsilon = 5.0$, $M = 0.0$ (d) $\varepsilon = 5.0$, $M = 0.5$ while $\mathrm{Pr} = 0.73$, and $\alpha = 0.3$.

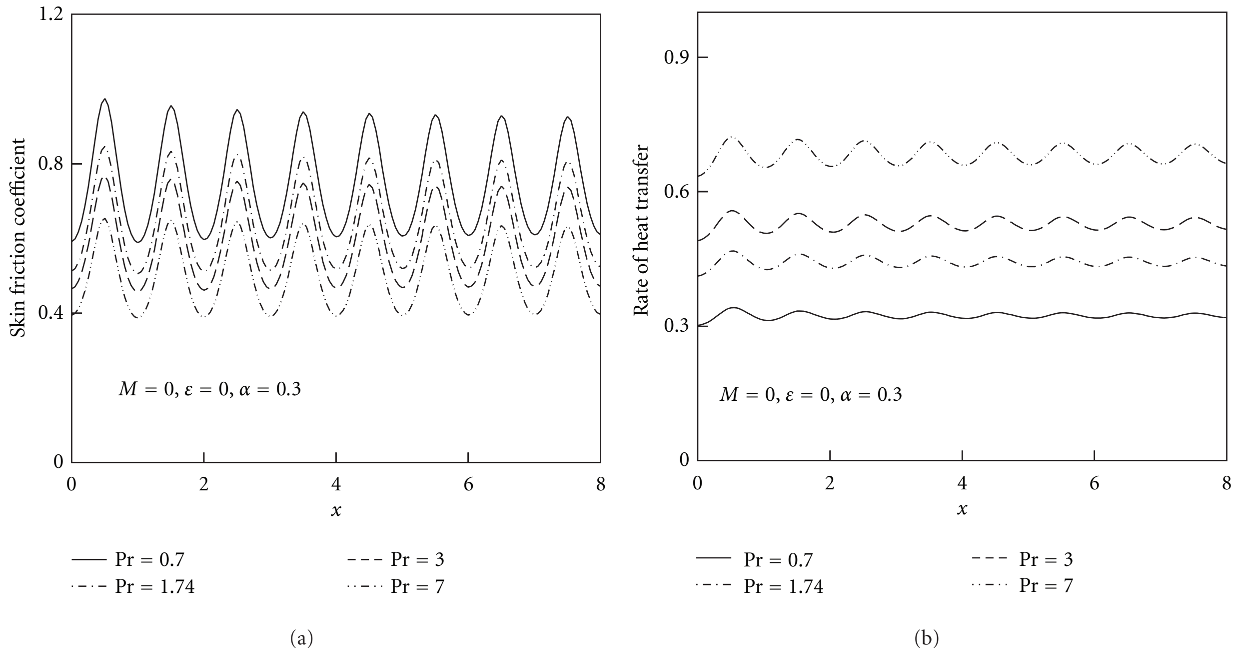


FIGURE 4: Variation of (a) skin friction coefficient and (b) rate of heat transfer against x for different values of Prandtl number Pr while $M = 0.0$, $\varepsilon = 0.0$, and $\alpha = 0.3$.

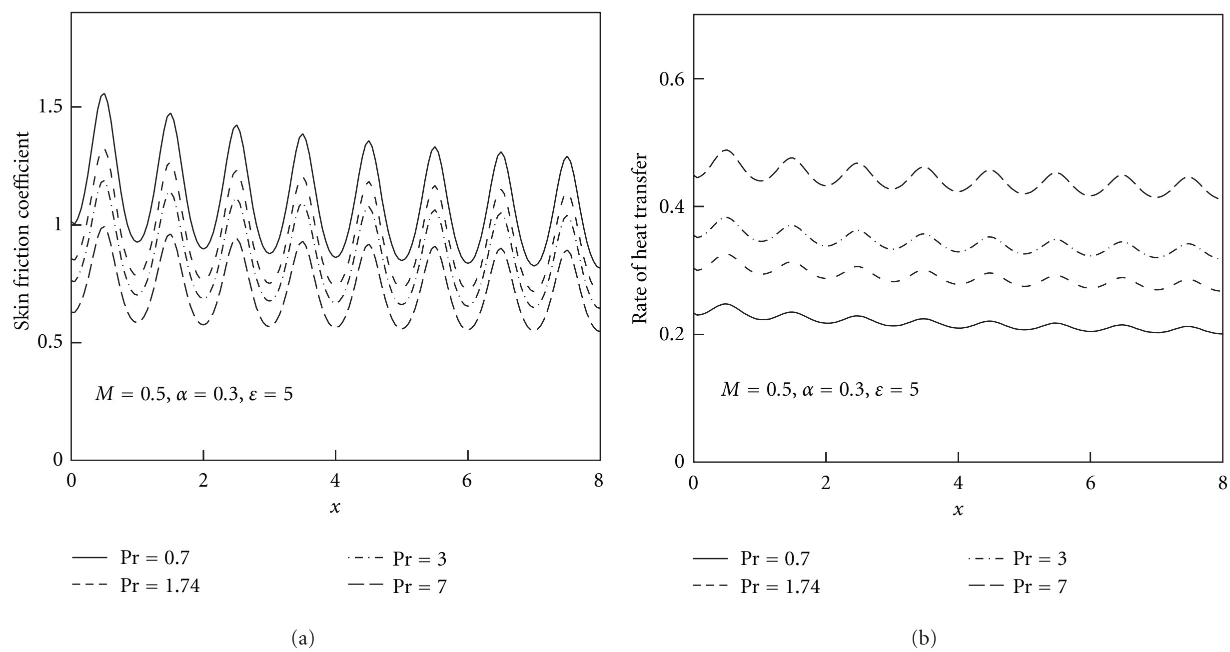
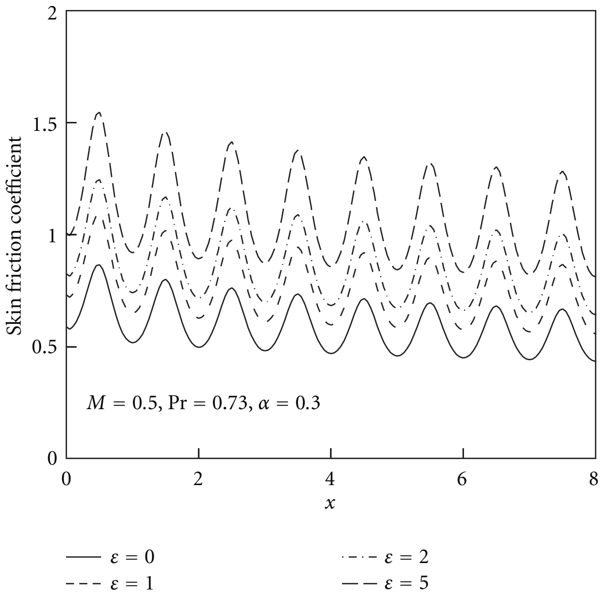
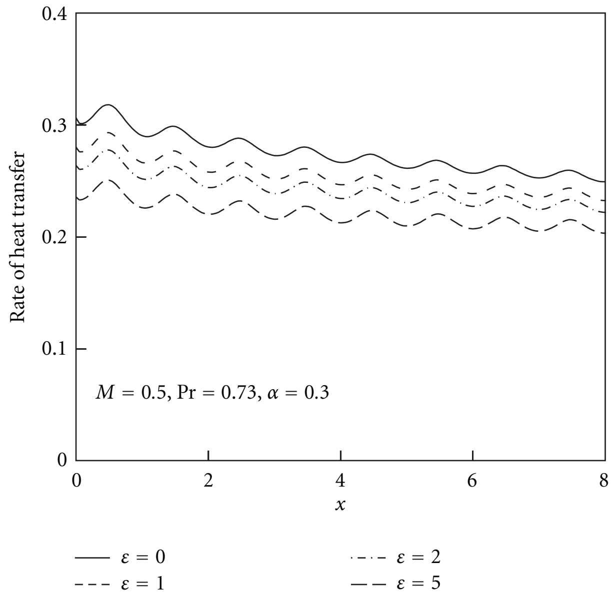


FIGURE 5: Variation of (a) skin friction coefficient and (b) rate of heat transfer against x for different values of Prandtl number Pr while $\varepsilon > 0$ and $M > 0$ ($M = 0.5, \varepsilon = 5.0$), and $\alpha = 0.3$.

The variation of the local skin friction coefficient C_{fx} and local rate of heat transfer Nu_x for different values of Prandtl number Pr for $\varepsilon = M = 0.0$ and $\varepsilon > 0$, $M > 0$ are shown in Figures 4 and 5 while $\alpha = 0.3$. The skin friction coefficient C_{fx} and local rate of heat transfer Nu_x varies according to the slope of the wavy surface. This is due to the alignment of the buoyancy force $1/(1+\sigma_x^2)$, as shown in (15), which drives the flow tangentially to the wavy surface. It can be observed from Figure 4 that without effects of variable viscosity parameter ε and the magnetic parameter M the skin friction coefficient and their amplitude reduce at a great extent for increasing values of the Prandtl number Pr and the rate of heat transfer increases. The local skin friction coefficient C_{fx} decreases by 32.94% and the rate of heat transfer increases by 52.65%



(a)



(b)

FIGURE 6: Variation of (a) skin friction coefficient and (b) rate of heat transfer against x for different values of ε while $\alpha = 0.3$, $M = 0.5$, and Pr $= 0.73$.

as Pr increases from 0.7 to 7.0. Figure 5 it is noted that for the influence of the parameters ε and M, the decreasing skin friction coefficient becomes slower in the downstream region. On the other hand, increasing the values of Prandtl number Pr speed up the decay of the temperature field away from the heated surface with a consequent increase in the rate of heat transfer and a reduction in the thermal boundary layer thickness. In this case the local skin friction coefficient C_{fx} decreases by 36.31% and the rate of heat transfer are increases by 49.31% as Pr increases from Pr = 0.7 to 7.0 which occur at point $x = 0.50$.

The effect of $\varepsilon = (0.0, 1.0, 2.0, 5.0)$ on the surface shear stress in terms of the local skin friction coefficient C_{fx} and the rate of heat transfer in terms of the local Nusselt number Nu_x are depicted graphically in Figure 6 when $\alpha = 0.3$, $M = 0.5$ and Pr $= 0.73$. From this figure it can be noted that an increase in the variable viscosity variation parameter ε, the skin friction coefficient increases monotonically along the upward direction of the plate and to decrease of the heat transfer rates. Here it is concluded that for high viscous fluid the skin friction is large and the corresponding rate of heat transfer is slow. In Figure 6(a), the maximum values of local skin friction coefficient C_{fx} are 0.86640 and 1.54688 for $\varepsilon = 0.0$ and 5.0, respectively, which occur at the same point $x = 0.50$ and it is seen that the local skin friction coefficient C_{fx} increases by 43.99% as ε increases from 0.0 to 5.0. Increasing values of ε lead to increase the amplitude of the C_{fx}. Again Figure 6(b) it is found that the rate of heat transfer decreases by 21.22% due to the increased value of ε.

The influence of the parameter M, on the reduced local skin friction coefficient C_{fx} and local rate of heat transfer Nu_x are illustrated in Figure 7 for Prandtl number Pr $= 0.73$, amplitude of wavy surface $\alpha = 0.3$ and viscosity parameter $\varepsilon = 5.0$. From Figure 7 it observed that an increase in the magnetic parameter $M = (0.0, 0.2, 1.0, 1.5, 2.0)$ leads to decrease the local skin friction coefficient C_{fx} and local rate of heat transfer Nu_x at different position of x. The skin friction coefficient and the rate of heat transfer coefficient decreases by 23.95% and 12.01%, respectively, as M increases from 0.0 to 2.0. The magnetic field acts against the flow and reduces the skin friction and the rate of heat transfer.

6. Conclusion

For different values of relevant physical parameters including the magnetic parameter M, the effect of natural convection flow of viscous incompressible fluid with temperature dependent variable viscosity along a uniformly heated vertical wavy surface has been studied. New variables to transform the complex geometry into a simple shape and were used a very efficient implicit finite difference method known as Keller-box scheme was employed to solve the boundary later equations. From the present investigation the following conclusions may be drawn:

(i) The skin friction coefficient decreases for an increase of the Prandtl number Pr, over the whole boundary layer but significantly increase the rate of heat transfer. When with and without the effects of magnetic parameter M and viscosity parameter ε are included.

(ii) The effect of increasing viscosity parameter ε results in decreasing the local rate of heat transfer Nu_x and increasing the local skin friction coefficient C_{fx}.

(iii) An increase in the values of M leads to decrease the local skin friction coefficient C_{fx} and the local rate of heat transfer Nu_x.

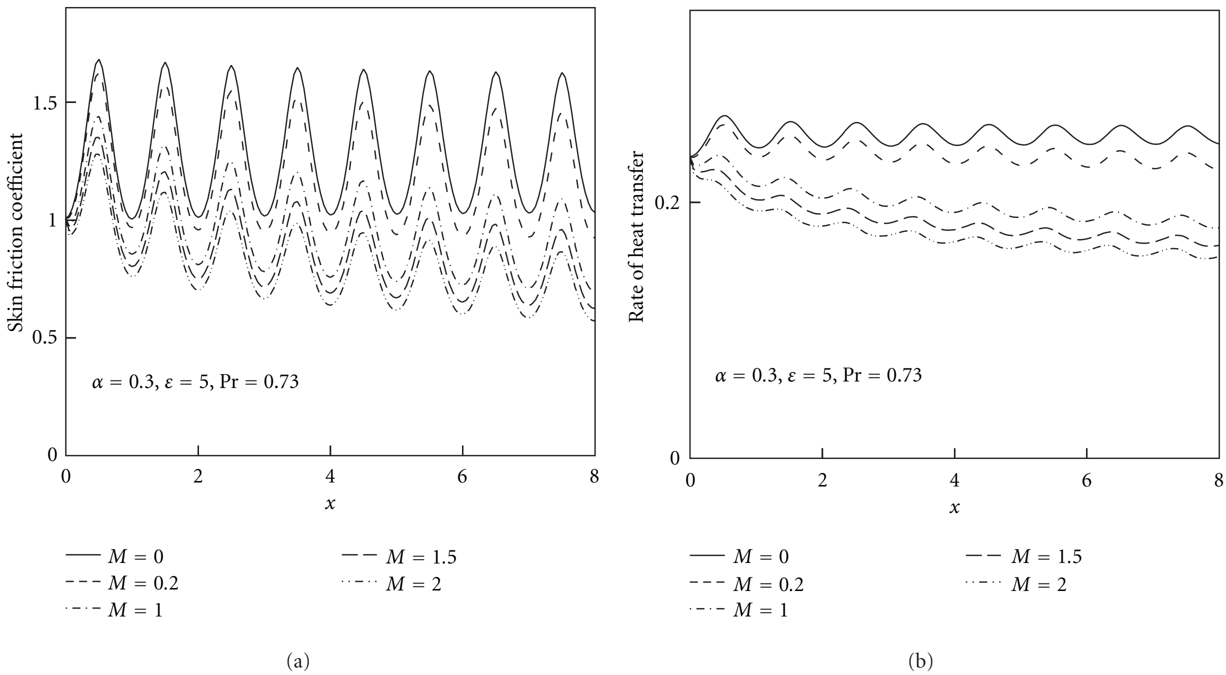


Figure 7: Variation of (a) skin friction coefficient and (b) rate of heat transfer against x for different values of magnetic parameter M while $\mathrm{Pr} = 0.73$, $\alpha = 0.3$, and $\varepsilon = 5$.

Nomenclature

C_{fx}: Local skin friction coefficient
C_p: Specific heat at constant pressure ($\mathrm{J \cdot kg^{-1} \cdot K^{-1}}$)
f: Dimensionless stream function
g: Acceleration due to gravity ($\mathrm{m \cdot s^{-2}}$)
Gr: Grashof number
k: Thermal conductivity ($\mathrm{W \cdot m^{-1} \cdot K^{-1}}$)
L: Characteristic length associated with the wavy surface (m)
M: Magnetic parameter
Nu_x: Local Nusselt number
P: Pressure of the fluid ($\mathrm{N \cdot m^{-2}}$)
Pr: Prandtl number
T: Temperature of the fluid in the boundary layer (K)
T_w: Temperature at the surface (K)
T_∞: Temperature of the ambient fluid (K)
u, v: Dimensionless velocity components along the (x, y) axes ($\mathrm{m \cdot s^{-1}}$)
x, y: Axis in the direction along and normal to the tangent of the surface.

Greek Symbols

α: Amplitude of the surface waves
β: Volumetric coefficient of thermal expansion ($\mathrm{K^{-1}}$)
β_0: Applied magnetic field strength
ε: Viscosity variation parameter
η: Dimensionless similarity variable
θ: Dimensionless temperature function
ψ: Stream function ($\mathrm{m^2 \cdot s^{-1}}$)
μ: Viscosity of the fluid ($\mathrm{kg \cdot m^{-1} \cdot s^{-1}}$)
μ_∞: Viscosity of the ambient fluid
ν: Kinematic viscosity ($\mathrm{m^2 \cdot s^{-1}}$)
ρ: Density of the fluid ($\mathrm{kg \cdot m^{-3}}$)
σ_0: Electrical conductivity
τ_w: Shearing stress
$\sigma(x)$: Surface profile function defined in (1).

Subscripts

w: Wall conditions
∞: Ambient conditions
x: Differentiation with respect to x.

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