Research Article
An Old Universe in K-Essence Cosmology

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1. Introduction

According to several different kinds of astrophysical studies including supernova (SN) Ia [1, 2], large-scale structure [3], and Cosmic Microwave Background (CMB) [4], the present Universe energy content seems to consist of approximately 4% baryons, 26% dark matter, and a little amount of radiation. The remaining around 70% of its energy, with a positive energy density but a negative pressure, is homogeneously distributed in the universe and is causing the accelerated expansion of the Universe. The simplest candidate of dark energy is the vacuum energy (or the cosmological constant Λ) [5]. This scenario is in general agreement with various data but suffers from two very difficult problems: first, why the vacuum energy is so much smaller than its expected natural value (the cosmological constant problem) [6]; second, why the matter and dark energy densities are of precisely the same order of magnitude today (the coincidence problem). Recently, it was shown that ΛCDM model may also suffer from age problem [7].

It is thus natural to pursue alternative possibilities to explain the mystery of dark energy, such as quintessence [8], phantom [9, 10], k-essence [11], tachyon [12–14], DGP [15], and spatial Ricci scalar dark energy model [16]. K-essence, a simple approach toward constructing a model for an accelerated expansion of the Universe, is to work with the idea that the unknown dark energy component is due to a minimally coupled scalar field φ with noncanonical kinetic energy [17]. A feature of k-essence models is that the negative pressure results from the nonlinear kinetic energy of the scalar field. Secondly, because of the dynamical attractor behavior, cosmic evolution is insensitive to initial conditions in k-essence theories. Thirdly, k-essence changes its speed of evolution in dynamic response to changes in the background EoS.

K-essence scenario has received much attention, it was originally proposed as a model for inflation [18] and then as a model for dark energy [17]. The stability of k-essence was studied in [19]. Dynamics of k-essence were discussed in [20]. Conditions for stable tracker solutions for k-essence in a general cosmological background were derived in [21]. Slow-roll conditions for thawing k-essence were obtained in [22]. The geometrical diagnostic for purely kinetic k-essence dark energy was discussed in [23]. In several cases, k-essence cannot be observationally distinguished from quintessence [24]. Models of k-essence unified dark matter were discussed in [25–27]. Theoretical and observational Constraints on k-essence dark energy models were discussed in [14, 28, 29]. A model-independent method to reconstruct the Lagrangian of the k-essence field by the parameterized Hubble parameter...
H(z) was studied in detail in [30]. With assumptions on the EoS of k-essence as functions of the scale factor a, the forms of the Lagrangians were discussed in [31]. With some assumptions on the EoS of k-essence as functions of the kinetic energy X, the evolution of purely k-essence is investigated in [32]. Ermakov invariant of k-essence and its observational probes for the early universe are discussed in [33].

In this paper, we will reconsider in k-essence cosmology the simplest k-essence models put forward by Scherrer [25] in which the Lagrangian contains only a kinetic factor and does not depend explicitly on the field itself. Sticking with a constant k-field potential, we obtain a general equation of state of k-essence under a simple hypothesis. This equation of state indicates that the acceleration expansion of the Universe can only be induced after the matter-dominated epoch in a very old Universe of about 33.5 $\pm$ 3.4 Gyr old, and the year where a transition from deceleration to acceleration expansion is about from 18.4 Gyr to 25.2 Gyr after the beginning of the Universe.

2. K-Essence Cosmology

As a candidate of dark energy, k-essence is defined as a scalar field $\phi$ with nonlinear kinetic terms which appears generically in the effective action in string and supergravity theories; its action minimally coupled with gravity generically is given by

$$S = \int \sqrt{g} \left[ -\frac{R}{2} + p(\phi, X) \right],$$

(1)

where $X = -\frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi$, and we take $8\pi G = 1$ throughout this paper. The Lagrangian $p$ depends on the specific-particle theory model. In this paper, we consider only factorial Lagrangian of the form

$$p_k = V(\phi) F(X),$$

(2)

and $F(X)$ is a function of the kinetic energy $X$. The name of the model suggests that the field should be driven only by its kinetic energy. For this to be strictly true, one should impose $F(0) = 0$. Using the perfect fluid analogy, the energy density is given by

$$\rho_k = V(\phi)[2XF_X - F],$$

(3)

where $F_X = dF/dX$. The corresponding equation of state parameter, $w_k = p_k/\rho_k$, is just

$$w_k = \frac{F}{2XF_X - F}. $$

(4)

The effective sound speed is given by

$$c_s^2 = \frac{\partial p_k/\partial X}{\partial \rho_k/\partial X} = \frac{F_X}{F_X + 2XF_{XX}},$$

(5)

with $F_{XX} \equiv d^2F/dX^2$. This definition comes from the equation describing the evolution of linear perturbations in a k-essence dominated Universe [34], and therefore is relevant when studying the stability of the theory.

From the conservation equation in a flat Friedmann-Robertson-Walker (FRW) spacetime, one finds the equation for the k-field [35]:

$$\dot{\phi} \left( \frac{\partial p}{\partial X} + \phi \frac{\partial^2 p}{\partial X^2} \right) + 3H \frac{\partial p}{\partial \phi} + \frac{\partial^2 p}{\partial X \partial \phi} \phi^3 - \frac{\partial p}{\partial \phi} = 0. $$

(6)

If $(\partial p/\partial X + \phi^2(\partial^2 p/\partial X^2)) = 0$ at some $X$, the sound speed must diverge, then the equation is singular and reduces to first-order equation which gives a constraint on the potential which must have the form: $V(\phi) = V_0/\phi^2$ [36]. As discussed in [37], this case may violates the causality.

In (2), a variety of functional forms for $V(\phi)$ and $F(X)$ have been investigated [11, 35]. Note that the sound speed, $c_s^2$, and the state parameter, $w_k$, do not depend explicitly on $V(\phi)$ in any case. Without losing generality, we take the $V(\phi)$ to be a constant which was extensively discussed in [14, 25, 28–32, 36]; in other words, we consider a purely kinetic model in which

$$p_k = F(X).$$

(7)

From (6), we get a new equation for $\phi$:

$$(F_X + 2XF_{XX})\dot{\phi} + 3HF_X \phi = 0. $$

(8)

In [36] (see also [25, 29]), a theoretical constraint on purely kinetic k-essence was obtained

$$XF_X^2 = k_0a^{-6}, $$

(9)

where $k_0$ is a constant of integration. This Equation can be rewritten as [29]

$$F_X = F_0a^{-3},$$

(10)

where $F_0$ is a constant of integration.

Like in [25], we take k-essence as unified dark matter. Then in a flat homogeneous FRW space-time, one of Einstein field equations can be written as

$$\dot{H} = -V(\phi)XF_X. $$

(11)

Inserting (9) or (10) into (11), we can have

$$\frac{dH}{d\phi} = F_1a^{-3},$$

(12)

where $F_1$ is a constant. This is an exact analytic solution for generic forms of $F(x)$. Assuming $a = a_0(t/t_0)^{\alpha}$ [38] and $\phi = \phi_0 t^\beta$ (during one certain epoch (see dust dominated era), $\alpha$ and $\beta$ can be seen as a constant), we obtain

$$\frac{dH}{d\phi} = F_2a^{-(\beta+1)/\alpha}, $$

(13)
here $\alpha > 0$, $a_0$ is the present value of the scale factor, and $F_3$ is a constant. Comparing (13) with (12), we get $\beta = 3\alpha - 1$. From (9), we can have
\[
\frac{dF}{da} = F_3 \frac{(3\alpha - 1)(3\alpha - 2)}{3\alpha} a^{-(2/\alpha)} - 1,
\]
where $F_3$ is a constant. For $\alpha = 1/3$ or $2/3$, the right side of (14) is zero, meaning that $F$ is a constant which is just zero (because $F(0) = 0$); these cases are not considered anymore in the rest of this paper. Integrating (14), we get the equation of state, $p_k$:
\[
p_k = F = (3\alpha - 1)(3\alpha - 2) k_0 a^{-(2/\alpha)},
\]
where $k_0$ is a constant. Inserting (15) into (3), (4), (5), we get, respectively,
\[
p_k = xF_k - F = -3\alpha (3\alpha - 1) k_0 a^{-(2/\alpha)},
\]
\[
w_k = -\frac{3\alpha - 2}{3\alpha},
\]
\[
c_i^2 = \frac{F_k}{xF_x} = \frac{3\alpha - 2}{3\alpha}.
\]

Consequently, we obtain a set of equations of purely kinetic k-essence without any assumption for the form of $F(x)$. To keep $p_k > 0$ at any time, we must require that if $k_0 > 0$, then $0 < \alpha < 1/3$, or, if $k_0 < 0$, then $\alpha > 1/3$. As explained above, we also require that $\alpha \neq 2/3$. An interesting feature of our result is that the sound speed, $c_i^2$, and the state parameter, $w_k$, are exactly equal, whose values only depend on the parameter $\alpha$. For any value of $\alpha$, one may always have $w_k > -1$, which means that purely kinetic k-essence does not violate the strong energy condition.

Obviously, for $\alpha \to 0$, the energy density runs to zero, but the pressure does not, so the equation of state and the sound speed diverge. This case can be considered as an unphysical state and does not violate causality [39]. The way to stop running into these divergences is if relativistic dynamics intervenes to stop the field from running to $w_k = \infty$ [40].

For $\alpha = 2/3 - \varepsilon (\varepsilon \to 0)$, one can have $p_k \to 0$ and $w_k \to 0$, which are the characteristics of the matter-dominated epoch. In this case-purely kinetic k-essence evolves asymptotically like a dark matter component. Another interesting result in this case is that the sound speed is near zero ($c_i^2 \to 0$). This property can suppress the integrated Sachs-Wolfe (ISW) effect at large angular scales [41].

When $\alpha > 2/3$, k-essence is characterized by negative pressure which means that Cosmic acceleration can only be induced after the matter-dominated epoch in our model. This transition of k-essence from positive to negative pressure is automatically triggered by dynamics.

Figure 1 shows the evolution of the equation of state parameter of k-essence $w_k$ as a function of redshift $z$ in purely kinetic k-essence cosmology and the evolution of the equation of state parameter of standard $\Lambda$CDM cosmology $w_\Lambda$ as a function of redshift $z$. Taking the present (zero redshift) value of the equation of state parameter of dark energy without a priori assumption of a flat Universe, $w_{\alpha 0} = -1.06_{-0.03}^{+0.13}$ [42], and the normalized density for matter $w_{m 0} = 0.31 \pm 0.08$ [43], we obtain $w_k(0) = -0.73$ for purely kinetic k-essence cosmology, while $w_\Lambda(0) = -0.69$ for standard $\Lambda$CDM cosmology; this difference can be tested to distinguish purely kinetic k-essence cosmology from standard $\Lambda$CDM cosmology. In this case, one may find $\alpha = 2.47$; this makes a definite prediction to the local expansion rate of our Universe and thus may be tested observationally. Any model which agrees with the SN Ia observations and has entered an accelerating phase in the recent epoch will have this feature [38]. Taking $H_0 = 72 \pm 8$ km s$^{-1}$ Mpc$^{-1}$ [44], one can easily obtain the age of the Universe which is about $33.5_{-2.8}^{+2.2}$ Gyr in purely kinetic k-essence cosmology; this result does not conflict with the stellar age bound: $t_0 > 11-12$ Gyr.

As the equation of state of k-essence takes the value of $-1/3$, the Universe is just under a transition from deceleration to acceleration of expansion, which is constrained at redshift $z_t = 0.46 \pm 0.13$ [45] or $z_t = 0.73 \pm 0.09$ [46], that is, about from 18.4 Gyr (corresponding to point a) to 25.2 Gyr (corresponding to point b) after the beginning of the Universe in purely kinetic k-essence cosmology.

From the observation of the very old quasar APM 0879 + 5255 the Universe is more than 2.1 Gyr old [47] at $z = 3.9$, which contradicts the age ($t < 1.7$ Gyr) predicted by standard $\Lambda$CDM cosmology [7]. In fact, the age of the quasar APM 0879 + 5255 is still in debate, which is dependent on the
Fe/O abundance ratio. The age of this quasar was initially estimated to be around 2.3 Gyr [48] and reevaluated to be 2.1 Gyr by using a chemodynamical model for the evolution of spheroids [47]. The age of 2.1 Gyr is set by the condition that Fe/O abundance ratio of the model reaches 3.3, which is the best-fitting value obtained in [49] (for detail see [7]). Of course, new and more reliable determination of the age of APM 08279 + 5255 are also needed. We mention in passing that when Dunlop et al. paper first came out, the 3.5 Gyr age of 53W091 at $z = 1.5$ was a significant problem for a non-$\Lambda$ cosmology [50]; however, soon thereafter papers came out that did spectral energy distribution (SED) fitting of its Keck spectrum that allowed new age estimates of around 1.8 Gyr [51, 52]. In addition, as discussed in [7], to understand the age problem better, we must be taking into account more recent observations on the value of the Hubble constant and more recent results on iron nucleosynthesis.

As seen in Figure 1, the equal-age line for 2.1 Gyr requires that $w_k(3.9) = 0.17$. Since physically $w_k \leq 0$ must be true, therefore the age of the Universe at $z = 3.9$ must be significantly older than 2.1 Gyr, in agreement with data. In fact, the equal-age line for 2.1 Gyr crosses $w_k = 0$ at $z = 5.3$, implying that astrophysical objects at the same age as the quasar APM 0879 + 5255 may be found up to $z = 5.3$, since at such high redshift the Universe should be dominated by dust-like matter, that is, $w_k \to 0$. This prediction may also be tested with high redshift observations of galaxies and quasars in the future.

We note, however, that after the dark-energy-dominated epoch, k-essence must be characterized by imaginary sound speed, this seems somewhat unreasonable. However, $c_s^2 > 0$ is not a sufficient condition for a theory to be stable [53]. When the generation of shear in the fluid is taken into account, the perturbation growth for $c_s^2 < 0$ can be stabilized [54]. This issue is worthy of further studying.

3. Conclusions and Discussions

To summarize, an equation of state of purely kinetic k-essence is obtained under a simple hypothesis but without any assumptions about the form of $F(x)$ in k-essence cosmology. This equation indicates that cosmic acceleration can only be induced after the dark-matter-dominated epoch. Because of dynamical reasons, there does not exist an epoch completely filled up by dark matter in purely kinetic k-essence cosmology. In this model of k-essence, the age of the Universe is about 33.5 Gyr now, the epoch when a transition from deceleration to acceleration expansion happened is about 18.4 to 25.2 Gyr after the beginning of the Universe, the Universe is much older than 2.1 Gyr at $z = 3.9$, and very old astrophysical objects with ages of around 2.1 Gyr exist up to redshift of 5.3. These results can be tested observationally in the future.

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References

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