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Research Article

Stability Analysis of Linear Discrete-Time Systems with Interval Delay: A Delay-Partitioning Approach

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This paper considers the problem of asymptotic stability of linear discrete-time systems with interval-like time-varying delay in the state. By using a delay partitioning-based Lyapunov functional, a new criterion for the asymptotic stability of such systems is proposed in terms of linear matrix inequalities (LMIs). The proposed stability condition depends on both the size of delay and partition size. The presented approach is compared with previously reported approaches.

1. Introduction

Mathematical models with time delays are frequently encountered in various physical, industrial, and engineering systems due to measurement and computational delays, transmission and transport lags. During the last two decades, there has emerged a considerable interest in the system theoretic problems of time-delay systems. In many applications, time delays must be taken into account in a realistic system design, for instance, chemical processes, thermal processes, echo cancellation, local loop equalization, multipath propagation in mobile communication, array signal processing, congestion analysis and control in high-speed networks, neural networks and long transmission line in pneumatic systems [1–6]. The presence of time delays may result in instability of the designed systems. An excellent survey on the stability results of time-delay systems has been presented in [7]. According to dependence of delay, the available stability criteria are generally classified into two types: delay-dependent criteria and delay-independent criteria. Delay-dependent stability criteria generally lead to less conservative results than delay-independent ones, especially if the size of time delay is small [8–12].

Many publications relating to the delay-dependent stability analysis of continuous time-delay systems have appeared (see, e.g., [4, 12–21] and the references cited therein).

In contrast, little effort has been made for studying the problem of stability of discrete time-delay systems. Utilizing Lyapunov functional method, several delay-dependent stability criteria for discrete-time systems have been reported in the literature [2, 22–27]. By employing novel techniques to estimate the forward difference of Lyapunov functional, stability criteria for linear discrete-time systems with interval-like time-varying delays have been proposed in [26]. The criteria proposed in [26] are less conservative with smaller numerical complexity than [22, 28–31]. The delay partitioning technique has been efficiently utilized in [32–36] to the stability analysis of systems with time-varying delays. Recently, by introducing free-weighting matrices and adopting the concept of delay partitioning, improved stability criteria have been established in [37]. The criteria reported in [37] are not only dependent on the delay but also dependent on the partitioning size. Though the approach in [37] provides less conservative results than [22, 29], it would lead to heavier computational burden and more complicated synthesis procedure.

Motivated by these developments, this paper studies the problem of stability analysis of linear discrete-time system with interval-like time-varying delay in the state. The paper is organized as follows. Section 2 introduces model description and preliminaries. By utilizing the delay partitioning idea of [37], a new linear-matrix-inequality- (LMI-) based criterion for the asymptotic stability of discrete-time state-delayed systems is proposed in Section 3. The proposed criterion depends on the size of delay as well as partition size. An example highlighting the usefulness of the presented criterion is given in Section 4.

2. Model Description and Preliminaries

The following notations are used throughout the paper:

$\mathbf{R}^{p\times q}$: set of $p \times q$ real matrices,

$\mathbf{R}^{p}$: set of $p \times 1$ real vectors,

$\mathbf{0}$: null matrix or null vector of appropriate dimension; the orders are specified in subscripts as the need arises,

$\mathbf{I}_p$: $p \times p$ identity matrix,

$\mathbf{B}^T$: transpose of the matrix (or vector) $\mathbf{B}$,

$\mathbf{B} > \mathbf{0}$: $\mathbf{B}$ is positive-definite symmetric matrix,

$\mathbf{B} < \mathbf{0}$: $\mathbf{B}$ is negative-definite symmetric matrix.

In this paper, we consider a linear, autonomous, multivariable discrete-time system with interval-like time-varying delay in the state. Specifically, the system under consideration is represented by the difference equation:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{A}_1\mathbf{x}(k - d(k)), \tag{2.1a}$$

$$\mathbf{x}(k) = \boldsymbol{\phi}(k), \quad k = -h_2, -h_2 + 1, \ldots, 0, \tag{2.1b}$$

where $\mathbf{x}(k) \in \mathbf{R}^n$ is the system state vector, $\mathbf{A}$ and $\mathbf{A}_1$ are constant matrices with appropriate dimensions, and $d(k)$ is a positive integer representing interval-like time-varying delay satisfying

$$1 \leq h_1 \leq d(k) \leq h_2, \tag{2.2}$$

where h_1 and h_2 are known positive integers representing the lower and upper delay bounds, respectively, and $\boldsymbol{\phi}(k)$ is an initial value at time k. Let the lower bound of the delay h_1 be divided into m number of partitions such that

$$h_1 = \tau m, \tag{2.3}$$

where τ is an integer representing partition size.

Definition 2.1. The equilibrium state $\mathbf{x}_e = \mathbf{0}$ of the system (2.1a) and (2.1b)–(2.3) is asymptotically stable if, for any $\varepsilon > 0$, there exists $\delta > 0$ such that if $\|\boldsymbol{\phi}(k)\| < \delta$, $k = -h_2, -h_2 + 1, \ldots, 0$, then $\|\mathbf{x}(k)\| < \varepsilon$, for every $k \geq 0$ and $\lim_{k\to\infty}\mathbf{x}(k) = \mathbf{0}$.

Lemma 2.2 (see [30]). *For any positive definite matrix $\mathbf{W} \in \mathbf{R}^{n\times n}$, two positive integers r and r_0 satisfying $r \geq r_0 \geq 1$, and vector function $\mathbf{x}(i) \in \mathbf{R}^n$, one has*

$$\left(\sum_{i=r_0}^{r}\mathbf{x}(i)\right)^T \mathbf{W}\left(\sum_{i=r_0}^{r}\mathbf{x}(i)\right) \leq (r - r_0 + 1)\sum_{i=r_0}^{r}\mathbf{x}^T(i)\ \mathbf{W}\mathbf{x}(i). \tag{2.4}$$

3. Main Result

In this section, an LMI-based criterion for the asymptotic stability of system (2.1a) and (2.1b)–(2.3) is established. The main result may be stated as follows.

Theorem 3.1. *For given positive integers τ, m, and h_2, the system described by* (2.1a) *and* (2.1b)–(2.3) *is asymptotically stable if there exist real matrices $\mathbf{P} = \mathbf{P}^T > \mathbf{0}$, $\mathbf{Q}_i = \mathbf{Q}_i{}^T > \mathbf{0}$ $(i = 1,2,3)$, $\mathbf{Z}_i = \mathbf{Z}_i^T > \mathbf{0}$ $(i = 1,2)$, $\mathbf{X}$, $\mathbf{Y}$ and $\mathbf{W}$ such that*

$$\begin{bmatrix} -\mathbf{Z}_2 & \mathbf{Y}^T & \mathbf{X} \\ \mathbf{Y} & \boldsymbol{\Psi} & \mathbf{W} \\ \mathbf{X}^T & \mathbf{W}^T & -\mathbf{Z}_2 \end{bmatrix} < \mathbf{0}, \tag{3.1}$$

where

$$\begin{aligned}\boldsymbol{\Psi} = {} & \boldsymbol{\Lambda}_1^T\boldsymbol{\Phi}_1\boldsymbol{\Lambda}_1 + \boldsymbol{\Lambda}_2^T\boldsymbol{\Phi}_2\boldsymbol{\Lambda}_2 + \boldsymbol{\Lambda}_3^T\boldsymbol{\Phi}_3\boldsymbol{\Lambda}_3 - \boldsymbol{\Lambda}_4^T\mathbf{Q}_2\boldsymbol{\Lambda}_4 + \boldsymbol{\Lambda}_1^T\boldsymbol{\Phi}_4\boldsymbol{\Lambda}_3 + \boldsymbol{\Lambda}_3^T\boldsymbol{\Phi}_4^T\boldsymbol{\Lambda}_1 - \boldsymbol{\Lambda}_5^T\mathbf{Z}_1\boldsymbol{\Lambda}_5 \\ & + \boldsymbol{\Lambda}_1^T\mathbf{Z}_1\boldsymbol{\Lambda}_5 + \boldsymbol{\Lambda}_5^T\mathbf{Z}_1\boldsymbol{\Lambda}_1 + \begin{bmatrix}\mathbf{0} & \mathbf{Y} & -\mathbf{Y}+\mathbf{W} & -\mathbf{W}\end{bmatrix} + \begin{bmatrix}\mathbf{0} & \mathbf{Y} & -\mathbf{Y}+\mathbf{W} & -\mathbf{W}\end{bmatrix}^T, \textit{where}\end{aligned}$$

$$\boldsymbol{\Lambda}_1 = \begin{bmatrix}\mathbf{I}_n & \mathbf{0}_{n\times(m+2)n}\end{bmatrix},$$

$$\begin{aligned}
&\mathbf{\Lambda}_2 = \begin{bmatrix} \mathbf{I}_{mn} & \mathbf{0}_{mn\times 3n} & \\ \mathbf{0}_{mn\times n} & \mathbf{I}_{mn} & \mathbf{0}_{mn\times 2n} \end{bmatrix},\\
&\mathbf{\Lambda}_3 = \begin{bmatrix} \mathbf{0}_{n\times(m+1)n} & \mathbf{I}_n & \mathbf{0}_n \end{bmatrix},\\
&\mathbf{\Lambda}_4 = \begin{bmatrix} \mathbf{0}_{n\times(m+2)n} & \mathbf{I}_n \end{bmatrix},\\
&\mathbf{\Lambda}_5 = \begin{bmatrix} \mathbf{0}_n & \mathbf{I}_n & \mathbf{0}_{n\times(m+1)n} \end{bmatrix},\\
&\mathbf{\Phi}_1 = \mathbf{A}^T\mathbf{P}\mathbf{A} - \mathbf{P} + \mathbf{Q}_2 + (h_2 - \tau m + 1)\mathbf{Q}_3 - \mathbf{Z}_1 + (\mathbf{A} - \mathbf{I}_n)^T\left(\tau^2\mathbf{Z}_1 + (h_2 - \tau m)^2\mathbf{Z}_2\right)(\mathbf{A} - \mathbf{I}_n),\\
&\mathbf{\Phi}_2 = \begin{bmatrix} \mathbf{Q}_1 & \mathbf{0} \\ \mathbf{0} & -\mathbf{Q}_1 \end{bmatrix},\\
&\mathbf{\Phi}_3 = \mathbf{A}_1^T\left(\mathbf{P} + \tau^2\mathbf{Z}_1 + (h_2 - \tau m)^2\mathbf{Z}_2\right)\mathbf{A}_1 - \mathbf{Q}_3,\\
&\mathbf{\Phi}_4 = \mathbf{A}^T\mathbf{P}\mathbf{A}_1 + (\mathbf{A} - \mathbf{I}_n)^T\left(\tau^2\mathbf{Z}_1 + (h_2 - \tau m)^2\mathbf{Z}_2\right)\mathbf{A}_1.
\end{aligned} \tag{3.2}$$

Proof. Choose a Lyapunov functional candidate as

$$\begin{aligned}
V(k) = {}& \mathbf{x}^T(k)\mathbf{P}\mathbf{x}(k) + \sum_{i=k-\tau}^{k-1} \mathbf{\Upsilon}^T(i)\mathbf{Q}_1\mathbf{\Upsilon}(i) + \sum_{i=k-h_2}^{k-1} \mathbf{x}^T(i)\mathbf{Q}_2\mathbf{x}(i) + \sum_{j=-h_2}^{-\tau m}\sum_{i=k+j}^{k-1} \mathbf{x}^T(i)\mathbf{Q}_3\mathbf{x}(i)\\
&+ \sum_{j=-\tau}^{-1}\sum_{i=k+j}^{k-1} \tau\Delta\mathbf{x}^T(i)\mathbf{Z}_1\Delta\mathbf{x}(i) + \sum_{j=-h_2}^{-\tau m-1}\sum_{i=k+j}^{k-1} (h_2 - \tau m)\Delta\mathbf{x}^T(i)\mathbf{Z}_2\Delta\mathbf{x}(i),
\end{aligned} \tag{3.3}$$

where

$$\mathbf{\Upsilon}(i) = \begin{bmatrix} \mathbf{x}^T(i) & \mathbf{x}^T(i-\tau) & \cdots & \mathbf{x}^T(i-(m-1)\tau) \end{bmatrix}^T, \tag{3.4}$$

$$\Delta\mathbf{x}(i) = \mathbf{x}(i+1) - \mathbf{x}(i). \tag{3.5}$$

Taking the forward difference of (3.3) along the solution of (2.1a) and (2.1b), we have

$$\begin{aligned}
\Delta V(k) &= V(k+1) - V(k)\\
&= [\mathbf{A}\mathbf{x}(k) + \mathbf{A}_1\mathbf{x}(k-d(k))]^T\mathbf{P}[\mathbf{A}\mathbf{x}(k) + \mathbf{A}_1\mathbf{x}(k-d(k))] - \mathbf{x}^T(k)\mathbf{P}\mathbf{x}(k)\\
&\quad + \mathbf{\Upsilon}^T(k)\mathbf{Q}_1\mathbf{\Upsilon}(k) - \mathbf{\Upsilon}^T(k-\tau)\mathbf{Q}_1\mathbf{\Upsilon}(k-\tau) + \mathbf{x}^T(k)\mathbf{Q}_2\mathbf{x}(k)\\
&\quad - \mathbf{x}^T(k-h_2)\mathbf{Q}_2\mathbf{x}(k-h_2) + (h_2 - \tau m + 1)\mathbf{x}^T(k)\mathbf{Q}_3\mathbf{x}(k)
\end{aligned}$$

$$
\begin{aligned}
&- \sum_{i=k-h_2}^{k-\tau m} \mathbf{x}^T(i)\mathbf{Q}_3\mathbf{x}(i) + \Delta\mathbf{x}^T(k)\left(\tau^2\mathbf{Z}_1 + (h_2-\tau m)^2\mathbf{Z}_2\right)\Delta\mathbf{x}(k) \\
&- \sum_{i=k-\tau}^{k-1} \tau\Delta\mathbf{x}^T(i)\mathbf{Z}_1\Delta\mathbf{x}(i) - \sum_{i=k-h_2}^{k-\tau m-1}(h_2-\tau m)\Delta\mathbf{x}^T(i)\mathbf{Z}_2\Delta\mathbf{x}(i).
\end{aligned} \tag{3.6}
$$

Using Lemma 2.2, we obtain

$$
-\sum_{i=k-\tau}^{k-1} \tau\Delta\mathbf{x}^T(i)\mathbf{Z}_1\Delta\mathbf{x}(i) \leq -(\mathbf{x}(k)-\mathbf{x}(k-\tau))^T\mathbf{Z}_1(\mathbf{x}(k)-\mathbf{x}(k-\tau)). \tag{3.7}
$$

Note that

$$
-\sum_{i=k-h_2}^{k-\tau m} \mathbf{x}^T(i)\mathbf{Q}_3\mathbf{x}(i) \leq -\mathbf{x}^T(k-d(k))\mathbf{Q}_3\mathbf{x}(k-d(k)). \tag{3.8}
$$

Now, we have the following relations:

$$
\begin{aligned}
0 &= 2\zeta^T(k)\mathbf{Y}\left[\mathbf{x}(k-\tau m) - \mathbf{x}(k-d(k)) - \sum_{i=k-d(k)}^{k-\tau m-1}\Delta\mathbf{x}(i)\right], \\
0 &= 2\zeta^T(k)\mathbf{W}\left[\mathbf{x}(k-d(k)) - \mathbf{x}(k-h_2) - \sum_{i=k-h_2}^{k-d(k)-1}\Delta\mathbf{x}(i)\right],
\end{aligned} \tag{3.9}
$$

where $\mathbf{Y}$ and $\mathbf{W}$ are constant matrices of appropriate dimensions and

$$
\zeta(k) = \left[\Upsilon^T(k) \quad \mathbf{x}^T(k-\tau m) \quad \mathbf{x}^T(k-d(k)) \quad \mathbf{x}^T(k-h_2)\right]^T. \tag{3.10}
$$

It follows from (2.1a) and (3.5) that

$$
\Delta\mathbf{x}(k) = (\mathbf{A}-\mathbf{I}_n)\mathbf{x}(k) + \mathbf{A}_1\mathbf{x}(k-d(k)). \tag{3.11}
$$

Employing (3.6)–(3.11), we have the following inequality:

$$
\begin{aligned}
\Delta V(k) \leq{}& \zeta^T(k)\mathbf{\Psi}\zeta(k) - 2\zeta^T(k)\mathbf{Y}\sum_{i=k-d(k)}^{k-\tau m-1}\Delta\mathbf{x}(i) - 2\zeta^T(k)\mathbf{W}\sum_{i=k-h_2}^{k-d(k)-1}\Delta\mathbf{x}(i) \\
&- \sum_{i=k-d(k)}^{k-\tau m-1}(h_2-\tau m)\ \Delta\mathbf{x}^T(i)\mathbf{Z}_2\Delta\mathbf{x}(i) - \sum_{i=k-h_2}^{k-d(k)-1}(h_2-\tau m)\ \Delta\mathbf{x}^T(i)\mathbf{Z}_2\Delta\mathbf{x}(i),
\end{aligned} \tag{3.12}
$$

where $\boldsymbol{\Psi}$ is given by (3.2). Equation (3.12) can also be rearranged as

$$\Delta V(k) \leq \frac{1}{(h_2-\tau m)} \sum_{i=k-d(k)}^{k-\tau m-1} \begin{bmatrix} \zeta(k) \\ -(h_2-\tau m)\Delta \mathbf{x}(i) \end{bmatrix}^T \begin{bmatrix} \boldsymbol{\Psi} & \mathbf{Y} \\ \mathbf{Y}^T & -\mathbf{Z}_2 \end{bmatrix} \begin{bmatrix} \zeta(k) \\ -(h_2-\tau m)\Delta \mathbf{x}(i) \end{bmatrix} + \frac{1}{(h_2-\tau m)} \sum_{i=k-h_2}^{k-d(k)-1} \begin{bmatrix} \zeta(k) \\ -(h_2-\tau m)\Delta \mathbf{x}(i) \end{bmatrix}^T \begin{bmatrix} \boldsymbol{\Psi} & \mathbf{W} \\ \mathbf{W}^T & -\mathbf{Z}_2 \end{bmatrix} \begin{bmatrix} \zeta(k) \\ -(h_2-\tau m)\Delta \mathbf{x}(i) \end{bmatrix}. \tag{3.13}$$

Using [17, Lemma 4.1], it can be shown that there exists a matrix $\mathbf{X}$ of appropriate dimensions satisfying (3.1) if and only if

$$\begin{bmatrix} \boldsymbol{\Psi} & \mathbf{Y} \\ \mathbf{Y}^T & -\mathbf{Z}_2 \end{bmatrix} < \mathbf{0}, \qquad \begin{bmatrix} \boldsymbol{\Psi} & \mathbf{W} \\ \mathbf{W}^T & -\mathbf{Z}_2 \end{bmatrix} < \mathbf{0}, \tag{3.14}$$

which together with (3.13) implies $\Delta V(k) < 0$ for all nonzero $\zeta(k)$. This completes the proof. □

Remark 3.2. A comparison of the number of the decision variables involved in several recent stability results is summarized in Table 1. It may be observed that the size of complexity in [22, 26, 31] is only related to state dimension n, whereas the complexity of [37] and Theorem 3.1 depends on both n and m.

The total number of scalar decision variables of Theorem 3.1 is $D_1 = (n/2)[n(m^2+4m+19)+(m+5)]$, and the total row size of the LMIs is $L_1 = 2n(m+5)$. The numerical complexity of Theorem 3.1 is proportional to $L_1 D_1^3$ [16]. In [37], the total number of scalar decision variables of Theorem 2 is $D_2 = (n/2)[n(3m^2+18m+41)+(3m+11)]$, the total row size of the LMIs is $L_2 = n(7m+31)$, and the numerical complexity is proportional to $L_2 D_2^3$. Therefore, Theorem 3.1 has much smaller numerical complexity than Theorem 2 of [37].

Remark 3.3. For a given h_1, the allowable maximum value of h_2 for guaranteeing the asymptotic stability of system (2.1a) and (2.1b)–(2.3) can be obtained by iteratively solving the LMI (3.1).

Remark 3.4. With $m = 1$, $\mathbf{Y} = [\mathbf{Y}_1^T \quad \mathbf{Y}_2^T \quad \mathbf{0}_{n\times 2n}]^T$, and $\mathbf{W} = [\mathbf{W}_1^T \quad \mathbf{W}_2^T \quad \mathbf{0}_{n\times 2n}]^T$, Theorem 3.1 reduces to an equivalent form of [26, Proposition 1]. Thus, as compared to [26, Proposition 1], Theorem 3.1 provides additional degrees of freedom in the selection of m, $\mathbf{Y}$, and $\mathbf{W}$ which would result in an enhanced stability region in the parameter space.

Remark 3.5. Using similar steps as in the proof of [37, Proposition 8], it is easy to establish that the conservatism of the stability result obtained via Theorem 3.1 is nonincreasing as the number of partitions increases.

It may be noted that, in the derivation of Theorem 3.1, the lower bound of the delay is assumed to be $h_1 = \tau m \geq 1$. For the situation where $h_1 = 0$, we have the following result.

Table 1: Comparison of the number of decision variables involved in various methods.

Methods	Number of the decision variables
Theorem 1 in [22]	$(23n^2 + 5n)/2$
Theorem 3 in [22]	$(67n^2 + 9n)/2$
Theorem 1 in [31]	$9n^2 + 3n$
Theorem 1 in [23]	$13n^2 + 5n$
Proposition 1 in [26]	$8n^2 + 3n$
Theorem 2 in [37]	$(n/2)[n(3m^2 + 18m + 41) + (3m + 11)]$
Theorem 3.1	$(n/2)[n(m^2 + 4m + 19) + (m + 5)]$

Theorem 3.6. *The system* (2.1a) *and* (2.1b) *with* $0 \le d(k) \le h_2$ *is asymptotically stable if there exist real matrices* $\mathbf{P} = \mathbf{P}^T > \mathbf{0}$, $\mathbf{Q}_i = \mathbf{Q}_i^T > \mathbf{0}$ $(i = 1, 2)$, $\mathbf{Z} = \mathbf{Z}^T > \mathbf{0}$, $\mathbf{X}$, $\widetilde{\mathbf{Y}}$, *and* $\widetilde{\mathbf{W}}$ *such that*

$$\begin{bmatrix} -\mathbf{Z} & \widetilde{\mathbf{Y}}^T & \mathbf{X} \\ \widetilde{\mathbf{Y}} & \widetilde{\boldsymbol{\Psi}} & \widetilde{\mathbf{W}} \\ \mathbf{X}^T & \widetilde{\mathbf{W}}^T & -\mathbf{Z} \end{bmatrix} < \mathbf{0}, \tag{3.15}$$

where

$$\begin{aligned} \widetilde{\boldsymbol{\Psi}} &= \widetilde{\boldsymbol{\Lambda}}_1^T \widetilde{\boldsymbol{\Phi}}_1 \widetilde{\boldsymbol{\Lambda}}_1 + \widetilde{\boldsymbol{\Lambda}}_1^T \widetilde{\boldsymbol{\Phi}}_2 \widetilde{\boldsymbol{\Lambda}}_2 + \widetilde{\boldsymbol{\Lambda}}_2^T \widetilde{\boldsymbol{\Phi}}_2^T \widetilde{\boldsymbol{\Lambda}}_1 + \widetilde{\boldsymbol{\Lambda}}_2^T \widetilde{\boldsymbol{\Phi}}_3 \widetilde{\boldsymbol{\Lambda}}_2 - \widetilde{\boldsymbol{\Lambda}}_3^T \mathbf{Q}_1 \widetilde{\boldsymbol{\Lambda}}_3 \\ &\quad + \begin{bmatrix} \widetilde{\mathbf{Y}} & -\widetilde{\mathbf{Y}} + \widetilde{\mathbf{W}} & -\widetilde{\mathbf{W}} \end{bmatrix} + \begin{bmatrix} \widetilde{\mathbf{Y}} & -\widetilde{\mathbf{Y}} + \widetilde{\mathbf{W}} & -\widetilde{\mathbf{W}} \end{bmatrix}^T, \ \textit{where} \\ \widetilde{\boldsymbol{\Lambda}}_1 &= \begin{bmatrix} \mathbf{I}_n & \mathbf{0}_{n\times 2n} \end{bmatrix}, \\ \widetilde{\boldsymbol{\Lambda}}_2 &= \begin{bmatrix} \mathbf{0}_n & \mathbf{I}_n & \mathbf{0}_n \end{bmatrix}, \\ \widetilde{\boldsymbol{\Lambda}}_3 &= \begin{bmatrix} \mathbf{0}_{n\times 2n} & \mathbf{I}_n \end{bmatrix}, \\ \widetilde{\boldsymbol{\Phi}}_1 &= \mathbf{A}^T \mathbf{P} \mathbf{A} - \mathbf{P} + {h_2}^2 (\mathbf{A} - \mathbf{I}_n)^T \mathbf{Z} (\mathbf{A} - \mathbf{I}_n) + \mathbf{Q}_1 + (h_2 + 1)\mathbf{Q}_2, \\ \widetilde{\boldsymbol{\Phi}}_2 &= \mathbf{A}^T \mathbf{P} \mathbf{A}_1 + {h_2}^2 (\mathbf{A} - \mathbf{I}_n)^T \mathbf{Z} \mathbf{A}_1, \\ \widetilde{\boldsymbol{\Phi}}_3 &= {\mathbf{A}_1}^T \left(\mathbf{P} + {h_2}^2 \mathbf{Z}\right) \mathbf{A}_1 - \mathbf{Q}_2. \end{aligned} \tag{3.16}$$

Proof. Choosing the Lyapunov functional as

$$\begin{aligned} V(k) &= \mathbf{x}^T(k) \mathbf{P} \mathbf{x}(k) + \sum_{i=k-h_2}^{k-1} \mathbf{x}^T(i) \mathbf{Q}_1 \mathbf{x}(i) \\ &\quad + \sum_{j=-h_2}^{0} \sum_{i=k+j}^{k-1} \mathbf{x}^T(i) \mathbf{Q}_2 \mathbf{x}(i) + \sum_{j=-h_2}^{-1} \sum_{i=k+j}^{k-1} h_2 \, \Delta \mathbf{x}^T(i) \mathbf{Z} \Delta \mathbf{x}(i) \end{aligned} \tag{3.17}$$

and employing the steps of the proof of Theorem 3.1 discussed earlier, one can easily arrive at the above theorem. The details of the proof of Theorem 3.6 are, therefore, omitted. □

Table 2: Admissible upper bound of h_2 for various h_1.

Method	$h_1 = 4$	$h_1 = 12$
Theorem 1 in [29]	8	13
Theorem 1 in [22]	13	16
Theorem 1 in [23]	17	21
Proposition 1 in [26]	17	21
Theorem 2 in [37]	17 ($m = 2, \tau = 2$) 18 ($m = 4, \tau = 1$)	21 ($m = 1, \tau = 12$) 22 ($m = 2, \tau = 6$)
Theorem 3.1	17 ($m = 2, \tau = 2$) 18 ($m = 4, \tau = 1$)	21 ($m = 1, \tau = 12$) 22 ($m = 2, \tau = 6$)

Remark 3.7. With $\widetilde{L}_1 = 9n$ rows and $\widetilde{D}_1 = 9n^2 + 2n$ scalar decision variables, LMI (3.15) in Theorem 3.6 has the numerical complexity proportional to $\widetilde{L}_1\widetilde{D}_1^3$ [16], while the condition in [37, Proposition 10] has the numerical complexity proportional to $\widetilde{L}_2\widetilde{D}_2^3$, where $\widetilde{L}_2 = 21n$ and $\widetilde{D}_2 = (n/2)(25n + 7)$. So, Theorem 3.6 has smaller numerical complexity than [37, Proposition 10].

4. A Numerical Example

To demonstrate the applicability of the presented results and compare them with previous results, we now consider a specific example of system (2.1a), (2.1b) with

$$\mathbf{A} = \begin{bmatrix} 0.8 & 0 \\ 0.05 & 0.9 \end{bmatrix}, \qquad \mathbf{A}_1 = \begin{bmatrix} -0.1 & 0 \\ -0.2 & -0.1 \end{bmatrix} \tag{4.1}$$

and $0 \leq d(k) \leq h_2$. This example has been considered in [22, 29, 37]. Using Matlab LMI toolbox [38, 39], it is found from Theorem 3.6 that the present system is stable for $0 \leq d(k) \leq 17$. In this case, the number of decision variables involved in Theorem 3.6 is 40. On the other hand, to obtain the upper bound $h_2 = 17$, [37, Proposition 10] requires 57 decision variables. This demonstrates the numerical efficiency of the proposed method.

Next, consider the system described by (2.1a), (2.1b)–(2.3), and (4.1). For different values of h_1, the admissible upper bound h_2 is listed in Table 2. From Table 2, it is clear that Theorem 3.1 can provide a larger upper bound h_2 than the previously reported stability results [22, 23, 26, 29]. It may also be observed that, for the example under consideration, Theorem 3.1 leads to upper bound h_2 which is identical to that arrived at via Theorem 2 in [37]. However, as discussed in Remark 3.2, Theorem 3.1 has much smaller numerical complexity than Theorem 2 in [37].

5. Conclusion

In this paper, the problem concerning the asymptotic stability of linear discrete-time systems with interval-like time-varying delay in the state has been considered. Using the concept of delay partitioning, an LMI-based criterion for the asymptotic stability of such systems has been established. The criterion depends on the size of delay as well as partition size. The presented approach may imply asymptotic stability for a broader class of time-varying state-delayed systems, as compared to previous approaches [22, 23, 26, 29]. The presented criterion is numerically less complex than [37]. The stability results discussed in this paper can easily be extended to delayed discrete-time systems with parameter uncertainties.

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