Research Article

Hopf Bifurcation Analysis in a Tabu Learning Neuron Model with Two Delays

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We consider the nonlinear dynamical behavior of a tabu learning neuron model with two delays. By choosing the sum of the two delays as a bifurcation parameter, we prove that Hopf bifurcation occurs in the neuron. Some numerical examples are also presented to verify the theoretical analysis.

1. Introduction

Starting with the work of Hopfield [1] on neural networks, the dynamical behaviors (including stability, instability, periodic oscillatory, bifurcation, and chaos) of the continuous-time neural networks have received increasing interest due to their promising potential applications in many fields, such as signal processing, pattern recognition, optimization, and associative memories.

Tabu learning [2, 3] applies the concept of tabu search [4–7] to neural networks for solving optimization problems. By continuously increasing the energy surface in a neighborhood of the current network state, it penalizes those states that have already been visited. This enables the state trajectory to climb out of local minima while tending toward those areas that have not yet been visited, thus performing an efficient search through the problem's solution space. Note that, unlike most existing neural-network-based methods for optimization, the goal of the tabu learning method is not to force the network state to converge to an optimal or nearly optimal solution, but rather, the network conducts an efficient search through the solution space. As pointed out in [8], it is very natural for one to ask what the state trajectory of the tabu learning neural network is like. In [8], the authors found and analyzed the Hopf bifurcation behavior in a tabu learning single-neuron model and also found chaotic behaviors in a tabu learning two-neuron model with both linear and

quadratic proximity functions. In [9], the authors found and analyzed the Hopf bifurcation behavior in a discrete-time tabu learning single-neuron model.

It is well known that there exist time delays in the information processing of neurons. The delayed axonal signal transmissions in the neural network models make the dynamical behaviors become more complicated and may destabilize the stable equilibria and admit periodic oscillation, bifurcation, and chaos. Therefore, the delay is an important control parameter in living nervous system: different ranges of delays correspond to different patterns of neural activities (see, e.g., [10, 11]).

In the present paper, we study the bifurcation behavior of a tabu learning neuron models with two delays. By choosing the sum of the two delays as a bifurcation parameter, we prove that Hopf bifurcation occurs in the neuron.

The organization of this paper is as follows. In the next section, we present the tabu learning neuron model with two delays that will be studied in this paper. In Section 3, we shall study the stability and the existence of bifurcation. To verify the theoretic analysis, numerical simulations are given in Section 4.

2. The Tabu Learning Neuron Model with Two Delays

Recently, Li et al. [8] have studied the following tabu learning single neuron model:

$$\begin{aligned} u'(t) &= -u(t) + af(u(t)) + J(t), \\ J'(t) &= -\alpha J(t) - \beta f(u(t)). \end{aligned} \tag{2.1}$$

u is the state of neuron, where $f(\cdot)$ is the activation function and a represents the strength of the connection from neuron to itself, and a cannot be a negative value in Hopfield-type neural network. J satisfied the second equation of system (2.1) which is a learning equation, where α is the memory decay rate and β is the learning rate. Both α and β are positive constants. In the tabu learning model, α, β should be carefully chosen [12] for they heavily influence the behavior of system (2.1).

In order to describe the model more clearly, we introduce two time delays in model (2.1)

$$\begin{aligned} u'(t) &= -u(t) + af(u(t)) + J(t - \tau_1), \\ J'(t) &= -\alpha J(t) - \beta f(u(t - \tau_2)). \end{aligned} \tag{2.2}$$

In the next section, we first consider the stability of equilibrium and then study the existence and conditions of bifurcation at the equilibrium by choosing $\tau = \tau_1 + \tau_2$ as the bifurcation parameter for system (2.2).

3. Stability and Existence of Hopf Bifurcation

For most of the models in the literature, including the ones in [13–15], the activation function f is $f(u) = \tan h(cu)$. However, we only make the following assumption on function f :

$$(H) \quad f \in C^1(R) \text{ and } f(0) = 0.$$

Clearly, $(u, J)^T = (0, 0)^T$ is equilibrium of system (2.2). Linearization of (2.2) at the zero equilibrium yields

$$\begin{aligned} u'(t) &= -u(t) + af'(0)u(t) + J(t - \tau_1), \\ J'(t) &= -\alpha J(t) - \beta f'(0)u(t - \tau_2), \end{aligned} \quad (3.1)$$

whose characteristic equation is

$$\lambda^2 + [\alpha + 1 - af'(0)]\lambda + [\alpha - \alpha af'(0)] + [\beta f'(0)]e^{-\lambda\tau} = 0, \quad (3.2)$$

where $\tau = \tau_1 + \tau_2$.

In [16, 17], the second-degree transcendental polynomial equation

$$\lambda^2 + p\lambda + r + (s\lambda + q)e^{-\lambda\tau} = 0 \quad (3.3)$$

has been extensively studied. In particular, we introduce the following results stated in [17] about the distributions of the roots of the characteristic (3.3).

Let

- (H_1) $p + s > 0$,
- (H_2) $q + r > 0$,
- (H_3) either $s^2 - p^2 + 2r < 0$ and $r^2 - q^2 > 0$ or $(s^2 - p^2 + 2r)^2 < 4(r^2 - q^2)$,
- (H_4) either $r^2 - q^2 < 0$ or $s^2 - p^2 + 2r > 0$ and $(s^2 - p^2 + 2r)^2 = 4(r^2 - q^2)$,
- (H_5) $r^2 - q^2 > 0$, $s^2 - p^2 + 2r > 0$ and $(s^2 - p^2 + 2r)^2 > 4(r^2 - q^2)$.

Lemma 3.1 (see [17]). *For (3.3), we have the following*

- (1) if (H_1) – (H_3) hold, then all roots of (3.3) have negative real parts for all $\tau \geq 0$;
- (2) if (H_1) , (H_2) , and (H_4) hold and $\tau = \tau_j^+$, then (3.3) has a pair of purely imaginary roots $\pm i\omega_+$. When $\tau = \tau_0^+$, then all roots of (3.3) except $\pm i\omega_+$ have negative real parts;
- (3) if (H_1) , (H_2) , and (H_5) hold and $\tau = \tau_j^+$ ($\tau = \tau_j^-$, resp.), then (3.3) has a pair of purely imaginary roots $\pm i\omega_+$ ($\pm i\omega_-$, resp.). Furthermore, when $\tau = \tau_j^+$ ($\tau = \tau_j^-$, resp.), then all roots of (3.3) except $\pm i\omega_+$ ($\pm i\omega_-$, resp.) have negative real parts.

Here,

$$\begin{aligned} \omega_{\pm} &= \frac{\sqrt{2}}{2} \left[s^2 - p^2 + 2r \pm \sqrt{(s^2 - p^2 + 2r)^2 - 4(r^2 - q^2)} \right]^{(1/2)}, \\ \tau_j^{\pm} &= \frac{1}{\omega_{\pm}} \arccos \left[\frac{q(\omega_{\pm}^2 - r) - ps\omega_{\pm}^2}{s^2\omega_{\pm}^2 + q^2} \right] + \frac{2j\pi}{\omega_{\pm}}, \quad j = 0, 1, 2, \dots \end{aligned} \quad (3.4)$$

From Lemma 3.1, the following theorem about stability and Hopf bifurcation of (2.2) are immediately obtained.

Theorem 3.2. For System (2.2), we have the following

- (1) if $\alpha + 1 - af'(0) > 0$ and $\alpha - \alpha\alpha f'(0) - \beta f'(0) > 0$, then the equilibrium $(0, 0)$ of the system (2.2) is asymptotically stable for all $\tau \geq 0$;
- (2) if $\alpha + 1 - af'(0) > 0$, $\alpha - \alpha\alpha f'(0) + \beta f'(0) > 0$, and $\alpha - \alpha\alpha f'(0) - \beta f'(0) < 0$, then $(0, 0)$ is asymptotically stable when $\tau \in [0, \tau_0^+)$ and unstable when $\tau > \tau_0^+$. System (2.2) undergoes Hopf bifurcations at $(0, 0)$ when $\tau = \tau_j^+$.

Here

$$\begin{aligned} \tau_j^+ &= \frac{1}{\omega_+} \arccos \left[\frac{\omega_+^2 - r}{\beta f'(0)} \right] + \frac{2j\pi}{\omega_+}, \quad j = 0, 1, 2, \dots, \\ \omega_+ &= \frac{\sqrt{2}}{2} \left\{ -\alpha^2 - 1 - af'(0) \right\} \\ &\quad + \sqrt{[-\alpha^2 - (1 - af'(0))]^2 - 4[(\alpha - \alpha\alpha f'(0))^2 - (\beta f'(0))^2]}^{(1/2)}. \end{aligned} \quad (3.5)$$

Corollary 3.3. When we set the activation function $f(u) = \tan h(u)$, notice that $f'(0) = 1$, then from Theorem 3.2 we get the following

- (1) if $0 < a < 1$ and $0 < \beta < \alpha(1 - a)$, then the equilibrium $(0, 0)$ of the system (2.2) is asymptotically stable for all $\tau \geq 0$;
- (2) if either $0 < a < 1$ and $\beta > \alpha(1 - a)$ or $a > 1$, $\alpha > a - 1$ and $\beta > \alpha(a - 1)$, then $(0, 0)$ is asymptotically stable when $\tau \in [0, \tau_0^+)$ and unstable when $\tau > \tau_0^+$. System (2.2) undergoes Hopf bifurcations at $(0, 0)$ when $\tau = \tau_j^+$.

Here τ_j^+ and ω_+ are the same as (3.5).

4. Numerical Simulation

In this section, we give an example of system (2.2) with $a = 1.5$, $\alpha = 0.8$, $\beta = 0.5$ and $f(\cdot) = \tan h(\cdot)$. Then $f(0) = 0$ and $f'(0) = 1$. The system (2.2) becomes

$$\begin{aligned} u'(t) &= -u(t) + 1.5 \tan h(u(t)) + J(t - \tau_1), \\ J'(t) &= -0.8J(t) - 0.5 \tan h(u(t - \tau_2)). \end{aligned} \quad (4.1)$$

In system (4.1), the case (2) of Corollary 3.3 is satisfied.

From the formulae (3.5), it follows that $\tau_0^+ = 0.6$. Thus, equilibrium $(0, 0)$ is stable when $\tau \in [0, \tau_0^+)$. Choose $\tau_1 = 0.1$, $\tau_2 = 0.3$; then $\tau_1 + \tau_2 < 0.6$. Figure 1(a)(A1)–(A3) shows that the origin is asymptotically stable. When τ passes through the critical value, τ_0^+ , $(0, 0)$ loses its stability and a Hopf bifurcation occurs. Choose $\tau_1 = 0.5$, $\tau_2 = 0.2$; then $\tau_1 + \tau_2 > 0.6$. Figure 1(b)(B1)–(B3) shows that a family of periodic solutions bifurcate from $(0, 0)$.

Figure 1: (a) (A1)–(A3): $\tau_1 + \tau_2 < 0.6$; (b) (B1)–(B3): $\tau_1 + \tau_2 > 0.6$.

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