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Research Article

Strategic Sensors and Regional Exponential Observability

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The aim of this paper is to introduce the concept of regional exponential observability in connection with the strategic sensors. Then, we give characterization of such sensors in order that regional exponential observability can be achieved. The obtained results are applied to two-dimensional systems, and various cases of sensors are considered. We also show that there exists a dynamical system for diffusion system which is not exponentially observable in the usual sense but it may be regionally exponentially observable.

1. Introduction

In system theory, the observability is related to the possibility of reconstruction of the state from the knowledge of system dynamics and the output [1–4]. The notion of regional analysis was extended by El Jai et al. [5, 6]. The study of this notion is motivated by certain concrete-real problem, in thermic, mechanic environment [7–9]. If a system is defined on a domain Ω and represented by the model as in (Figure 1), then we are interested in the regional state on ω of the domain Ω.

The concept of regional asymptotic analysis was introduced recently by Al-Saphory and El Jai in [10–12], consisting in studying the behaviour of the system not in all the domain Ω but only on particular region ω of the domain.

The purpose of this paper is to give some results related to the link between regional exponential observability and strategic sensors. We consider a class of distributed system and we explore various results connected with the different types of measurements, domains, and boundary conditions.

The paper is organized as follows. Section 2 devotes to the introduction of exponential regional observability problem. We give the formulation problem and preliminaries. We need

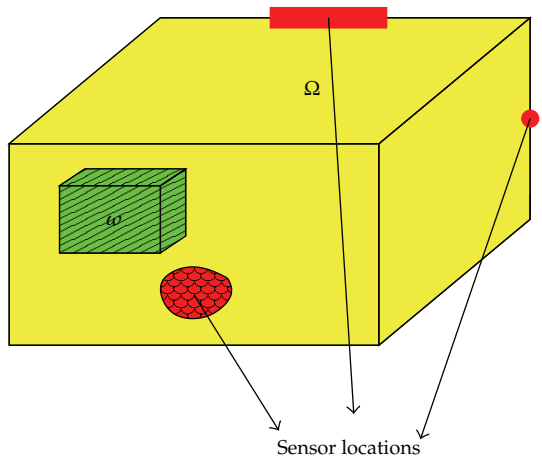

Figure 1: The domain of Ω, the subregion ω, and the sensors locations.

some notions concerning the exponential behaviour (ω-strategic sensor, ω-detectability, and ω-observer). Section 3 is related to the characterization notion of ω_E-observable by the use of strategic sensors. In Section 4, we illustrate applications with many situations of sensor locations.

2. Regional Exponential Observability

2.1. Problem Statement

Let Ω be an open bounded subset of R^n, with boundary $\partial\Omega$ and let $[0,T]$, $T > 0$ be a time measurement interval. Suppose that ω be a nonempty given subregion of Ω. We denote $\Theta = \Omega \times (0,\infty)$ and $\prod = \partial\Omega \times (0,\infty)$. The considered distributed parameter systems is described by the following parabolic systems:

$$\begin{aligned} \frac{\partial x}{\partial t}(\xi,t) &= Ax(\xi,t) + \mathrm{Bu}(t) \quad \Theta \\ x(\xi,0) &= x_\circ(\xi) \quad \Omega \\ x(\eta,t) &= 0 \quad \prod \end{aligned} \tag{2.1}$$

augmented with the output function

$$y(\cdot,t) = Cx(\cdot,t), \tag{2.2}$$

where A is a second-order linear differential operator, which generates a strongly continuous semigroup $(S_A(t))_{t\geq 0}$ on the Hilbert space $X = L^2(\Omega)$ and is self-adjoint with compact resolvent. The operators $B \in L(R^p, X)$ and $C \in L(R^q, X)$ depend on the structures of actuators and sensors [13, 14]. The spaces X, U, and O are separable Hilbert spaces where X is the state space, $U = L^2(0,\infty,R^p)$ is the control space, and $O = L^2(0,\infty,R^q)$ is the observation space,

where p and q are the numbers of actuators and sensors. Under the given assumption [15], the system (2.1) has a unique solution:

$$x(\xi,t) = S_A(t)x_\circ(\xi) + \int_0^t S_A(t-\tau)\mathrm{Bu}(\tau)d\tau. \tag{2.3}$$

The problem is that how to observe exponentially the current state in a given subregion ω (see Figure 1), using convenient sensors and to give a sufficient condition for the existence of a regional exponential observability.

2.2. ω-Strategic Sensor

The purpose of this subsection is to give the characterization for sensors in order that the system (2.1) is regionally exponentially observable in ω.

(i) Sensors are any couple $(D_i, f_i)_{1\le i\le q}$ where D_i denote closed subsets of $\overline{\Omega}$, which is spatial supports of sensors and $f_i \in L^2(D_i)$ define the spatial distributions of measurements on D_i.

According to the choice of the parameters D_i and f_i, we have various types of sensors. These sensors may be types of zones when $D_i \subset \Omega$. The output function (2.2) can be written in the form

$$y(\cdot,t) = Cx(\cdot,t) = \int_{D_i} x(\xi,t)f_i(\xi)d\xi. \tag{2.4}$$

Sensors may also be pointwise when $D_i = \{b_i\}$ and $f_i = \delta_{b_i}(x-b_i)$ where δ_{b_i} is Dirac mass concentrated in b_i. Then, the output function (2.2) can be given by the form

$$y(\cdot,t) = Cx(\cdot,t) = \int_{\Omega} x(\xi,t)\delta_{b_i}(\xi-b_i)d\xi. \tag{2.5}$$

In the case of internal pointwise sensors, the operator C is unbounded and some precaution must be taken in [13, 14]. In the case when (2.1) is autonomous system, (2.3) allows to give the following equation:

$$x(\xi,t) = S_A(t)x_\circ(\xi). \tag{2.6}$$

(ii) Define the operator $K : X \to O$,

$$x \longrightarrow CS_A(\cdot)x \tag{2.7}$$

which is in the case of internal zone sensors is linear and bounded [16]. The adjoint operator K^* of K is defined by

$$K^*y = \int_0^t S_A^*(s)C^*y(s)ds. \tag{2.8}$$

(iii) For the region ω of the domain Ω, the operator χ_ω is defined by

$$\begin{aligned} \chi_\omega : L^2(\Omega) &\longrightarrow L^2(\omega) \\ x &\longrightarrow \chi_\omega x = x|_\omega, \end{aligned} \tag{2.9}$$

where $x|_\omega$ is the restriction of x to ω.

(iv) An autonomous system associated to (2.1)-(2.2) is exactly (resp., weakly) ω-observable if

$$\operatorname{Im} \chi_\omega K^* = L^2(\omega) \qquad \left(\text{resp. } \overline{\operatorname{Im} \chi_\omega K^*(\cdot)} = L^2(\omega)\right). \tag{2.10}$$

(v) A sequence of sensors $(D_i, f_i)_{1\le i\le q}$ is ω-strategic if the system (2.1)-(2.2) is weakly ω-observable [5].

The concept of ω-strategic has been extended to the regional boundary case as in [17]. Assume that the set (φ_{n_j}) of eigenfunctions of $L^2(\Omega)$ orthonormal in $L^2(\omega)$ is associated with eigenvalues λ_n of multiplicity r_n and suppose that the system (2.1) has J unstable modes. Then, we have the following result.

Proposition 2.1. *The sequence of sensors $(D_i, f_i)_{1\le i\le q}$ is ω-strategic if and only if*

(1) $q \ge r$,

(2) rank $G_n = r_n$, *for all* n, $n = 1, \ldots, J$ *with*

$$G_n = (G_n)_{ij} = \begin{cases} \langle \varphi_{nj}, f_i(\cdot)\rangle_{L^2(D_I)}, & \textit{in the zone case}, \\ \varphi_{nj}(b_i), & \textit{in the pointwise case}, \end{cases} \tag{2.11}$$

where sup $r_n = r$ *and* $J = 1, \ldots, r_n$.

Proof. The proof of this proposition is similar to the rank condition in [16]; the main difference is that the rank condition is as follows

$$\text{rank } G_n = r_n, \quad \forall n. \tag{2.12}$$

For Proposition 2.1., we need only to hold for rank $G_n = r_n$, for all n, $n = 1, \ldots, J$. □

2.3. ω_E-Observability

Regional exponential observability characterization needs some notions which are related to the exponential behaviour (stability, detectability, and observer). The concept of exponential behaviour has been extended recently by Al-Saphory and El Jai as in [12].

Definition 2.2. A semigroup is exponentially regionally stable in $L^2(\omega)$ (or ω_E-stable) if, for every initial state $x_\circ(\cdot) \in L^2(\Omega)$, the solution of the autonomous system associated with (2.1) converges exponentially to zero when $t \to \infty$.

Definition 2.3. The system (2.1) is said to be exponentially stable on ω (or ω_E-stable) if the operator A generates a semigroup which is exponentially stable in $L^2(\omega)$. It is easy to see that the system (2.1) is ω_E-stable if and only if, for some positive constants M_ω and α_ω,

$$\left\|\chi_\omega S_A(\cdot)\right\|_{L^2(\omega)} \leq M_\omega e^{-\alpha_\omega t} \quad t \geq 0. \tag{2.13}$$

If $(S_A(t))_{t\geq 0}$ is ω_E-stable, then, for all $x_\circ(\cdot) \in L^2(\Omega)$, the solution of autonomous system associated with (2.1) satisfies

$$\|x_\circ(t)\|_{L^2(\omega)} = \left\|\chi_\omega S_A(\cdot)x_\circ\right\|_{L^2(\omega)} \leq M_\omega e^{-\alpha_\omega t}\|x_\circ\|_{L^2(\omega)} \tag{2.14}$$

and then

$$\lim_{t\to\infty}\|x(t)\|_{L^2(\omega)} = 0. \tag{2.15}$$

Definition 2.4. The system (2.1) together with output (2.2) is said to be exponentially detectable on ω (or ω_E-detectable) if there exists an operator $H_\omega : R^q \to L^2(\omega)$ such that $(A - H_\omega C)$ generates a strongly continuous semigroup $(S_{H_\omega}(t))_{t\geq 0}$ which is ω_E-stable.

Definition 2.5. Consider the system (2.1)-(2.2) together with the dynamical system

$$\begin{aligned} \frac{\partial z}{\partial t}(\xi,t) &= F_\omega x(\xi,t) + G_\omega u(t) + H_\omega y(t) \quad \Theta \\ z(\xi,0) &= z_\circ(\xi) \quad \Omega \\ z(\eta,t) &= 0 \quad \prod, \end{aligned} \tag{2.16}$$

where F_ω generates a strongly continuous semigroup $(S_{F_\omega}(t))_{t\geq 0}$ which is stable on Hilbert space $Z, G_\omega \in L(R^p, Z)$ and $H_\omega \in L(R^q, Z)$. The system (2.16) defines an ω_E-estimator for $\chi_\omega Tx(\xi,t)$ if

(1) $\lim_{t\to\infty}\|z(\cdot,t) - \chi_\omega Tx(\cdot,t)\|_{L^2(\omega)} = 0$,

(2) $\chi_\omega T$ maps $D(A)$ in $D(F_\omega)$ where $z(\xi,t)$ is the solution of the system (2.16).

Definition 2.6. The system (2.16) specifies an ω_E-observer for the system (2.1)-(2.2) if the following conditions hold:

(1) there exist $M_\omega \in L(R^q, L^2(\omega))$ and $N_\omega \in L(L^2(\omega))$ such that

$$M_\omega C + N_\omega \chi_\omega T = I_\omega, \tag{2.17}$$

(2) $\chi_\omega TA + F_\omega \chi_\omega T = G_\omega C$ and $H_\omega = \chi_\omega TB$,

(3) the system (2.16) defines an ω_E-observer.

Definition 2.7. The system (2.16) is said to be ω_E-observer for the system (2.1)-(2.2) if $X = Z$ and $\chi_\omega T = I_\omega$. In this case, we have $F_\omega = A - G_\omega C$ and $H_\omega = B$. Then, the dynamical system (2.16) becomes

$$\begin{aligned} \frac{\partial z}{\partial t}(\xi,t) &= Az(\xi,t) + \mathrm{B}u(t) - G_\omega\big(\mathrm{C}z(\xi,t) - y(\cdot,t)\big) \quad \Theta \\ z(\xi,0) &= 0 \quad \Omega \\ z(\eta,t) &= 0 \quad \prod. \end{aligned} \tag{2.18}$$

Definition 2.8. The system (2.1)-(2.2) is ω_E-observable if there exists a dynamical system which is exponential ω_E-observer, for the original system. Now, the approach which is observed is that the current state $x(\xi,t)$ exponentially is given by the following result.

3. Strategic Sensors and ω_E-Observability

In this section, we give an approach which allows to construct an ω_E-estimator of $x(\xi,t)$. This method avoids the consideration of initial state [6]; it enables to observe exponentially the current state in ω without needing the effect of the initial state of the considered system.

Theorem 3.1. *Suppose that the sequence of sensors $(D_i, f_i)_{1\le i\le q}$ is ω-strategic and the spectrum of A contain J eigenvalues with nonnegative real parts. Then, the system* (2.1)-(2.2) *is ω_E-observable by the following dynamical system:*

$$\begin{aligned} \frac{\partial z}{\partial t}(\xi,t) &= Az(\xi,t) + \mathrm{B}u(t) - G_\omega C\big(z(\xi,t) - y(\cdot,t)\big) \quad \Theta \\ z(\xi,0) &= z_\circ(\xi) \quad \Omega \\ z(\eta,t) &= 0 \quad \prod. \end{aligned} \tag{3.1}$$

Proof. The proof is limited to the case of zone sensors in the following steps.

Step 1. Under the assumptions of Section 2.1, the system (2.1) can be decomposed by the projections P and $I - P$ on two parts, unstable and stable. The state vector may be given by $x(\xi,t) = [x_1(\xi,t) + x_2(\xi,t)]^{\mathrm{tr}}$ where $x_1(\xi,t)$ is the state component of the unstable part of the system (2.1) and may be written in the form

$$\begin{aligned} \frac{\partial x_1}{\partial t}(\xi,t) &= A_1 x_1(\xi,t) + P\mathrm{B}u(t) \quad \Theta \\ x_1(\xi,0) &= x_{\circ 1}(\xi) \quad \Omega \\ x_1(\eta,t) &= 0 \quad \prod \end{aligned} \tag{3.2}$$

and $x_2(\xi,t)$ is the component state of the part of the system (2.1) given by

$$\begin{aligned} \frac{\partial x_2}{\partial t}(\xi,t) &= A_2 x_2(\xi,t) + (I-P)\mathrm{Bu}(t) \quad \Theta \\ x_2(\xi,0) &= x_{\circ 2}(\xi) \quad \Omega \\ x_2(\eta,t) &= 0 \quad \prod. \end{aligned} \tag{3.3}$$

The operator A_1 is represented by matrix of order $(\sum_{n=1}^{J} r_n, \sum_{n=1}^{J} r_n)$ given by

$$\begin{aligned} A_1 &= \mathrm{diag}\lfloor \lambda_1,\ldots,\lambda_1,\lambda_2,\ldots,\lambda_2,\ldots,\lambda_{\mathrm{j}},\ldots,\lambda_{\mathrm{j}}\rfloor, \\ \mathrm{PB} &= \left[\mathrm{G}_1^{\mathrm{tr}},\mathrm{G}_2^{\mathrm{tr}},\ldots,\mathrm{G}_J^{\mathrm{tr}}\right]. \end{aligned} \tag{3.4}$$

Step 2. Since the sequence suite of sensors $(D_i, f_i)_{1\le i\le q}$ is ω-strategic for the unstable part of the system (2.1). The subsystem (3.2) is weakly ω-observable [5], and since it is of finite dimensional, it is exactly ω-observable [2]. Therefore, it is ω_E-detectable and hence there exists an operator H_ω^1 such that $A_1 - H_\omega^1 C$ which satisfies the following: $\exists M_\omega^1, \alpha_\omega^1 > 0$ such that $\|e^{(A_1-H_\omega^1 C)t}\| \le M_\omega^1 e^{-\alpha_\omega^1 t}$ and, then, we have

$$\|x_1(\cdot,t)\|_{L^2(\omega)} \le M_\omega^1 e^{-\alpha_\omega^1 t}\|Px_\circ\|_{L^2(\omega)}. \tag{3.5}$$

Since the semigroup generated by the operator A_2 is ω_E-stable, there exists $M_\omega^2, \alpha_\omega^2 > 0$ such that

$$\|x_2(\cdot,t)\|_{L^2(\omega)} \le M_\omega^1 e^{-\alpha_\omega^1}\|(I-P)x_{\circ 2}(\cdot)\|_{L^2(\omega)} + \int_0^t M_\omega^2 e^{-\alpha_\omega^2(t-\tau)}\|(I-P)x_{\circ 2}(\cdot)\|_{L^2(\omega)}\|u(\tau)\|d\tau \tag{3.6}$$

and therefore $\|x(\xi,t)\|_{L^2(\omega)} \to 0$ when $t \to \infty$. Finally, the system (2.1)-(2.2) is ω_E-detectable.

Step 3. Let $e(\xi,t) = x(\xi,t) - z(\xi,t)$ where $z(\xi,t)$ is solution of the system (3.1). Driving the above equation and using (2.1) and (3.1), we obtain

$$\begin{aligned} \frac{\partial e}{\partial t}(\xi,t) &= \frac{\partial x}{\partial t}(\xi,t) - \frac{\partial z}{\partial t}(\xi,t) \\ &= Ax(\xi,t) + \mathrm{Bu}(t) - Az(\xi,t) - \mathrm{Bu}(t) + H_\omega C(z(\xi,t) - x(\cdot,t)) \\ &= (A - H_\omega C)e(\xi,t). \end{aligned} \tag{3.7}$$

Since the system (2.1)-(2.2) is ω_E-detectable, there exists an operator $H_\omega \in L(R^q, L^2(\omega))$, such that the operator $(A - H_\omega C)$ generates exponentially regionally stable, strongly continuous semigroup $(S_{H_\omega}(t))_{t\ge 0}$ on $L^2(\omega)$ which satisfies the following relations:

$$\exists M_\omega, \alpha_\omega > 0 \text{ such that } \left\|\chi_\omega S_{H_\omega}(t)\right\|_{L^2(\omega)} \le M_\omega e^{-\alpha_\omega t}. \tag{3.8}$$

Finally, we have

$$\|e(\cdot,t)\|_{L^2(\omega)} \leq \left\|\chi_\omega S_{H_\omega}(t)\right\|_{L^2(\omega)} \|e_\circ(\cdot)\| \leq M_\omega e^{-\alpha_\omega t}\|e_\circ(\cdot)\| \tag{3.9}$$

with $e_\circ(\cdot) = x_\circ(\cdot) - z_\circ(\cdot)$ and therefore $e(\xi,t)$ converges exponentially to zero as $t \to \infty$. Thus, the dynamical system (3.1) observes exponentially the regional state $x(\xi,t)$ of the system original system and (2.1)-(2.2) is ω_E-observable. □

Remark 3.2. We can deduce that

(1) a system which is exactly ω-observable is exponentially ω-observable,

(2) a system which is exponentially observable is exponentially ω-observable,

(3) a system which is exponentially ω-observable is exponentially ω_1-observable, in every subset ω_1 of ω, but the converse is not true. This may be proven in the following example.

Example 3.3. Consider the system

$$\begin{aligned} \frac{\partial x}{\partial t}(\xi,t) &= \Delta x(\xi,t) + x(\xi,t) \quad \Theta \\ x(\xi,0) &= x_\circ(\xi) \quad \Omega \\ z(\eta,t) &= 0 \quad \prod \end{aligned} \tag{3.10}$$

augmented with the output function

$$y(t) = \int_\Omega x(\xi,t)\delta(\xi - b_i)d\xi, \tag{3.11}$$

where $\Omega = (0,1)$ and $b_i \in \Omega$ are the location of sensors (b_i, δ_{b_i}) as in (Figure 2). The operator $A = (\Delta + 1)$ generates a strongly continuous semigroup $(S_A(t))_{t\geq 0}$ on the Hilbert space $L^2(\omega)$ [15]. Consider the dynamical system

$$\begin{aligned} \frac{\partial z}{\partial t}(\xi,t) &= \Delta z(\xi,t) + z(\xi,t) - HC(z(\xi,t) - x(\xi,t)) \quad (0,1),\ t > 0, \\ z(\xi,0) &= z_\circ(\xi) \quad (0,1), \\ z(0,t) &= z(1,t) = 0 \quad t > 0, \end{aligned} \tag{3.12}$$

where $H \in L(R^q, Z), Z$ is the Hilbert space, and $C : Z \to R^q$ is linear operator. If $b_i \in Q$, then the sensors (b_i, δ_{b_i}) are not strategic for the unstable subsystem (3.10) [1] and therefore the system (3.10)-(3.11) is not exponentially detectable in Ω [14]. Then, the dynamical system (3.12) is not observer and then (3.10)-(3.11) is not exponentially observable [16].

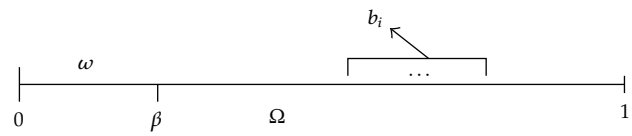

Figure 2: The domain Ω, the subregion ω, and locations b_i of internal pointwise sensors.

Now, we consider the region $\omega = [0, \beta] \subset (0, 1)$ and the dynamical system

$$\begin{gathered}\frac{\partial z}{\partial t}(\xi, t) = \Delta z(\xi, t) + z(\xi, t) - H_\omega C(z(\xi, t) - x(\xi, t)) \quad (0, 1), \ \mathrm{t} > 0, \\ z(\xi, 0) = z_\circ(\xi) \quad (0, 1), \\ z(0, t) = z(1, t) = 0 \quad t > 0,\end{gathered} \tag{3.13}$$

where $H_\omega \in L(R^q, L^2(\omega))$. If $b_i/\beta \notin Q$, then the sensors (b_i, δ_{b_i}) are ω-strategic for the unstable subsystem of (3.10) [7] and then the system (3.10)-(3.11) is ω_E-detectable. Therefore, the system (3.10)-(3.11) is ω_E-observable by ω_E-observer [12].

4. Application to Sensor Location

In this section, we present an application of the above results to a two-dimensional system defined on $\Omega = (0, 1) \times (0, 1)$ by the form

$$\begin{gathered}\frac{\partial x}{\partial t}(\xi_1, \xi_2, t) = \Delta x(\xi_1, \xi_2 t) + \mathrm{Bu}(t) \quad \Theta \\ x(\xi_1, \xi_2, 0) = x_\circ(\xi_1, \xi_2) \quad \Omega \\ x(\eta_1, \eta_2, t) = 0 \quad \prod\end{gathered} \tag{4.1}$$

together with output function by (2.4), (2.5). Let $\omega = (\alpha_1, \beta_1) \times (\alpha_2, \beta_2)$ be the considered region which is subset of (0,1) × (0,1). In this case, the eigenfunctions of system (4.1) are given by

$$\varphi_{ij}(\xi_1, \xi_2) = \frac{2}{\sqrt{(\beta_1 - \alpha_1)(\beta_2 - \alpha_2)}} \sin i\pi\left(\frac{\xi_1 - \alpha_1}{\beta_1 - \alpha_1}\right) \sin j\pi\left(\frac{\xi_2 - \alpha_2}{\beta_2 - \alpha_2}\right) \tag{4.2}$$

associated with eigenvalues

$$\lambda_{ij} = -\left(\frac{i^2}{(\beta_1 - \alpha_1)^2} + \frac{j^2}{(\beta_2 - \alpha_2)^2}\right). \tag{4.3}$$

The following results give information on the location of internal zone or pointwise ω-strategic sensors.

4.1. Internal Zone Sensor

Consider the system (4.1) together with output function (2.2) where the sensor supports D are located in Ω. The output (2.2) can be written by the form

$$y(t) = \int_D x(\xi_1, \xi_2, t) f(\xi_1, \xi_2) d\xi_1 d\xi_2, \tag{4.4}$$

where $D \subset \Omega$ is location of zone sensor and $f \in L^2(D)$. In this case of Figure 3, the eigenfunctions and the eigenvalues are given by (4.2) and (4.3). However, if we suppose that

$$\frac{(\beta_1 - \alpha_1)^2}{(\beta_2 - \alpha_2)^2} \notin Q, \tag{4.5}$$

then $r = 1$ and one sensor may be sufficient to achieve ω_E-observability [18]. In this case, the dynamical system (3.1) is given by

$$\begin{aligned} \frac{\partial z}{\partial t}(\xi_1, \xi_2, t) &= \Delta z(\xi_1, \xi_2, t) + z(\xi_1, \xi_2, t) + \mathrm{Bu(t)} - H_\omega < x(\cdot, t), f_i(\cdot) > -Cz(\xi, t) \quad \Theta \\ z(\xi_1, \xi_2, 0) &= z_\circ(\xi_1, \xi_2) \quad \Omega \\ z(\eta_1, \eta_2, t) &= 0 \quad \prod. \end{aligned} \tag{4.6}$$

Let the measurement support be rectangular with

$$D = [\xi_1 - l_1, \xi_1 + l_2] \times [\xi_2 - l_2, \xi_2 + l_2] \in \Omega, \tag{4.7}$$

then we have the following result.

Corollary 4.1. *If f_1 is symmetric about $\xi_1 = \xi_{\circ_1}$ and f_2 is symmetric about $\xi_2 = \xi_{\circ_2}$, then the system* (4.1)–(4.4) *is ω_E-observable by the dynamical system* (4.6) *if*

$$\frac{i(\xi_{\circ_1} - \alpha_1)}{(\beta_1 - \alpha_1)}, \quad \frac{i(\xi_{\circ_2} - \alpha_2)}{(\beta_2 - \alpha_2)} \notin N \quad \textit{for some } i = 1, 2, \ldots, J. \tag{4.8}$$

4.2. Internal Pointwise Sensor

Let us consider the case of pointwise sensor located inside of Ω. The system (4.1) is augmented with the following output function:

$$y(t) = \int x(\xi_1, \xi_2, t) \delta(\xi_1 - b_1, \xi_2 - b_2) d\xi_1 d\xi_2, \tag{4.9}$$

where $b = (b_1, b_2)$ is the location of pointwise sensor as defined in Figure 4.

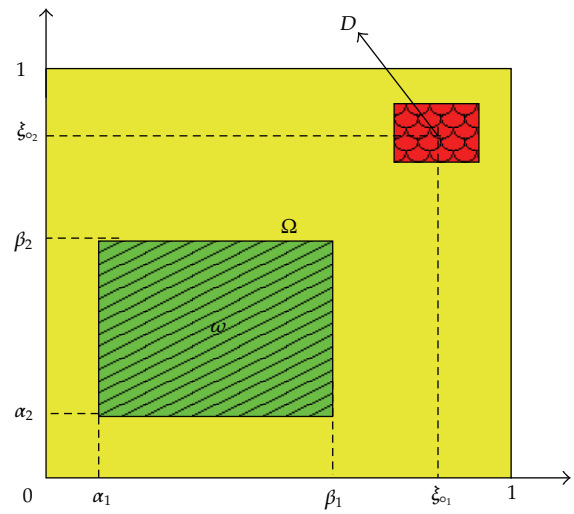

Figure 3: Domain Ω, subregion ω, and location D of internal zone sensor.

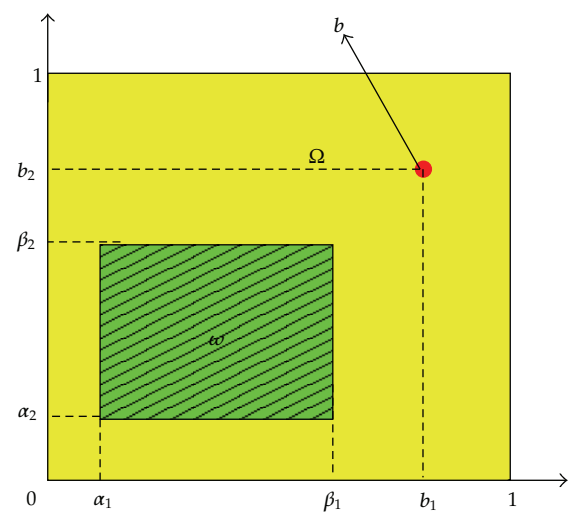

Figure 4: Rectangular domain Ω, region ω, and location b of internal pointwise sensor.

If $(\beta_1 - \alpha_1)/(\beta_2 - \alpha_2) \notin \mathrm{Q}$, then $m = 1$ and one sensor (b, δ_b) may be sufficient for ω_E-observability. Then, the dynamical system is given by

$$\begin{aligned} \frac{\partial z}{\partial t}(\xi_1, \xi_2, t) &= \Delta z(\xi_1, \xi_2, t) + z(\xi_1, \xi_2, t) + \mathrm{Bu}(t) + H_\omega\big(x(b_1, b_2, t) - y(t)\big) \quad \Theta \\ z(\xi_1, \xi_2, 0) &= z_\circ(\xi_1, \xi_2) \quad \Omega \\ z(\eta_1, \eta_2, t) &= 0 \quad \prod. \end{aligned} \tag{4.10}$$

Thus, we obtain the following.

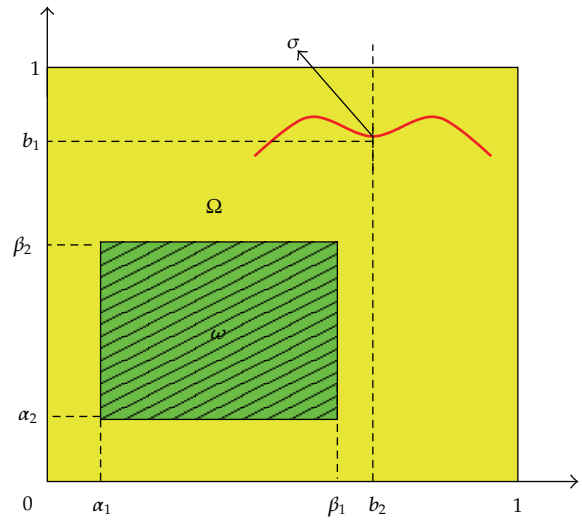

Figure 5: Rectangular domain and location σ of internal filament sensors.

Corollary 4.2. *The system* (4.1)–(4.9) *is not* ω_E*-observable by the dynamical system* (4.10) *if* $i(b_1 - \alpha_1)/(\beta_1 - \alpha_1)$ *and* $i(b_2 - \alpha_2)/(\beta_2 - \alpha_2) \in N$, *for every* $i, 1 \leq i \leq J$.

4.3. Internal Filament Sensor

Consider the case of the observation on the curve $\sigma = \mathrm{Im}(\gamma)$ with $\gamma \in C^1(0,1)$ (see Figure 5), then we have the following.

Corollary 4.3. *If the observation recovered by filament sensor* (σ, δ_σ) *such that it is symmetric with respect to the line* $\xi = \xi_\circ$, *then the system* (4.1)–(4.9) *is not* ω_E*-observable by* (4.10) *if* $i(\xi_{\circ 1} - \alpha_1)/(\beta_1 - \alpha_1)$ *and* $i(\xi_{\circ 2} - \alpha_2)/(\beta_2 - \alpha_2) \in N$ *for all* $i = 1, \ldots, q$.

Remark 4.4. These results can be extended to the following:

(1) case of Neumann or mixed boundary conditions [1, 2],

(2) case of disc domain $\Omega = (D, 1)$ and $\omega = (0, r_\omega)$ where $\omega \subset \Omega$ and $0 < r_\omega < 1$ [10],

(3) case of boundary sensors where $C \notin L(X, R^q)$; we refer to see [13, 14].

5. Conclusion

The concept developed in this paper is related to the regional exponential observability in connection with the strategic sensors. It permits us to avoid some "bad" sensor locations. Various interesting results concerning the choice of sensors structure are given and illustrated in specific situations. Many questions still opened. This is the case of, for example, the problem of finding the optimal sensor location ensuring such an objective. The dual result of regional controllability concept is under consideration.

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