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Research Article

Estimation of Failure Probability and Its Applications in Lifetime Data Analysis

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Since Lindley and Smith introduced the idea of hierarchical prior distribution, some results have been obtained on hierarchical Bayesian method to deal with lifetime data. But all those results obtained by means of hierarchical Bayesian methods involve complicated integration compute. Though some computing methods such as Markov Chain Monte Carlo (MCMC) are available, doing integration is still very inconvenient for practical problems. This paper introduces a new method, named *E-Bayesian estimation method*, to estimate failure probability. In the case of one hyperparameter, the definition of E-Bayesian estimation of the failure probability is provided; moreover, the formulas of E-Bayesian estimation and hierarchical Bayesian estimation and the property of E-Bayesian estimation of the failure probability are also provided. Finally, calculation on practical problems shows that the provided method is feasible and easy to perform.

1. Introduction

In the area related to the reliability of industrial products, engineers often deal with the truncated data in life testing of products, where the data sometimes have small sample size or have been censored, and the products of interest have high reliability. In the literature, Lindley and Smith [1] first introduced the idea of hierarchical prior distribution. Han [2] developed some methods to construct hierarchical prior distribution. Recently, hierarchical Bayesian methods have been applied to data analysis [3]. However, complicated integration compute is a hard work by using hierarchical Bayesian methods in practical problems, though some computing methods such as Markov Chain Monte Carlo (MCMC) methods are available [4, 5].

Han [6] introduced a new method—E-Bayesian estimation method—to estimate failure probability in the case of two hyperparameters, proposed the definition of E-Bayesian of failure probability, and provided formulas for E-Bayesian estimation of the failure probability under the cases of three different prior distributions of hyperparameters. But we did not provide formulas for hierarchical Bayesian estimation of the failure probability nor discuss the relations between the two estimations. In this paper, we will introduce the definition for E-Bayesian estimation of the failure probability, provide formulas both for E-Bayesian estimation and hierarchical Bayesian estimation, and also discuss the relations between the two estimations in the case of only one hyperparameter. We will see that the E-Bayesian estimation method is really simple.

Conduct type I censored life testing m time, denote the censored times as t_i ($i = 1, 2, \dots, m$), the corresponding sample numbers as n_i, and the corresponding failure sample numbers observed in the testing process as r_i ($r_i = 0, 1, 2, \dots, n_i$).

In the situation where no information about the life distribution of tested products is available, Mao and Luo [7] introduce a so-called curves fitting distribution method to give the estimation of the failure probability p_i at time t_i, $p_i = P(T < t_i)$ for $i = 1, 2, \dots, m$, where T is the life of a product.

This paper introduces a new method, called *E-Bayesian estimation method,* to estimate failure probability. The definition and formulas of E-Bayesian estimation of the failure probability are described in Sections 2 and 3, respectively. In Section 4, formulas of hierarchical Bayesian estimation of the failure probability are proposed. In Section 5, the properties of E-Bayesian estimation are discussed. In Section 6, an application example introduces given. Section 7 is the conclusions.

2. Definition of E-Bayesian Estimation of p_i

Suppose that there are X failures among n samples, then X can be viewed as a random variable with binomial distribution Bin (n, p_i). We take the conjugate prior of p_i, Beta (a, b), with density function

$$\pi(p_i \mid a, b) = \frac{p_i^{a-1}(1-p_i)^{b-1}}{B(a,b)}, \tag{1}$$

where $0 < p_i < 1$, $B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}dt$ is the beta function, and hyperparameters $a > 0$ and $b > 0$.

The derivative of $\pi(p_i \mid a, b)$ with respect to p_i is

$$\frac{d\pi(p_i \mid a,b)}{dp_i} = \frac{p_i^{a-2}(1-p_i)^{b-2}[(a-1)(1-p_i)-(b-1)p_i]}{B(a,b)}. \tag{2}$$

According to Han [2], a and b should be chosen to guarantee that $\pi(p_i \mid a, b)$ is a decreasing function of p_i. Thus $0 < a < 1$, $b > 1$.

Given $0 < a < 1$, the larger the value of b, the thinner the tail of the density function. Berger had shown in [8] that the thinner tailed prior distribution often reduces the robustness of the Bayesian estimate. Consequently, the hyperparameter b should be chosen under the restriction $1 < b < c$, where c is a given upper bound. How to determine the constant c would be described later in an example.

Since $a \in (0,1)$ and $1/2$ is an expectation of uniform on (0, 1), that we take $a = 1/2$. When $1 < b < c$ and $a = 1/2$, $\pi(p_i \mid a, b)$ also is a decreasing function of p_i.

In this paper we only consider the case when $a = 1/2$. Then the density function $\pi(p_i \mid a, b)$ becomes

$$\pi(p_i \mid b) = \frac{p_i^{(1/2)-1}(1-p_i)^{b-1}}{B((1/2),b)}. \tag{3}$$

The definition of E-Bayesian estimation was originally addressed by Han [6] in the case of two hyperparameters. In the case of one hyperparameter, the E-Bayesian estimation of failure probability is defined as follows.

Definition 1. With $\hat{p}_{iB}(b)$ being continuous,

$$\hat{p}_{iEB} = \int_D \hat{p}_{iB}(b)\pi(b)db \tag{4}$$

is called the expected Bayesian estimation of p_i (briefly E-Bayesian estimation), which is assumed to be finite, where D is the domain of b, $\hat{p}_{iB}(b)$ is the Bayesian estimation of p_i with hyperparameter b, and $\pi(b)$ is the density function of b over D.

Definition 1 indicates that the E-Bayesian estimation of p_i,

$$\hat{p}_{iEB} = \int_D \hat{p}_{iB}(b)\pi(b)db = E[\hat{p}_{iB}(b)], \tag{5}$$

is the expectation of the Bayesian estimation of p_i for the hyperparameter.

3. E-Bayesian Estimation of p_i

Theorem 2. *For the testing data set $\{(n_i, r_i, t_i),\ i = 1,\ldots,m\}$ with type I censor, where $r_i = 0,1,2,\ldots,n_i$, let $s_i = \sum_{j=i}^m n_j$ and $e_i = \sum_{j=1}^i r_j$ $(i = 1,2,\ldots,m)$. If the prior density function $\pi(p_i \mid b)$ of p_i is given by (3), then, we have the following.*

(i) *With the quadratic loss function, the Bayesian estimation of p_i is*

$$\hat{p}_{iB}(b) = \frac{e_i + (1/2)}{s_i + b + (1/2)}. \tag{6}$$

(ii) *If prior distribution of b is uniform on (1, c), then E-Bayesian estimation of p_i is*

$$\hat{p}_{iEB} = \frac{e_i + (1/2)}{c-1}\ln\left(\frac{s_i + c + (1/2)}{s_i + (3/2)}\right). \tag{7}$$

Proof. (i) For the testing data set $\{(n_i, r_i, t_i),\ i = 1,\ldots,m\}$ with type I censor, where $r_i = 0,1,2,\ldots,n_i$, according to Han [6], the likelihood function of samples is

$$L(r_i \mid p_i) = \binom{s_i}{e_i} p_i^{e_i}(1-p_i)^{s_i-e_i}, \quad 0 < p_i < 1, \tag{8}$$

where $s_i = \sum_{j=i}^m n_j$ and $e_i = \sum_{j=1}^i r_j$ $(i = 1,2,\ldots,m)$.

Combined with the prior density function $\pi(p_i \mid b)$ of p_i given by (3), the Bayesian theorem leads to the posterior density function of p_i,

$$\begin{aligned} h_1(p_i \mid s_i, b) &= \frac{\pi(p_i \mid b)L(r_i \mid p_i)}{\int_0^1 \pi(p_i \mid b)L(r_i \mid p_i)dp_i} \\ &= \frac{p_i^{e_i+(1/2)-1}(1-p_i)^{s_i+b-e_i-1}}{\int_0^1 p_i^{e_i+(1/2)-1}(1-p_i)^{s_i+b-e_i-1}dp_i} \\ &= \frac{p_i^{e_i+(1/2)-1}(1-p_i)^{s_i+b-e_i-1}}{B(e_i+(1/2), s_i+b-e_i)}, \quad 0 < p_i < 1. \end{aligned} \tag{9}$$

Thus, with the quadratic loss function, the Bayesian estimation of p_i is

$$\begin{aligned} \hat{p}_{iB}(b) &= \int_0^1 p_i h_1(p_i \mid s_i, b)dp_i \\ &= \frac{\int_0^1 p_i^{e_i+(3/2)-1}(1-p_i)^{s_i+b-e_i-1}dp_i}{B(e_i+(1/2), s_i+b-e_i)} \\ &= \frac{B(e_i+(3/2), s_i+b-e_i)}{B(e_i+(1/2), s_i+b-e_i)} \\ &= \frac{e_i+(1/2)}{s_i+b+(1/2)}. \end{aligned} \tag{10}$$

(ii) If prior distribution of b is uniform on $(1, c)$, then, by Definition 1, the E-Bayesian estimation of p_i is

$$\begin{aligned}\hat{p}_{i\mathrm{EB}} &= \int_D \hat{p}_{iB}(b)\pi(b)db = \frac{1}{c-1}\int_1^c \frac{e_i+(1/2)}{s_i+b+(1/2)}db \\ &= \frac{e_i+(1/2)}{c-1}\ln\left(\frac{s_i+c+(1/2)}{s_i+(3/2)}\right).\end{aligned} \tag{11}$$

This concludes the proof of Theorem 2. □

4. Hierarchical Bayesian Estimation

If the prior density function $\pi(p_i \mid b)$ of p_i is given by (3), how can the value of hyperparameter a be determined? Lindley and Smith [1] addressed an idea of hierarchical prior distribution, which suggested that one prior distribution may be adapted to the hyperparameters while the prior distribution includes hyperparameters.

If the prior of p_i, $\pi(p_i \mid b)$, is given by (3), prior distribution of b is uniform on $(1, c)$, and the density function is $\pi(b) = (1/(c-1))$, $1 < b < c$, then, hierarchical prior density function of p_i is

$$\begin{aligned}\pi(p_i) &= \int_1^c \pi(p_i \mid b)\pi(b)db \\ &= \frac{1}{c-1}\int_1^c \frac{p_i^{(1/2)-1}(1-p_i)^{b-1}}{B((1/2),b)}db, \quad 0 < p_i < 1.\end{aligned} \tag{12}$$

Theorem 3. *For the testing data set $\{(n_i, r_i, t_i),\ i = 1, \ldots, m\}$ with type I censor, where $r_i = 0, 1, 2, \ldots, n_i$, let $s_i = \sum_{j=i}^m n_j$ and $e_i = \sum_{j=1}^i r_j$ $(i = 1, 2, \ldots, m)$. If the hierarchical prior density function $\pi(p_i)$ of p_i is given by (12), then, using the quadratic loss function, the hierarchical Bayesian estimation of p_i is*

$$\hat{p}_{i\mathrm{HB}} = \frac{\int_1^c (B(e_i+(3/2), s_i+b-e_i)/B((1/2),b))db}{\int_1^c (B(e_i+(1/2), s_i+b-e_i)/B((1/2),b))db}. \tag{13}$$

Proof. According to the course of the proof of Theorem 2, the likelihood function of samples is

$$L(r_i \mid p_i) = \binom{s_i}{e_i} p_i^{e_i}(1-p_i)^{s_i-e_i}, \quad 0 < p_i < 1, \tag{14}$$

where $s_i = \sum_{j=i}^m n_j$ and $e_i = \sum_{j=1}^i r_j (i = 1, 2, \ldots, m)$.

From the hierarchical prior density function of p_i given by (12), the Bayesian theorem leads to the hierarchical posterior density function of p_i,

$$\begin{aligned}&h_2(p_i \mid s_i) \\ &= \frac{\pi(p_i)L(r_i \mid p_i)}{\int_0^1 \pi(p_i)L(r_i \mid p_i)dp_i} \\ &= \frac{\int_1^c \left(p_i^{e_i+(1/2)-1}(1-p_i)^{s_i+b-e_i-1}/B((1/2),b)\right)db}{\int_1^c (1/B((1/2),b))\left\{\int_0^1 p_i^{e_i+(1/2)-1}(1-p_i)^{s_i+b-e_i-1}dp_i\right\}db} \\ &= \frac{\int_1^c \left(p_i^{e_i+(1/2)-1}(1-p_i)^{s_i+b-e_i-1}/B((1/2),b)\right)db}{\int_1^c (B(e_i+(1/2), s_i+b-e_i)/B((1/2),b))db}, \quad 0 < p_i < 1.\end{aligned} \tag{15}$$

With the quadratic loss function, the hierarchical Bayesian estimation of p_i is

$$\begin{aligned}&\hat{p}_{i\mathrm{HB}} \\ &= \int_0^1 p_i h_2(p_i \mid s_i)dp_i \\ &= \frac{\int_1^c (1/B((1/2),b))\left\{\int_0^1 p_i^{e_i+(3/2)-1}(1-p_i)^{s_i+b-e_i-1}dp_i\right\}db}{\int_1^c (B(e_i+(1/2), s_i+b-e_i)/B((1/2),b))db} \\ &= \frac{\int_1^c (B(e_i+(3/2), s_i+b-e_i)/B((1/2),b))db}{\int_1^c (B(e_i+(1/2), s_i+b-e_i)/B((1/2),b))db}.\end{aligned} \tag{16}$$

Thus, the proof is completed. □

5. Property of E-Bayesian Estimation of p_i

Now we discuss the relations between $\hat{p}_{i\mathrm{EB}}$ and $\hat{p}_{i\mathrm{HB}}$ in Theorems 2 and 3.

Theorem 4. *In Theorems 2 and 3, $\hat{p}_{i\mathrm{EB}}$ and $\hat{p}_{i\mathrm{HB}}$ satisfy $\lim_{s_i \to \infty} \hat{p}_{i\mathrm{EB}} = \lim_{s_i \to \infty} \hat{p}_{i\mathrm{HB}}$.*

Proof. According to the course of the proof of Theorem 2, we have that

$$\hat{p}_{i\mathrm{EB}} = \frac{1}{c-1}\int_1^c \frac{e_i+(1/2)}{s_i+b+(1/2)}db. \tag{17}$$

When $b \in (1, c)$, $(e_i + (1/2)/s_i + b + (1/2))$ is continuous; by the mean value theorem for definite integrals, there is at least one number $b_1 \in (1, c)$ such that

$$\begin{aligned}\int_1^c \frac{e_i+(1/2)}{s_i+b+(1/2)}db &= \frac{e_i+(1/2)}{s_i+b_1+(1/2)}\int_1^c db \\ &= \frac{(e_i+(1/2))(c-1)}{s_i+b_1+(1/2)}.\end{aligned} \tag{18}$$

According to (17) and (18), we have that

$$\lim_{s_i \to \infty} \hat{p}_{i\mathrm{EB}} = \left(e_i + \frac{1}{2}\right)\lim_{s_i \to \infty}\frac{1}{s_i+b_1+(1/2)} = 0. \tag{19}$$

According to the relations of Beta function and Gamma function, we have that

$$B\left(e_i+\left(\frac{3}{2}\right), s_i+b-e_i\right)$$
$$=\frac{\Gamma(e_i+(3/2))\Gamma(s_i+b-e_i)}{\Gamma(s_i+b+(3/2))}$$
$$=\frac{(e_i+(1/2))\Gamma(e_i+(1/2))\Gamma(s_i+b-e_i)}{(s_i+b+(1/2))\Gamma(s_i+b+(1/2))} \tag{20}$$
$$=\frac{e_i+(1/2)}{s_i+b+(1/2)}B(e_i+(1/2), s_i+b-e_i),$$

where the Gamma function is $\Gamma(a)=\int_0^\infty x^{a-1}e^{-x}dx$.

When $b\in(1,c)$, we have that $B(e_i+(1/2), s_i+b-e_i)>0$, and $(e_i+(1/2)/s_i+b+(1/2))$ is continuous; by the generalized mean value theorem for definite integrals, there is at least one number $b_2\in(1,c)$ such that

$$\frac{\int_1^c(B(e_i+(3/2),s_i+b-e_i)/B((1/2),b))db}{\int_1^c(B(e_i+(1/2),s_i+b-e_i)/B((1/2),b))db}$$
$$=\frac{\int_1^c(e_i+(1/2)/s_i+b+(1/2))\cdot(B(e_i+(1/2),s_i+b-e_i)/B((1/2),b))db}{\int_1^c(B(e_i+(1/2),s_i+b-e_i)/B((1/2),b))db}$$
$$=\frac{e_i+(1/2)}{s_i+b_2+(1/2)}\cdot\frac{\int_1^c(B(e_i+(1/2),s_i+b-e_i)/B((1/2),b))db}{\int_1^c(B(e_i+(1/2),s_i+b-e_i)/B((1/2),b))db}$$
$$=\frac{e_i+(1/2)}{s_i+b_2+(1/2)}. \tag{21}$$

According to Theorem 3, we have that

$$\lim_{s_i\to\infty}\hat{p}_{i\mathrm{HB}}=\lim_{s_i\to\infty}\frac{\int_1^c(B(e_i+(3/2),s_i+b-e_i)/B((1/2),b))db}{\int_1^c(B(e_i+(1/2),s_i+b-e_i)/B((1/2),b))db}$$
$$=\left(e_i+\frac{1}{2}\right)\lim_{s_i\to\infty}\frac{1}{s_i+b_2+(1/2)}=0. \tag{22}$$

According to (19) and (22), we have that $\lim_{s_i\to\infty}\hat{p}_{i\mathrm{EB}}=\lim_{s_i\to\infty}\hat{p}_{i\mathrm{HB}}$.

Thus, the proof is completed. □

Theorem 4 shows that $\hat{p}_{i\mathrm{EB}}$ and $\hat{p}_{i\mathrm{HB}}$ are asymptotically equivalent to each other as s_i tends to infinity. In application, $\hat{p}_{i\mathrm{EB}}$ and $\hat{p}_{i\mathrm{HB}}$ are close to each other, when s_i is sufficiently large.

6. Application Example

Han [6] provided a testing data of type I censored life testing for a type of engine, which is listed in Table 1 (time unit: hour).

By Theorems 2, 3, and Table 1, we can obtain $\hat{p}_{i\mathrm{EB}}$, $\hat{p}_{i\mathrm{HB}}$, and $\hat{p}_{i-B}=|\hat{p}_{i\mathrm{EB}}-\hat{p}_{i\mathrm{HB}}|$. Some numerical results are listed in Table 2.

From Table 2, we find that for some c ($c=2,3,4,5,6$), $\hat{p}_{i\mathrm{EB}}$, and $\hat{p}_{i\mathrm{HB}}$ ($i=1,2,\ldots,9$) are very close to each other and satisfy Theorem 4; for different c ($c=2,3,4,5,6$), $\hat{p}_{i\mathrm{EB}}$ and $\hat{p}_{i\mathrm{HB}}$ ($i=1,2,\ldots,9$) are all robust (when $i=9$, there exists some difference). In application, the author suggests selecting a value of c in the middle point of interval [2, 6], that is, $c=4$.

When $c=4$, some numerical results are listed in Table 3 and Figure 1.

Range: in Figure 1, $*$ is the results of $\hat{p}_{i\mathrm{EB}}$ and o is the results of $\hat{p}_{i\mathrm{HB}}$ ($i=1,2,\ldots,9$).

From Table 3 and Figure 1, we find that $\hat{p}_{i\mathrm{EB}}$ and $\hat{p}_{i\mathrm{HB}}$ are very close to each other and consistent with Theorem 4.

From Table 3, we find that the results of $\hat{p}_{i\mathrm{EB}}$ and $\hat{p}_{i\mathrm{HB}}$ are very close to the corresponding results of Han [6].

According to Han [6], we may assume that the lifetime of these products obeys Weibull distribution with distribution function

$$F(t)=1-\exp\left\{-\left(\frac{t}{\eta}\right)^m\right\},\quad \eta>0,\ m>0,\ t>0. \tag{23}$$

According to Mao and Luo [7], the least square estimates of η and m are, respectively,

$$\hat{\eta}=\exp(\hat{\mu}),\qquad \hat{m}=\frac{1}{\hat{\sigma}}, \tag{24}$$

where $\hat{\mu}=(BC-AD)/(mB-A^2)$, $\hat{\sigma}=(mD-AC)/(mB-A^2)$, $A=\sum_{i=1}^m x_i$, $B=\sum_{i=1}^m x_i^2$, $C=\sum_{i=1}^m y_i$, $D=\sum_{i=1}^m x_iy_i$, $x_i=\ln\{(1-\hat{p}_i)^{-1})\}$, $\hat{p}_i$ is the estimate of p_i and $y_i=\ln t_i$, $i=1,2,\ldots,m$.

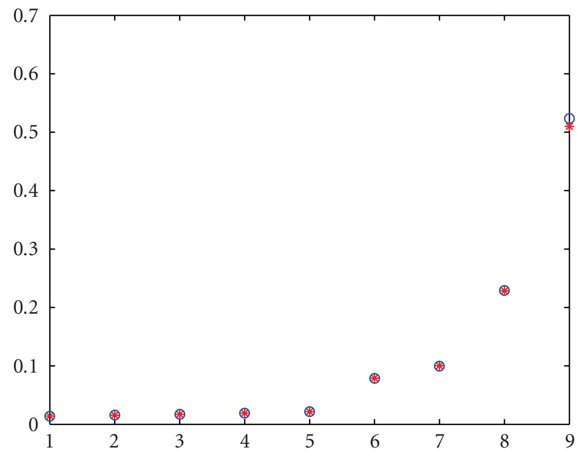

Figure 1: Results of $\hat{p}_{i\mathrm{EB}}$ and $\hat{p}_{i\mathrm{HB}}$.

Table 1: Test data of the engine.

i	1	2	3	4	5	6	7	8	9
t_i	250	450	650	850	1050	1250	1450	1650	1850
n_i	3	3	3	3	4	4	4	4	4
r_i	0	0	0	0	0	1	0	1	1
e_i	0	0	0	0	0	1	1	2	3
s_i	32	29	26	23	20	16	12	8	4

Table 2: Results of $\widehat{p}_{i\mathrm{EB}}$ and $\widehat{p}_{i\mathrm{HB}}$ $(i = 1, 2, \ldots, 9)$.

c	2	3	4	5	6	Range
$\widehat{p}_{1\mathrm{EB}}$	0.014707	0.014497	0.014294	0.014099	0.013911	0.000796
$\widehat{p}_{1\mathrm{HB}}$	0.014693	0.014458	0.014228	0.014004	0.013788	0.000905
$\widehat{p}_{1-B}$	0.000014	0.000039	0.000066	0.000095	0.000123	0.000109
$\widehat{p}_{2\mathrm{EB}}$	0.016130	0.015878	0.015636	0.015404	0.015181	0.000949
$\widehat{p}_{2\mathrm{HB}}$	0.016114	0.015832	0.015557	0.015291	0.015035	0.001079
$\widehat{p}_{2-B}$	0.000016	0.000046	0.000079	0.000113	0.000146	0.000130
$\widehat{p}_{3\mathrm{EB}}$	0.017859	0.017551	0.017256	0.016975	0.016705	0.001154
$\widehat{p}_{3\mathrm{HB}}$	0.017839	0.017495	0.017160	0.016838	0.016530	0.001309
$\widehat{p}_{3-B}$	0.000020	0.000056	0.000096	0.000137	0.000175	0.000155
$\widehat{p}_{4\mathrm{EB}}$	0.020002	0.019618	0.019252	0.018904	0.018572	0.001430
$\widehat{p}_{4\mathrm{HB}}$	0.019978	0.019548	0.019133	0.018736	0.018358	0.001620
$\widehat{p}_{4-B}$	0.000024	0.000070	0.000119	0.000168	0.000214	0.000190
$\widehat{p}_{5\mathrm{EB}}$	0.022731	0.022237	0.021770	0.021328	0.020909	0.001822
$\widehat{p}_{5\mathrm{HB}}$	0.022699	0.022148	0.021619	0.021118	0.020642	0.001620
$\widehat{p}_{5-B}$	0.000032	0.000089	0.000151	0.000210	0.000267	0.000235
$\widehat{p}_{6\mathrm{EB}}$	0.083355	0.081160	0.079112	0.077194	0.075394	0.007961
$\widehat{p}_{6\mathrm{HB}}$	0.083237	0.080856	0.078634	0.076573	0.074661	0.008576
$\widehat{p}_{6-B}$	0.000118	0.000304	0.000478	0.000621	0.000733	0.000615
$\widehat{p}_{7\mathrm{EB}}$	0.107188	0.103613	0.100335	0.097317	0.094524	0.012664
$\widehat{p}_{7\mathrm{HB}}$	0.107011	0.103177	0.099683	0.096509	0.093620	0.013391
$\widehat{p}_{7-B}$	0.000177	0.000436	0.000652	0.000808	0.000904	0.000727
$\widehat{p}_{8\mathrm{EB}}$	0.250209	0.238819	0.228697	0.219623	0.211428	0.030781
$\widehat{p}_{8\mathrm{HB}}$	0.250016	0.238721	0.229136	0.220951	0.213898	0.036118
$\widehat{p}_{8-B}$	0.000193	0.000099	0.000439	0.000672	0.002470	0.002277
$\widehat{p}_{9\mathrm{EB}}$	0.584689	0.542771	0.507871	0.478226	0.452640	0.132050
$\widehat{p}_{9\mathrm{HB}}$	0.589643	0.559105	0.528191	0.523250	0.512187	0.077456
$\widehat{p}_{9-B}$	0.004954	0.016334	0.020320	0.045024	0.059547	0.054594

Table 3: Results of $\widehat{p}_{i\mathrm{EB}}$ and $\widehat{p}_{i\mathrm{HB}}$.

i	1	2	3	4	5
$\widehat{p}_{i\mathrm{EB}}$	0.014294	0.015636	0.017256	0.019252	0.021770
$\widehat{p}_{i\mathrm{HB}}$	0.014228	0.015557	0.017160	0.019133	0.021619
$\widehat{p}_{i-B}$	0.000066	0.000079	0.000096	0.000119	0.000151
i	6	7	8	9	
$\widehat{p}_{i\mathrm{EB}}$	0.079112	0.100335	0.228697	0.507871	
$\widehat{p}_{i\mathrm{HB}}$	0.078634	0.099683	0.229136	0.528191	
$\widehat{p}_{i-B}$	0.000478	0.000652	0.000439	0.020320	

According to (24), we can obtain the estimate of the reliability at moment t,

$$\widehat{R}(t) = \exp\left\{-\left(\frac{t}{\widehat{\eta}}\right)^{\widehat{m}}\right\}, \tag{25}$$

where $\widehat{\eta}$ and $\widehat{m}$ are given by (24).

From (24) and Table 3, we can obtain $\widehat{\eta}$ and $\widehat{m}$. Some numerical results of $\widehat{\eta}$ and $\widehat{m}$ are listed in Table 4.

Table 4: Results of $\widehat{\eta}$ and $\widehat{m}$.

Parameter estimate	$\widehat{m}$	$\widehat{\eta}$
E-Bayesian estimate	2.640846864	2738.461813
Hierarchical Bayesian estimate	2.678276374	2639.899868
Difference between the above two estimates	0.037429506	98.56194500

Table 5: Results of $\hat{R}_{EB}(t)$ and $\hat{R}_{HB}(t)$.

t	200	600	1000	1400	1800	2000
$\hat{R}_{EB}(t)$	0.999003	0.982019	0.932467	0.843643	0.718796	0.646553
$\hat{R}_{HB}(t)$	0.999056	0.982247	0.932059	0.840921	0.712032	0.637398
$\hat{R}_{-B}(t)$	0.000053	0.000228	0.000408	0.002722	0.006764	0.009155

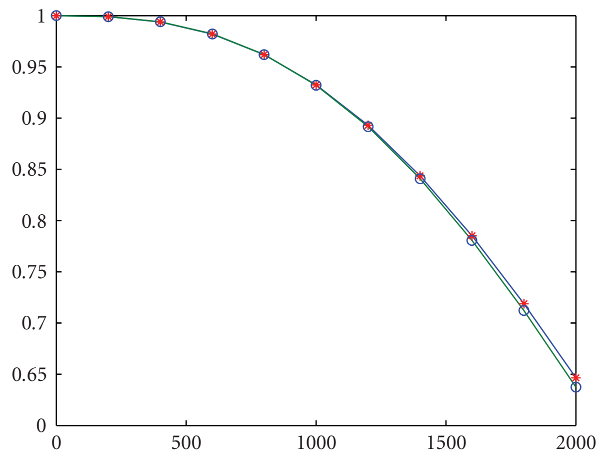

Figure 2: Results of $\hat{R}_{EB}(t)$ and $\hat{R}_{HB}(t)$.

From Table 4, we find that the results of $\hat{\eta}$ and $\hat{m}$ are very close to the corresponding results of Han [6].

From (25) and Table 4, we can obtain the estimate of the reliability with some numerical results of $\hat{R}_{EB}(t)$ and $\hat{R}_{HB}(t)$ listed in Table 5 and Figure 2.

Note that $\hat{R}_{EB}(t)$ is the estimate of reliability at moment t with regard to $\hat{p}_{iEB}$, $\hat{R}_{HB}(t)$ is the estimate of reliability at moment t with regard to $\hat{p}_{iHB}$, and $\hat{R}_{-B}(t) = |\hat{R}_{EB}(t) - \hat{R}_{HB}(t)|$.

Range: in Figure 2, $*$ is the results of $\hat{R}_{EB}(t)$ and o is the results of $\hat{R}_{HB}(t)$.

From Table 5, we find that $\hat{R}_{-B}(t) < 0.0092$ when $t = 200, 600, 1000, 1400, 1800, 2000$, and the results of $\hat{R}_{EB}(t)$ and $\hat{R}_{HB}(t)$ are very close to those of Han [6].

7. Conclusions

This paper introduces a new method, called *E-Bayesian estimation,* to estimate failure probability. The author would like to put forward the following two questions for any new parameter estimation method: (1) how much dependence is there between the new method and the other already-made ones? (2) In which aspects is the new method superior to the old ones?

For the E-Bayesian estimation method, Theorem 4 has given a good answer to the above question (1) and, in addition, the application example shows that $\hat{p}_{iEB}$ and $\hat{p}_{iHB}$ satisfy Theorem 4.

To question (2), from Theorems 2 and 3, we find that the expression of the E-Bayesian estimation is much simple, whereas the expression of the hierarchical Bayesian estimation relies on beta function and complicated integrals expression, which is often not easy.

Reviewing the application example, we find that the E-Bayesian estimation method is both efficient and easy to operate.

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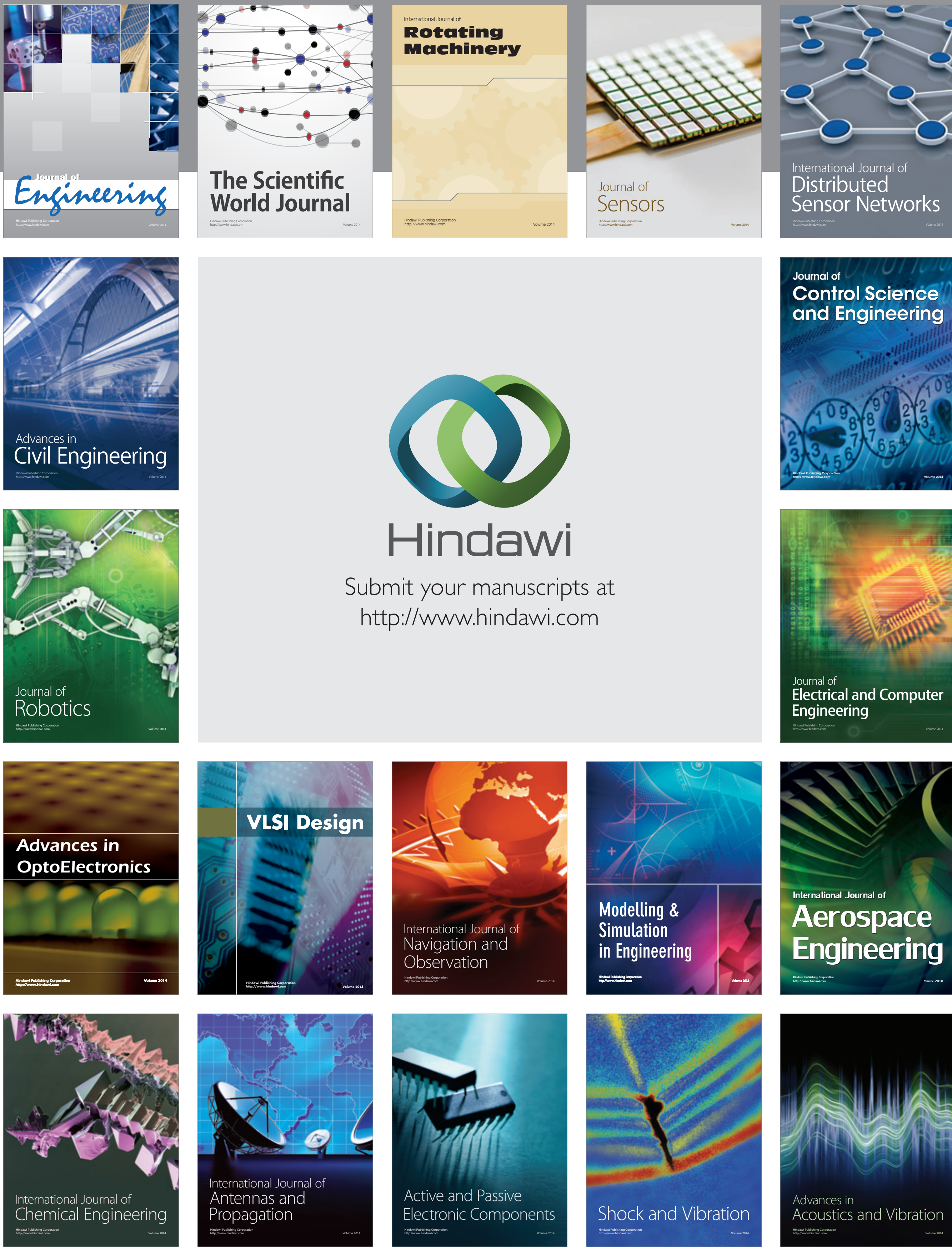
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