Research Article

Endogenous Fertility and Development Traps with Endogenous Lifetime

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Received 26 February 2011; Revised 24 March 2011; Accepted 27 March 2011

Academic Editor: Ali M. Kutun

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We extend the literature on endogenous lifetime and economic growth by Chakraborty (2004) and Bunzel and Qiao (2005) to endogenous fertility. We show that development traps due to underinvestments in health cannot appear when fertility is an economic decision variable and the costs of children are represented by a constant fraction of the parents' income used for their upbringing.

1. Introduction

When endogenous mortality based on public health investments is introduced in the Diamond's [1] overlapping generations (OLG) model with exogenous fertility, development traps due to scarce investments in health can appear and long-run differences in output (As regards the literature on cross-country income and growth differentials see, amongst others, Durlauf and Johnson [2] and Fiaschi and Lavezzi [3, 4]) and longevity across countries emerge when production is relatively capital oriented (see [5]) and the level of technological development is fairly low (see [6]). Unlike previous studies, we show that, when both fertility and the length of life are endogenous, an economy monotonically converges to a unique long-run outcome. Therefore, to the extent that during the stages of development fertility becomes an individual decision variable influenced by economic incentives and constraints, as suggested by the new home economics literature (e.g., Becker [7]), development traps due to scarce health investments are avoided if the costs of children are represented by a fixed fraction of income which reflects expenditures on commodities and services necessary for their upbringing.

On the one hand, the importance of endogenous fertility on the process of economic growth is well established, at least starting from the seminal papers by Becker and Barro [8], Barro and Becker [9], and Becker et al. [10]. On the other

hand, a recent literature on endogenous mortality, through public and/or private health spending, and economic growth is emerging (see, Chakraborty [5], Bhattacharya and Qiao [11], Leung and Wang [12]) (While the first author shows that endogenous lifetime thought public investments in health may explain the occurrence of poverty traps when health spending is relatively scarce, the second and third authors introduce both private and public expenditures [11], or only the private one Leung and Wang [12], as a determinant of the individual length of life. In particular, the model by [11] is concerned with the study of endogenous fluctuations and chaotic motions around the *unique* equilibrium of the economy, which can occur however only when the longevity function is convex and the private and public inputs are fairly complementary. Indeed, merely introducing fertility as an economic decision variable does not alter any of the conclusions of that paper, and thus development traps appear neither under exogenous nor endogenous fertility in such a case.), but without assuming fertility as an economic decision variable. (Indeed, an exception is represented by Blackburn and Cipriani [13], where fertility and adult mortality are both endogenously determined in a dynamic general equilibrium OLG model. However, they assume human capital accumulation through education, instead of public health spending, as the main determinant of life expectancy. Their model can explain the existence of multiple regimes of development and accord with the stylised

fact of the so-called Demographic Transition, especially as regards the historical decline in fertility and mortality.) The present paper contributes to these two strands of literature and shows the importance of endogenous fertility as the main determinant of a dramatic change in the long-run demoeconomic outcomes with respect to the case of exogenous population growth.

The remainder of the paper is organised as follows. Section 2 presents the OLG model à la Chakraborty [5] extended with endogenous fertility, introduced according to the hypotheses of the so-called weak form of altruism towards children (see Zhang and Zhang [14]), that is, parents derive utility directly from the *number* of children they have, and shows that multiple equilibria can never appear. Section 3 concludes.

2. The Model

Consider a general equilibrium OLG closed economy populated by identical individuals, identical firms, and a government that runs a public health programme at a balanced budget. The lifetime of the typical agent is divided into childhood and adulthood, the latter being, in turn, divided into work time (youth) and retirement time (old age). As a child, each individual does not take economic decisions. When adult, he/she draws utility from material consumption and the number of children, as in Eckstein and Wolpin [15], Eckstein et al. [16], and Galor and Weil [17], which are assumed to be a normal good. This is the so-called weak form of altruism towards children (see Zhang and Zhang [14]), because parents derive utility from the number of children they have but do not enjoy from the utility derived by their offspring. Alternatively, under the hypothesis of “strong” altruism towards children, parents derive utility from both the number and well-being of their descendants (see Barro [18], Razin and Ben-Zion [19], Caballé [20, 21], Zhang [22]). (Indeed, there exists a strand of literature in which parents choose to have children to secure material support when old (this is essentially the case for several under-developed and developing countries), i.e. non-altruistic models of endogenous fertility (see Bental [23], Cigno [24] and Zhang and Nishimura [25]). In such a case, therefore, “Children are treated as (poor man’s) capital goods. It is postulated that children transfer a fraction of their income to their old parents and this fraction is determined endogenously.” (Zhang and Zhang [14], p. 1226.) In any case, however, the choice of the quantity (as well as the quality) of children is evaluated by parents according to economic incentives and constraints, namely, benefits and costs, of having and caring for descendants.

Young individuals of generation t (N_t) are endowed with one unit of labour which is inelastically supplied to firms and receive wage income at the rate w_t . We assume that the probability of surviving from youth to old age is endogenous and determined by the individual health level, which is in turn augmented with the public provision of health care services (see Chakraborty [5]). The survival probability at the end of youth of an individual started working at t , π_t , therefore depends upon her health capital, h_t , and is given

by a nondecreasing concave function $\pi_t = \pi(h_t)$, where $\pi(0) = 0$, $\lim_{h \rightarrow \infty} \pi(h) = \beta \leq 1$ and $\lim_{h \rightarrow 0} \pi'_h(h) = \gamma < \infty$.

We assume that the per worker health investment at t (h_t) is financed at a balanced budget with a (constant) wage income tax $0 < \tau < 1$, that is,

$$h_t = \tau w_t. \quad (1)$$

Moreover, the costs of children are assumed to be given by a fixed amount $e > 0$ of resources per child (see Van Groezen [26], van Groezen and Meijdam [27]). (See also Deaton and Muellbauer [28] and Raut and Srinivasan [29], which assume the costs of children as a combination of time- and income-based activities.) This assumption implies that each parent derives utility from having a child only whether he/she receives at least a certain amount of commodities and services for his/her upbringing. These costs are captured by the fixed price e , equal for all children, and can reflect—in a broad sense—the quality of each descendant. (See van Praag and Warneer [30] for a survey on estimates on the costs of children.)

Therefore, the budget constraint of an individual of the working-age (child-bearing) generation at t reads as

$$c_{1,t} + s_t + en_t = w_t(1 - \tau), \quad (2)$$

that is, wage income—net of the contribution to finance health expenditure—is divided into material consumption when young, $c_{1,t}$, savings, s_t , and the cost of raising n descendants.

Old individuals retire and live with the amount of resources saved when young plus the expected interest accrued from t to $t + 1$ at the rate r_{t+1}^e . (We assume that the individuals have perfect foresight with respect to the level of the future interest rate.) The existence of a perfect annuity market (where savings are intermediated through mutual funds) implies that old survivors will benefit not only from their own past saving plus interest but also from the saving plus interest of those who have deceased. Hence, the budget constraint of an old retired individual started working at t can be expressed as

$$c_{2,t+1} = \frac{1 + r_{t+1}^e}{\pi_t} s_t, \quad (3)$$

where $c_{2,t+1}$ is old-age consumption. The representative individual of generation t chooses fertility and saving to maximise the lifetime utility function:

$$U_t = \ln(c_{1,t}) + \pi_t \ln(c_{2,t+1}) + \phi \ln(n_t), \quad (4)$$

subject to (2) and (3), where $\phi > 0$ captures the parents’ relative taste for children. Therefore, fertility and saving are, respectively, given by

$$\begin{aligned} n_t &= \frac{\phi w_t(1 - \tau)}{(1 + \pi_t + \phi)e}, \\ s_t &= \frac{\pi_t w_t(1 - \tau)}{1 + \pi_t + \phi}. \end{aligned} \quad (5)$$

2.1. Production, Equilibrium, and Dynamics. Firms are identical and markets are competitive. Aggregate production is determined as $Y_t = AK_t^\alpha L_t^{1-\alpha}$, where Y_t , K_t , and $L_t = N_t$ are output, capital, and the labour input at time t , respectively, $A > 0$ and $0 < \alpha < 1$. Profit maximisation implies that factor inputs are paid their marginal products, that is,

$$\begin{aligned} r_t &= \alpha A k_t^{\alpha-1} - 1, \\ w_t &= (1 - \alpha) A k_t^\alpha, \end{aligned} \quad (6)$$

where $k_t := K_t/N_t$ is capital per worker (The output is sold at unit price and capital totally depreciates at the end of every period.)

Knowing that $N_{t+1} = n_t N_t$, market clearing in the capital market implies that $n_t k_{t+1} = s_t$, which is combined with (5) to obtain

$$k_{t+1} = \frac{e}{\phi} \pi(k_t), \quad (7)$$

where $\pi(k_t) = \pi[\tau(1 - \alpha) A k_t^\alpha]$. Analysis of (7) gives the following proposition.

Proposition 1. *The dynamic system described by (7) possesses two steady states $\{0, \bar{k}\}$, with $\bar{k} > 0$ (only the positive one being asymptotically stable).*

Proof. Let first the following lemma be established.

Lemma 1. *Define the right-hand side of (7) as $G(k)$. Then, one has that: (1.i) $G(0) = 0$, (1.ii) $G'_k(k) > 0$ for any $k > 0$, (1.iii) $\lim_{k \rightarrow +\infty} (G(k)/k) < 1$, and (1.iv) $\lim_{k \rightarrow 0^+} G'_k(k) = +\infty$.*

Proof. From (7), (1.i) and (1.ii) follow immediately. Now, since $\lim_{k \rightarrow +\infty} \pi(k) = \beta$, then

$$\lim_{k \rightarrow +\infty} \frac{G(k)}{k} = \frac{e}{\phi} \lim_{k \rightarrow +\infty} \frac{\pi(k)}{k} = \frac{e\beta}{\phi} \lim_{k \rightarrow +\infty} \frac{1}{k} = 0, \quad (8)$$

which proves (1.iii). Moreover,

$$\begin{aligned} \lim_{k \rightarrow 0^+} G'_k(k) &= \frac{e}{\phi} \alpha \tau (1 - \alpha) A \lim_{k \rightarrow 0^+} \frac{\pi'_k(k)}{k^{1-\alpha}} \\ &= \frac{e}{\phi} \alpha \tau (1 - \alpha) A \gamma \lim_{k \rightarrow 0^+} \frac{1}{k^{1-\alpha}} = +\infty, \end{aligned} \quad (9)$$

where we used $\lim_{k \rightarrow 0^+} \pi'_k(k) = \gamma$. This proves (1.iv). $\square$

Proposition 1 therefore follows. In fact, by properties (1.i) and (1.iv), zero is always an unstable steady state of (7). By (1.ii) and (1.iii), $G(k)$ is a monotonic increasing function of k and eventually falls below the 45° line. Since $\pi(k)$ is a nondecreasing concave function of k , then one and only one positive stable steady state exists for any $k > 0$. $\square$

Comparing Proposition 1 with the results of the existing literature gives the importance of our findings. In fact, Chakraborty ([5], Proposition 1 (i), p. 126) shows, in an OLG context with exogenous fertility, that $\alpha > 1/2$ represents a necessary condition for the existence of multiple steady

FIGURE 1: Capital accumulation function.

states if health investments are fairly scarce, while Bunzel and Qiao [6] find that a large enough level of technological development (high values of the index A in the Cobb-Douglas function) is indeed sufficient for the existence of at least one positive stable steady state when $\alpha > 1/2$, otherwise an economy is permanently entrapped into poverty. Unlike previous studies, Proposition 1 shows that scarce public investment in health does not cause the appearance of poverty traps when fertility is endogenous and the costs of children are fixed, that is, the unique equilibrium scenario of the standard Diamond's growth models is restored, as Figure 1 shows. Note that the assumption of fixed costs of children is crucial for the results. Indeed, if such costs were assumed to be either proportional to labour income or time based, the results by Chakraborty ([5], Proposition 1 (i), p. 126) would remain valid.

3. Conclusions

Recently Chakraborty [5] and Bunzel and Qiao [6] showed that development traps can appear in Diamond's overlapping generations model with exogenous fertility and endogenous lifetime. This may explain long-run differences in output and longevity across countries. In this paper we found that the poverty trap result by Chakraborty [5] is sensitive to the exclusion of endogenous fertility. Indeed, to the extent that during the stages of development individuals rationally choose the desired family size by comparing benefits and costs of children, we showed that development traps due to underinvestments in health are avoided, that is, the economy always converges towards a unique long-run outcome, if the costs of children are represented by a fixed share of income used for their upbringing. Our results seem to contrast the results by Moav [31] who in a model with the quantity-quality trade-off, that is, poor individuals choose to have more children of lower quality, finds that endogenous fertility can actually increase the likelihood of a poverty trap.

Of course, we are aware of the limits of our analysis, especially because our conclusions hold only under the

hypothesis of fixed costs of children. However, it is interesting to observe how the endogenisation of fertility dramatically changes the final outcomes in comparison with the case of exogenous fertility studied by Chakraborty [5].

Acknowledgment

The authors gratefully acknowledge an anonymous referee for useful comments on an earlier draft. The usual disclaimer applies.

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