Research Article

Carrier Frequency Offsets Problem in DCT-SC-FDMA System: Investigation and Compensation

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The Single-Carrier Frequency Division Multiple Access (SC-FDMA) system is a well-known system, which has recently become a preferred choice for uplink channels. In this system, the Carrier Frequency Offsets (CFOs) disrupt the orthogonality between subcarriers and give rise to Inter-carrier Interference (ICI), and Multiple Access Interference (MAI) among users. In this paper, the impact of the CFOs on the performance of the Discrete Cosine Transform (DCT) SC-FDMA (DCT-SC-FDMA) system is investigated. Then, a new low-complexity joint equalization and CFOs compensation scheme is proposed to cancel the interference in frequency domain. The Minimum Mean Square Error (MMSE) equalizer is utilized in the proposed scheme. A hybrid scheme comprising MMSE equalization, CFOs compensation, and Parallel Interference Cancellation (PIC) is also suggested and investigated for further enhancement of the performance of the DCT-SC-FDMA system with interleaved subcarriers assignment. For simplicity, this scheme will be referred to as the MMSE+PIC scheme. From the obtained simulation results, it is found that the proposed schemes are able to enhance the system performance, even in the presence of the estimation errors.

1. Introduction

In the SC-FDMA system, each user employs a different set of orthogonal subcarriers as in the Orthogonal Frequency Division Multiple Access (OFDMA) system. For perfect time and frequency synchronization, the different orthogonal subcarriers sets for the different users make it possible to avoid the MAI [1, 2]. However, similar to the OFDMA system, the SC-FDMA system is sensitive to CFOs, which are mainly due to oscillator mismatches and/or Doppler shifts. As a result, in this system, the CFOs disrupt the orthogonality between subcarriers and give rise to ICI and MAI among users [3, 4]. Moreover, CFOs compensation is difficult in uplink communications since the CFOs compensation for a certain user may result in the misalignment of the other synchronized users [4].

The existing SC-FDMA system uses the Discrete Fourier Transform (DFT) in its implementation. A method for using the DCT as an alternative to the DFT is presented in [5], where we have introduced an improved DCT-SC-FDMA system and compared its performance with that of the DFT-SC-FDMA and the OFDMA systems.

The issue of CFOs in multicarrier systems was extensively studied in the literature [6–10]. However, for the uplink SC-FDMA system, there have been a few papers that addressed this issue [3, 4, 11, 12]. In [3], the impact of the CFOs compensation on the performance of Multiple-Input Multiple-Output (MIMO) DFT-SC-FDMA system was investigated. Frequency offsets estimation for high-speed users in E-UTRA uplink was proposed and investigated in [4]. In [11], the authors proposed an equalizer to mitigate the impact of the residual MAI after the CFOs compensation process in the DFT-SC-FDMA system. A joint suppression method for the phase noise and CFOs by block-type pilots for the MIMO DFT-SC-FDMA system was discussed in [12].
2. DCT-SC-FDMA System Model

This section describes the uplink DCT-SC-FDMA system. We consider $U$ users communicating at the same time with a fixed base station through independent multipath Rayleigh-fading channels as shown in Figure 1. The received signals from all users at the base station are assumed to be synchronized in the time domain.

At the transmitter side, the modulated symbols are grouped into blocks and an $N$-points DCT is performed. Then, the subcarriers are mapped in the frequency domain. After that, an Inverse DCT (IDCT) is performed and a Cyclic Prefix (CP) of length $N_C$ is added to the resulting signal. The length of the CP must be greater than the maximum excess delay of the channel to accommodate the Interblock Interference (IBI). Finally, the resulting signal is transmitted through the wireless channel.

In matrix notation, the transmitted signal from the $u$th user ($u = 1, 2, \ldots, U$) can be formulated as follows:

$$
\mathbf{x}^u = \mathbf{P}_{\text{add}} \mathbf{D}_M^{-1} \mathbf{M}_T^u \mathbf{D}_N \mathbf{x}^u,
$$

where $\mathbf{x}^u$ is an $N \times 1$ vector containing the modulated symbols, $\mathbf{D}_N$ is the $N \times N$ DCT matrix, $\mathbf{M}_T^u$ is an $M \times N$ matrix describing the subcarriers mapping of the $u$th user. $\mathbf{D}_M^u$ is the $M \times M$ IDCT matrix. $M = Q \cdot N$, where $Q$ is the maximum number of users that can transmit simultaneously. $\mathbf{P}_{\text{add}}$ is an $(M + N_C) \times M$ matrix, which adds a CP of length $N_C$. The entries of $\mathbf{M}_T^u$ for both DCT-LFDMA and DCT-IFDMA are given, respectively, in

$$
\mathbf{M}_T^u = \left[ \mathbf{0}_{(u-1)\times N}; \mathbf{I}_N; \mathbf{0}_{(M-uN)\times N} \right],
$$

$$
\mathbf{M}_T^u = \left[ \mathbf{0}_{(u-1)\times N}; \mathbf{u}_1^T; \mathbf{0}_{(Q-u)\times N}; \ldots; \mathbf{0}_{(u-1)\times N}; \mathbf{u}_1^T; \mathbf{0}_{(Q-u)\times N} \right],
$$

where the $\mathbf{I}_N$ and $\mathbf{0}_{Q \times N}$ matrices denote the $N \times N$ identity matrix and the $Q' \times N$ all-zero matrix, respectively.
are the noise and the transmitted signal after the CP removal, where

\[ P_{\text{add}} = [C, I_M]^T, \]  

(4)

assuming perfect time synchronization at the receiver side, the received signal can be expressed as follows:

\[ r = \sum_{u=1}^{U} E^u H^u x^u + n, \]  

(6)

where \( E^u \) is an \((M + N_C) \times (M + N_C)\) diagonal matrix with elements \( [E^u]_{m,m} = e^{j\pi\zeta_u(2m+1)/2M} \), \( m = 0, \ldots, M + N_C - 1 \), which describes the CFO matrix of the \( u \)th user for the DCT-SC-FDMA system. \( \zeta_u \) is the normalized CFO of the \( u \)th user with respect to the subcarriers frequency spacing. \( H^u \) is an \((M + N_C) \times (M + N_C)\) matrix describing the channel of the \( u \)th user. \( n \) is an \((M + N_C) \times 1\) vector containing the noise. After the removal of the CP, the received signal becomes

\[ r = P_{\text{rem}} r = \sum_{u=1}^{U} E^u H^u x^u + \tilde{n}, \]  

(7)

where \( P_{\text{rem}} \) is an \( M \times (M + N_C)\) matrix, which removes the CP. It is given by

\[ P_{\text{rem}} = [0_{M \times N_C}, I_M]. \]  

(8)

\( E^u \) is an \( M \times M \) diagonal matrix, which describes the CFO of the \( u \)th user after the CP removal. \( \tilde{n} = P_{\text{rem}} n \) and \( \tilde{x}^u = P_{\text{rem}} x^u \) are the noise and the transmitted signal after the CP removal, respectively. \( H_{\Pi}^u \) is an \( M \times M \) circulant matrix describing the channel of the \( u \)th user.

After that, an \( M \)-points DFT is performed on the received signal as follows:

\[ R = F_M r = \sum_{u=1}^{U} \Pi_{\text{circ}}^u \Lambda^u \tilde{x}^u + N, \]  

(9)

where \( \Pi_{\text{circ}}^u = F_M E^u F_M^H \) is a circulant matrix representing the interference from the \( u \)th user. \( \tilde{x}^u = F_M \tilde{x}^u \) is an \( M \times 1 \) vector. \( N \) is the DFT of \( n \). The simplification of (9) depends on the fact that \( H_{\Pi}^u = F_M \Lambda^u F_M^H \), where \( \Lambda^u \) is an \( M \times M \) diagonal matrix containing the DFT of the circulant sequence of \( H_{\Pi}^u \).

After that, the Frequency Domain Equalization (FDE), the \( M \)-points IDFT, and the DCT-SC-FDMA demodulation processes are performed to provide the estimate of the modulated symbols as follows:

\[ \hat{x}^u = D_{\Pi}^H M_{\text{R}}^u D_M F_M^H W^u R, \]  

(10)

taking the transpose of (2) and (3), respectively, that is, \( M_{\text{R}}^u = M_{\text{R}}^H \).

Equation (10) can be written as follows:

\[ \hat{x}^u = A^k x^k + \tilde{A}^k x^k + \sum_{u=1}^{U} n^u x^u + \tilde{n}. \]  

(11)

The structures of all components of (11) are given as follows:

\[ A^k = \text{diag} \left( D_{\Pi}^{-1} M_{\text{R}}^k D_M F_M^H W^k \Pi_{\text{circ}}^k \Lambda^k F_M D_{\Pi}^{-1} M_{\text{R}}^H \right), \]

\[ \tilde{A}^k = D_{\Pi}^{-1} M_{\text{R}}^k D_M F_M^H W^k \Pi_{\text{circ}}^k \Lambda^k F_M D_{\Pi}^{-1} M_{\text{R}}^H D_N - A^k, \]

\[ \tilde{n} = D_{\Pi}^{-1} M_{\text{R}}^k D_M F_M^H W^k N, \]

\[ B^u = D_{\Pi}^{-1} M_{\text{R}}^u D_M F_M^H W^u \Pi_{\text{circ}}^u \Lambda^u F_M D_{\Pi}^{-1} M_{\text{R}}^H D_N. \]

Finally, the demodulation and the decoding processes take place in the time domain.

3. The Proposed Equalization Scheme

3.1. Mathematical Model. To derive the MMSE equalization matrix of the proposed scheme, (9) must be rearranged as follows:

\[ R = \Pi_{\text{circ}}^k \hat{x}^k + N_p, \]  

(13)

where \( \Pi_{\text{circ}}^k = \Pi_{\text{circ}}^k \Lambda^k \) is the \( M \times M \) interference matrix of the \( k \)th user. \( N_p = \sum_{u=1, u \neq k}^{U} \Pi_{\text{circ}}^k \hat{x}^u + N \) is the MAI plus noise matrix. Now, we define the error \( e \) between the estimated symbols \( \hat{x}^k = W_{\text{prop}}^k R \) and the transmitted symbols \( X^k \) as follows:

\[ e^k = W_{\text{prop}}^k R - \hat{x}^k. \]  

(14)

The equalization matrix of the \( k \)th user is determined by the minimization of the following Mean Square Error (MSE) cost function:

\[ J^k = E \left\{ ||e^k||^2 \right\} = E \left\{ ||W_{\text{prop}}^k R - \hat{x}^k||^2 \right\}, \]  

(15)

where \( E \{ \cdot \} \) is the expectation. Solving \( \partial J^k / \partial W_{\text{prop}}^k = 0 \), we obtain that

\[ W_{\text{prop}}^k = R_{\text{X}}^k \Pi_{\text{circ}} \left( \Pi_{\text{circ}}^k R_{\text{X}}^k \Pi_{\text{circ}}^H + R_{\text{N}_p}^k \right)^{-1}. \]  

(16)

where \( R_{\text{X}}^k \) and \( R_{\text{N}_p}^k \) are the data and the overall noise (MAI plus noise) covariance matrices of the \( k \)th user. We assume that the noise is additive white Gaussian with zero mean and covariance \( \sigma_n^2 \), such that \( E[N_d N_d^H] = \sigma_n^2 I_N \). Then, \( R_{\text{N}_p}^k \) can be obtained as follows:

\[ R_{\text{N}_p}^k = E \left\{ \sum_{u=1, u \neq k}^{U} \Pi_{\text{circ}}^u \hat{x}^u + N_d \right\} \left\{ \sum_{u=1, u \neq k}^{U} \Pi_{\text{circ}}^u \hat{x}^u + N_d \right\}^H. \]  

(17)

\[ = \sum_{u=1, u \neq k}^{U} \Pi_{\text{circ}}^u \Pi_{\text{circ}}^H + \sigma_n^2 I_M. \]
frequency spacing between these subcarriers increases. As a result, a threshold \( r \) for the number of subcarriers, beyond which the interference is neglected, can be introduced as a design parameter. From Figure 2, we can see that \( r = 10 \) is the best choice to give a good performance in the proposed equalization scheme with an acceptable complexity.

3.3. Complexity Evaluation. For the proposed MMSE equalization scheme, the inversion of an \( M \times M \) matrix for each user is required, which is practically difficult for a large \( M \). The full system implementation requires a complexity of \( O(M^3) \), which is large for a system with a large number of active subcarriers. However, the structure of the approximated interference matrix is a banded structure. Thus, the banded matrix implementation is considered to reduce the complexity of the proposed equalization scheme. The total number of operations required in our banded matrix implementation for all users is approximately \( MU[16r^2 + 26r + 5] \) [13, 14]. Hence, for large values of \( M \), the overall complexity is lower than that of the full system implementation.

4. The Proposed MMSE + PIC Scheme

It is known that the DCT-IFDMA system is more sensitive to CFOs than the DCT-LFDMA system. So, at high CFOs and SNR values, the residual MAI after the proposed MMSE equalization may degrade the Bit Error Rate (BER) performance of the DCT-IFDMA system. To solve this problem, the proposed MMSE equalizer is combined with PIC to further reduce the effect of the residual MAI on the DCT-IFDMA system, as shown in Figure 3. In the proposed MMSE + PIC scheme, the MMSE equalizer is used to estimate the MAI interference, which is then regenerated and removed from the original received signal using the PIC in the frequency domain.

5. Simulation Results

To evaluate the performance of the proposed schemes, some simulation experiments are carried out. We consider an uplink DCT-SC-FDMA system with 512 subcarriers. In this system, there are four users with 128 subcarriers allocated to each user. The users employ Quadrature Phase Shift Keying (QPSK) mapping for their data symbols. The channel model used for simulations is the vehicular A model [15]. A convolutional code with rate 1/2, constraint length 7, and octal generator polynomial (133,171) is used. Each frequency offset is a random variable with uniform distribution in \([-0.3, 0.3]\). The CFOs are chosen randomly to simulate a more practical scenario. For the comparison purpose, single-user detector [7], and the circular convolution detector [7, 10] are simulated for the uplink DCT-SC-FDMA system.

Figure 4 shows a comparison in the BER performance between the single-user detector, the circular convolution detector, and the proposed MMSE equalization scheme for the DCT-SC-FDMA system. The DCT-SC-FDMA system without CFOs and the DCT-SC-FDMA system with CFOs but without compensation are also studied for comparison.
It is clear that the proposed MMSE scheme significantly outperforms the conventional schemes, especially at high SNR values. Although the performance of the proposed scheme for the DCT-IFDMA system is superior to all conventional schemes, the performance loss is about 2.5 dB at a BER $= 10^{-3}$. Adding a PIC stage to the proposed scheme can avoid this loss and provide a better BER performance, especially at large CFOs values.

Figure 5 shows the BER performance of the proposed MMSE and MMSE + PIC schemes for the DCT-IFDMA system. From this figure, it is clear that the PIC can avoid the MAI and provide better BER performance than the MMSE scheme. It is observed from Figure 5 that the performance loss due to the proposed MMSE + PIC is 0.5 dB at a BER $= 10^{-3}$, which is acceptable.

Figure 6 illustrates the BER performance of the proposed schemes versus the maximum normalized CFO $\zeta_{\text{max}}$ for the DCT-IFDMA system. It can be seen that the performance of the proposed MMSE + PIC scheme is always better than the MMSE scheme, especially at large CFOs values.

The impact of the channel estimation errors on the performance of the proposed schemes for the DCT-SC-FDMA system is studied and shown in Figures 7 and 8. SNR = 20 dB is considered. The estimated CFOs are obtained by adding the true values of the CFOs to
a zero-mean independent Gaussian random variable. The estimated channels are obtained by the same manner.

Figure 7 shows that the performance of the DCT-LFDMA system with the proposed MMSE scheme starts to degrade as the standard deviation of the CFOs estimation error ($\delta_{\text{CFO}}$) becomes larger than 0.05 and the standard deviation of the channel coefficients estimation error ($\delta_{\text{Ch}}$) becomes larger than 0.01. Figure 8 shows that the performance of the DCT-IFDMA system with the proposed MMSE and the proposed MMSE + PIC schemes starts to degrade as $\delta_{\text{CFO}}$ becomes larger than 0.01 and $\delta_{\text{Ch}}$ becomes larger than 0.001. This indicates that the proposed compensation schemes are robust to the estimation errors.

6. Conclusions

In this paper, the issue of the CFOs in the uplink DCT-SC-FDMA system is investigated and compensated. Simulation results show that CFOs destroy the orthogonality of the sub-carriers and result in ICI and MAI, which degrades the BER performance. This paper presented two new compensation schemes. Simulation results show that the proposed MMSE and MMSE + PIC schemes are able to mitigate the impact of CFOs to provide a better BER performance for DCT-SC-FDMA system. Also, simulation results demonstrate that the proposed MMSE and MMSE + PIC schemes outperform both the circular convolution detector and the single-user detector. Moreover, it is found that the proposed MMSE and MMSE + PIC schemes are robust to the estimation errors.

References


