

Review Article

Lie Group of Spacetime

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A brief review is presented of de Sitter-Fantappiè relativity, and we propose some cosmological reflections suggested by this theory. Compared to the original works, some deductions have been very simplified, and only the physical meaning of the equations has been analyzed.

1. Introduction

When Albert Einstein and Willem de Sitter applied the theory of general relativity (GR) to the spacetime of the universe, cosmology gained a foundation in physics that it had lacked previously, when it was only a subject in philosophy. Predictions of relativistic cosmology include the initial abundance of chemical elements formed in a period of primordial nucleosynthesis, the large-scale structure of the universe, and the existence and properties of a thermal echo from the early cosmos, the cosmic background radiation. Despite this, we have to pay close attention to general relativity, where, inevitably, the application of Einstein's equations to cosmological problems requires an extreme extrapolation of their validity to very far regions of spacetime. We can look for solutions which present a cosmological interest, as long as we take into account, for this type of problem, that such equations could be little more than a good model. As it is known, minor changes to the equations, while exhibiting all the classical verifications, produce completely different cosmologically interesting solutions. It is useful to observe that even though cosmology accepts GR as a definitive theory of gravitation, there are still some uncertain aspects due to a baffling pluralism. Possible universes are numerous and differ from each other substantially. The relativistic cosmology is unable to provide an explanation as to why the density of the universe should be so close to the critical value. In fact we have

$$\Omega - 1 = \frac{k}{H^2 a^2} = \frac{k}{\dot{a}^2}, \quad (1)$$

where Ω is the density parameter, H is the Hubble constant, k is the curvature, and a is the scale factor. We see that as \dot{a}^2 decreases with time, and $|\Omega - 1|$ must increase if k is nonzero. This means that the universe diverges from the flat case if $k \neq 0$, and the fact that it appears to be almost flat today means that Ω must have been very close to one in the early universe. Besides at present, the cosmic microwave background is observed to be extremely homogeneous and isotropic on large scales, with temperature fluctuations of only $10^{-5}K$. This suggests that all regions of the sky were in casual contact at some time in the past, but is contradicted as follows. The horizon size is the distance light has travelled since the beginning of the universe and is given by

$$d(t) = a(t) \int_{t_1}^{t_2} \frac{dt}{a(t)} \quad (2)$$

which remains finite as $a(t_1) \rightarrow 0$ if $\ddot{a} > 0$. When the microwave background was formed the region in casual contact would have been approximately 0.09 Mpc. With the subsequent expansion this corresponds to a patch of the present microwave background subtending an angle of only 2 degrees. The most principal problem is the singularity problem, and, according to Hawking-Penrose theorems, the appearance of singularity in cosmological solutions of general relativity is inevitable [1, 2]. Many physicists and cosmologists are inclined to believe that classical general relativity must be revised in the case of extremely high energy densities, pressures, and temperatures. The singularity must mean for cosmology that the classical Einsteinian theory is inapplicable in the beginning of cosmological expansion

of the universe. Finally recent observations of Type Ia supernovae indicate that the universe is in an accelerating expansion phase and its geometry is flat [3–5]. These observations give rise to the search for a field which can be responsible for accelerated expansion, and Dark Energy is the most popular way to explain these recent observations. The simplest Dark Energy candidate is the cosmological constant Λ introduced by Einstein to obtain a static universe but later on he himself rejected it. This vacuum energy density is equivalent to a perfect fluid obeying the equation of state $p_\Lambda = -\rho_\Lambda$. However, the nature and cosmological origin of Dark Energy still remain enigmatic at present, and it is not clear yet whether Dark Energy can be described by a cosmological constant which is independent of time or by dynamical scalar fields such as quintessence. Due to these facts alternative theories have been considered as, for example, Extended Theories of Gravity which have become a sort of paradigm in the study of gravitational interaction based on the enlargement and the correction of the traditional Einstein scheme [6, 7] role in the development of GR, as, for example, the Brans-Dicke theory in which the gravitational interaction is mediated by a scalar field as well as the tensor field of GR, and the gravitational constant G is not presumed to be constant but instead $1/G$ is replaced by a scalar field which can vary from place to place and with time [8]. Another theory in accord with Mach's Principle is de Sitter-Fantappiè relativity initially proposed by Fantappiè and subsequently developed by Arcidiacono and other authors [9–22]. At present, both Brans-Dicke theory and de Sitter-Fantappiè relativity are generally held to be in agreement with observation. Let us remember that inflation has been very successful in solving the problems in the standard big bang cosmology such as the horizon and flatness problems. When the universe is dominated by the material whose equation of state satisfies $p < -\rho/3$ the accelerating universe is realized. A natural candidate of matter with negative pressure necessary to drive inflation is a scalar field, so-called “inflaton.” A homogeneous classical scalar field $\phi = \phi(t)$ is characterized by the energy density $\rho(\phi)$ and the pressure $p(\phi)$

$$\begin{aligned}\rho(\phi) &= \frac{1}{2}\dot{\phi}^2 + V(\phi), \\ p(\phi) &= \frac{1}{2}\dot{\phi}^2 - V(\phi),\end{aligned}\tag{3}$$

where $V(\phi)$ is the potential of the scalar field. From the Klein-Gordon equation, the equation of motion for the homogeneous scalar field is given by

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0,\tag{4}$$

where H is the Hubble constant and $V_\phi = dV/d\phi$. Let us remember that $H = (\dot{a}/a) \Rightarrow \dot{H} = (\ddot{a}a - \dot{a}^2)/a^2 = (\ddot{a}/a) - H^2$,

and therefore in the scalar field dominant universe, the Friedmann equations become

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2} \left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right),\tag{5}$$

$$\dot{H} = -\frac{4\pi G}{c^2} \dot{\phi}^2.\tag{6}$$

Inflation is realized when $(1/2)\dot{\phi}^2 \ll V$. Let us define two parameters, so-called “slow-roll parameters,” as

$$\epsilon = \frac{1}{16\pi G} \left(\frac{V_\phi}{V}\right)^2, \quad \eta = \frac{1}{8\pi G} \frac{V_{\phi\phi}}{V},\tag{7}$$

where $V_{\phi\phi} = d^2V/d\phi^2$. The conditions $\epsilon \ll 1$ and $\eta \ll 1$ are satisfied in the slow-roll approximation. Under these conditions, the homogeneous background equations (6) and (7) are reduced to

$$\begin{aligned}3H\dot{\phi} &\simeq -V_\phi, \\ H^2 &\simeq \frac{8\pi G}{3} V.\end{aligned}\tag{8}$$

Under the slow-roll conditions, slow-roll parameter can be rewritten in terms of Hubble parameter as

$$\epsilon \simeq \frac{1}{16\pi G} \frac{\dot{\phi}^2}{H^2} = -\frac{\dot{H}}{H^2} = -\frac{\ddot{a}}{aH^2} + 1.\tag{9}$$

From this expression, $\epsilon < 1$ is equivalent to the accelerated expansion and the universe expands as if dominated by the cosmological constant, that is, the universe behaves as de Sitter universe, expanding exponentially. The horizon problem is solved because the different regions we see used to be close enough to communicate, but during inflation, space expanded so rapidly that these close regions were spread out to cover all of the visible universe. As inflation causes the ball to expand, the curvature of the ball lessens as well. Thus, as the universe expands, it starts to appear flatter. For this reason the picture of an early period of inflation followed by standard decelerated expansion is widely accepted today by cosmologists as an accurate description of the early evolution of our universe. The problem is that there is no fundamental theory that can explain why inflation happened in our universe. It is just proposed as an ad hoc solution.

The paper is organized as follows. In Section 2 we introduce the Lie groups, while in Section 3 we analyze the group structure of spacetime and its consequences; finally in Section 4 we give the conclusion.

2. Lie Groups and Lie Algebras

Symmetry is one of the most fundamental properties of nature. The branch of mathematics dealing with symmetry is the group theory. Lie groups lie at the intersection of these two fundamental fields of mathematics: algebra and geometry. A Lie group is first of all a group, and secondly it is a differentiable manifold. A Lie group is a group which is also a differentiable manifold, with the property that the group

operations are compatible with the differential structure. That is, the applications

- (a) $G \times G \rightarrow G : (a, b) \rightarrow a \cdot b$,
- (b) $G \rightarrow G : a \rightarrow a^{-1}$

are differentiable. Informally, a Lie group is a group of symmetries where the symmetries are continuous. A Lie algebra is a vector space V over some field K together with a binary operation $[\cdot, \cdot] : V \times V \rightarrow V$ called the Lie bracket, which satisfies the following axioms.

- (a) Bilinearity:

$$\begin{aligned} [a + b, c] &= [a, c] + [b, c], \\ [a, b + c] &= [a, b] + [a, c], \\ [\lambda a, b] &= [a, \lambda b] = \lambda [a, b], \end{aligned}$$

for all $a, b \in V$ and for all $\lambda \in K$.

- (b) Anticommutativity:

$$[a, b] = -[b, a] \text{ for all } a, b \in V.$$

- (c) The Jacobi identity:

$$[[a, b], c] + [[b, c], a] + [[c, a], b] = 0 \text{ for all } a, b, c \in V.$$

Besides from the second axiom we deduce

$$\begin{aligned} [a, b] + [b, a] = 0 &\Rightarrow [a, a] + [a, a] = 2[a, a] = \\ 0 &\Rightarrow [a, a] = 0 \text{ for all } a \in V. \end{aligned}$$

For any associative algebra $(A, *)$ one can construct a Lie algebra where the Lie bracket is defined by commutator $[a, b] = a * b - b * a$.

If $\{e_1, \dots, e_n\}$ is a basis of V we can introduce the so-called structure constants defined by $[e_i, e_j] = c_{kij}e_k$, and these constants are useful to calculate the commutator of any $a, b \in V$. In fact $[a, b] = a_i b_j [e_i, e_j] = a_i b_j c_{kij} e_k$. Any vector space V endowed with the identically zero Lie bracket becomes a Lie algebra. Such Lie algebras are called Abelian. The three-dimensional Euclidean space R^3 with the Lie bracket given by the cross product of vectors becomes a three-dimensional Lie algebra.

After these premises, we analyze some groups of transformation which are particularly important in the study of physics. In an Euclidean n -dimensional space, the linear and homogeneous transformations

$$x'_i = \sum_k a_{ik} x_k \quad (10)$$

that leave invariable the quadratic form $s^2 = x_1^2 + x_2^2 + \dots + x_n^2$ are said to be orthogonal transformations and form a group and precisely the orthogonal group O_n . In this case the matrix $[a_{ik}]$ is orthogonal and, that is, its inverse is equal to its transpose. Besides the determinant of $[a_{ik}]$ is ± 1 , and the orthogonal transformations whose matrices have determinant equal to 1 form a subgroup called special orthogonal group $SO(n)$ which is the group of all rotations about the origin of n -dimensional Euclidean space.

The numbers of parameters of the orthogonal and special orthogonal group is $n(n-1)/2$. The orthogonal group is a subgroup of the general linear group GL_n with determinant of $[a_{ik}] \neq 0$, and this transformations group has n^2 parameters. We also quote the special linear group written SL_n , and this is a subgroup of GL_n consisting of matrices with determinant equal to 1. Instead $U(n)$ is the unitary group, and that is the group of $n \times n$ unitary complex matrices of dimension n^2 . Obviously this is a subgroup of the general linear group and finally we quote the special unitary group denoted $SU(n)$ consisting of all $n \times n$ unitary complex matrices with unit determinant, and its dimension is $n^2 - 1$ standard model of particle physics; in fact $SU(2)$ is the basic group of the electroweak interaction and $SU(3)$ of quantum chromodynamics that describes the strong interaction. The mathematician Sophus Lie had the idea to consider in the study of continuous groups of transformations only the infinitesimal elements of group, that is, the elements that are in the proximities of the identity element. The properties of these infinitesimal elements characterize the properties of the group. We consider a group of transformations with one parameter t

$$x'_i = x_i(x_1, x_2, \dots, x_n, t), \quad (11)$$

and we introduce the infinitesimal operator X of the group. It is possible to show that [23]

$$x'_i = x_i + tXx_i + \frac{1}{2}t^2X^2x_i + \dots = e^{tX}x_i. \quad (12)$$

When the parameter t is considered infinitely small, we obtain the infinitesimal transformations

$$x'_i = x_i + tXx_i. \quad (13)$$

For example, the infinitesimal operator of rotations group on a plan is $X = x_1\partial_2 - x_2\partial_1$, and we have the infinitesimal transformations

$$\begin{aligned} x'_1 &= x_1 - tx_2, \\ x'_2 &= x_2 + tx_1. \end{aligned} \quad (14)$$

Assigned a continuous group to r parameters, it is possible to calculate its infinitesimal operators and its infinitesimal transformations. Vice versa, assigning r infinitesimal operators X_1, \dots, X_r and introducing the Lie bracket (related to the Poisson bracket) $[X_i, X_k] = X_iX_k - X_kX_i$ we have " r infinitesimal operators, among them independent, that can produce a group with r parameters if and only if $[X_i, X_k] = \sum_s c_{iks}X_s$ ".

Two groups with the same structure constants are isomorphic in the neighborhood of the identity element.

Cartan has given a complete classification of the simple groups, and he has found that there are four infinite families.

- (1) A_n groups; a model of these groups is $SU(n+1)$.
- (2) B_n groups; a model of these groups is $SO(2n+1)$.
- (3) C_n groups; a model of these groups is $Sp(2n)$.
- (4) D_n groups; a model of these groups is $SO(2n)$.

$Sp(2n)$ denotes the symplectic group. Then there are other five possible simple groups and that is the so-called exceptional cases G_2, F_4, E_6, E_7 , and E_8 . These cases are said exceptional because they do not fall into infinite series of groups of increasing dimension. G_2 has 14 dimensions, F_4 has 52 dimensions, E_6 has 78 dimensions, E_7 has 133 dimensions, and E_8 has 248 dimensions. The E_8 algebra is the largest and most complicated of these exceptional cases and is often the last case of various theorems to be proved. E_8 is a very beautiful group; in fact it is the symmetries of a particular 57-dimensional object while E_8 itself is 248-dimensional.

3. SpaceTime Group

As reported in [19], Fantappiè's starting point was the study of classical and relativistic physics spacetime. Spacetime of classical physics is a 4-dimensional manifold endowed with the following geometrical structures:

- (1) absolute time;
- (2) absolute space;
- (3) absolute spatial distances;
- (4) absolute temporal distances.

Spacetime of special relativity, instead, is a 4-dimensional manifold endowed with the following geometrical structures:

- (1) relative time;
- (2) relative space;
- (3) absolute spacetime distances.

Fantappiè noted that general relativity follows an extra-neous approach to the tradition of mathematical physics in that it does not follow the group structure of physics. This is different from classical mechanics and special relativity.

As it is well known let us remember that Galileo's group is the main group of classical physics and is formed by the composition of the following transformations.

- (a) Spatial Rotations characterized by three parameters

$$x'_\mu = a_{\mu\nu}x_\nu, \quad t' = t, \quad (15)$$

where $[a_{\mu\nu}]$ is an orthogonal matrix whose determinant is +1.

- (b) Inertial movements characterized by the three components of velocity

$$x'_\mu = x_\mu + v_\mu t, \quad t' = t. \quad (16)$$

- (c) Spatial translations characterized by three parameters

$$x'_\mu = x_\mu + a_\mu, \quad t' = t. \quad (17)$$

- (d) Temporal translations characterized by only one parameter

$$x'_\mu = x_\mu, \quad t' = t + t_0. \quad (18)$$

Galileo's group has order 10 and expresses Galileo's well-known relativity principle. Moving on to relativistic physics, spatial rotations and inertial movements are blended in a unique operation, the rotations of an Euclidian space M_4 , characterized by 6 parameters,

$$x'_i = a_{ik}x_k, \quad (19)$$

where $|a_{ik}| = 1$, $x_1 = x$, $x_2 = y$, $x_3 = z$, $x_4 = ict$.

These transformations, called Lorentz's special transformations, form Lorentz's proper group and joining the reflections, form Lorentz's extended group. Then we need to add the translations of M_4

$$x'_i = x_i + a_i, \quad (20)$$

characterized by 4 parameters, which comprise spatial and temporal translations. By composing the transformations of these two groups, we obtain Lorentz's general transformations which form Poincaré's group of 10 parameters

$$x'_i = a_{ik}x_k + a_i. \quad (21)$$

Poincaré's group mathematically translates Einstein's relativity principle. When $c \rightarrow \infty$ so that $v/c \ll 1$, Minkowski's spacetime reduces to that of Newton's and Poincaré's group reduces to Galileo's group.

Fantappiè went on this direction and tried to understand if Poincaré's group could be the limit of a more general group, in the same manner as Galileo's group is the limit of Poincaré's group. In [9] He wrote a new group of transformations which had as limit Poincaré's group, and He was able also to demonstrate that his group was not able to be the limit of any continuous group of 10 parameters. That is, by limiting to groups of 10 parameters and to 4-dimensional spaces, what happened with Galileo's and Poincaré's groups cannot be repeated. For this reason this group is called the final group.

Fantappiè's group is characterized by two constants: speed of light c and a radius of spacetime r . This group determines a universe endowed with a perfect symmetry: De Sitter's universe. Let us remember that de Sitter's universe is obtained from Einstein's equations with a positive cosmological constant and the solution of motion equation is the following:

$$r(t) = r(0)e^{ct\sqrt{\Lambda/3}}. \quad (22)$$

This model is a 4-dimensional hyperboloid in real time and a 4-dimensional sphere in imaginary time.

Fantappiè, moving from a spacetime with hyperbolic structure, showed that, through a flat projective representation, one could obtain a spacetime which generalizes Minkowski's spacetime.

The de Sitter-Fantappiè relativity consists of two distinct theories: special relativity (DFSR) and general relativity (DFGR). The relationship between DFSR and DFGR is the same that exists between Einsteinian special relativity (SR) and general relativity. DFSR is a generalization of SR obtained by requiring the laws of physics to be invariant

with respect to the Fantappiè group, instead of the Poincaré group. When the radius of the universe r tends to infinity, the Fantappiè group degenerates into the Poincaré group, and DFSR becomes the usual SR. DFSR coincides locally with SR, and its only difference from it lies in the predictions relating to the observation of objects that are very distant in space or events that are very distant in time; thus, crucial experiments (or, rather, observations) capable of confuting or verifying DFSR can only be carried out in a cosmological context. Let us remember that, to have a flat representation of hyperbolic geometry, we fix a circle in the plane, with center O and radius r , called the absolute of Cayley-Klein. By introducing a system of orthogonal coordinates with origin in the center of the circle, it is not possible to represent the distance of two points $A(x, y)$ and $B(x', y')$ in the form $\sqrt{(x-x')^2 + (y-y')^2}$ because it is not invariant for the projective transformations. An expression of the coordinates of A and B , which remains invariable for all the projective transformations which leave the limit circle fixed, is the harmonic ratio of the four points A, B, M, N

$$(ABMN) = \left(\frac{AM}{BM}\right) : \left(\frac{BN}{AN}\right), \quad (23)$$

where M and N are the extremes of chord AB .

We assume as distance

$$\text{dist}(AB) = k \log(ABMN). \quad (24)$$

Therefore every point of the hyperbolic plane always remain at infinite distance from the points of the *absolute*. So the hyperbolic plane is finite and limited if we make the measures in an Euclidian sense. On the contrary, if the measures are not Euclidian, the hyperbolic plane is infinite and unlimited.

Fantappiè chooses, as the absolute quadric, the hypersphere

$$x^2 + y^2 + z^2 - c^2 t^2 + r^2 = 0. \quad (25)$$

He showed that Minkowski's spacetime can be considered as a limit case of the projected spacetime when $r \rightarrow \infty$. Poincaré's group proves the limit of the group of motions of new spacetime in itself. To determine the transformations of the new group, we observe that the spacetime motions are represented by the projections that transform the absolute circumference in itself. Such absolute, with the introduction of imaginary time, can be written as

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 + r^2 = 0, \quad (26)$$

and, by considering the homogeneous coordinates \bar{x}_A so defined

$$x_k = r \frac{\bar{x}_k}{\bar{x}_5}, \quad (27)$$

it becomes

$$\bar{x}_1^2 + \bar{x}_2^2 + \bar{x}_3^2 + \bar{x}_4^2 + \bar{x}_5^2 = 0. \quad (28)$$

It follows that the motions we are searching for are those which leave the prior quadratic form invariable and are

the orthogonal substitutions on the five variables \bar{x}_A . These transformations form the group of 5-dimensional rotations. Fantappiè transformations can be thought of as rotations in a five-dimensional pseudo-Euclidian spacetime:

$$\bar{x}^C = \Lambda_D^C \bar{x}^D. \quad (29)$$

Let us synthesize the geometric structure of Fantappiè's group in the following scheme:

$$\begin{aligned} \text{GALILEO'S GROUP} &\implies \text{Rotations } S_3 \\ &\implies \text{Pullings} \\ &\implies \text{Translations } S_3 \\ &\implies \text{Time Translations} \quad (30) \\ \text{POINCARÉ'S GROUP} &\implies \text{Rotations } S_4 \\ &\implies \text{Translations } S_4 \end{aligned}$$

$$\text{FANTAPPIÉ'S GROUP} \implies \text{Rotations } S_5$$

The connection between the metric approach to Einstein gravitation and Fantappiè-Arcidiacono group one is the aim of DFGR, which describes a universe globally at constant curvature and locally at variable curvature. In DFGR we have the following five-dimensional gravitational equations that generalize the four-dimensional Einstein equations:

$$R_{AB} - \frac{1}{2} R \gamma_{AB} = \chi T_{AB} \quad (A, B = 0, 1, 2, 3, 4), \quad (31)$$

where γ_{AB} are the coefficients of the five-dimensional metric and

$$T_{AB} = \left(\mu + \frac{p}{c^2}\right) (u_A u_B - h^2 x_A x_B) + p \gamma_{AB}. \quad (32)$$

In these equations, R_{AB} is the projective curvature contracted tensor. The energy tensor T_{AB} is defined as a function of the velocity of the cosmic fluid, of its pressure, and of its mass density. It has been assumed that $h = c/r$. While in classical GR the curvature tensor equal to zero means Minkowski spacetime, in DFGR curvature tensor equal to zero means de Sitter spacetime. In the DFGR scheme, the projective metrics

$$ds^2 = \gamma_{AB} d\bar{x}^A d\bar{x}^B \quad (33)$$

satisfies the normalization condition

$$\gamma_{AB} \bar{x}^A \bar{x}^B = r^2. \quad (34)$$

For simplicity we set $r = 1$ getting

$$d\bar{x}^i = x^i d\bar{x}^0 + \bar{x}^0 dx^i, \quad (35)$$

and we have

$$\begin{aligned} ds^2 &= (\gamma_{ik} x^i x^k) (\bar{x}^0)^2 + 2(\gamma_{ik} x^i x^k + \gamma_{i0}) dx^i (\bar{x}^0 d\bar{x}^0) \\ &+ (\gamma_{ik} x^i x^k + 2\gamma_{i0} x^i + \gamma_{00}) (d\bar{x}^0)^2. \end{aligned} \quad (36)$$

Substantially a 4-dimensional space with projective connections is a space which, in the infinitesimal neighbours of every of its points, is a projective space and is also provided with a law of projective representation between neighbours of two of its infinitely close points. In this space we have a field of quadrics

$$Q = g_{AB} dx^A dx^B = 0. \quad (37)$$

For every point the quadric is the absolute of a local non-Euclidean metric. The conservation of the quadric field requires that

$$\nabla_A g_{BC} = 0, \quad (38)$$

where the covariant derivative is built from the connections coefficients

$$\pi_{BC}^A = \frac{1}{2} g^{AK} (\partial_C g_{BK} + \partial_B g_{KC} - \partial_K g_{CB}). \quad (39)$$

From these coefficients we can construct a projective curvature tensor

$$R_{BLM}^A = \partial_L \pi_{BM}^A - \partial_M \pi_{BL}^A + \pi_{RL}^A \pi_{BM}^R - \pi_{KM}^A \pi_{BL}^K. \quad (40)$$

In [16] the authors have solved the gravitational equations in the following case:

$$\lim_{t_0 \rightarrow \infty} \left(R_{AB} - \frac{1}{2} R g_{AB} - \chi T_{AB} = 0 \right) \quad (41)$$

obtaining the Fridman cosmological model having the spatial curvature $k = 0$. As it can be seen, unlike as in conventional GR, *the spatial section is necessarily flat* in DFGR. Consequently, there is no critical value of at which space is flat. This removes the flatness problem afflicting the standard model.

The varying speed of light cosmology has been proposed independently by variable authors [24–29] to explain the horizon problem of cosmology and propose an alternative to cosmic inflation. In fact in varying speed of light theories near the beginning of the universe, all points of the expanding space will have been in communication with one another, solving the horizon problem. It is possible to write the Friedmann relations in the presence of time-dependent speed of light

$$\begin{aligned} \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3} \left(\rho + \frac{3p}{c^2(t)} \right) + \frac{\Lambda c^2(t)}{3}, \\ \left(\frac{\dot{a}}{a} \right)^2 + \frac{kc^2(t)}{a^2} &= \frac{8\pi G}{3} \rho + \frac{\Lambda c^2(t)}{3}, \end{aligned} \quad (42)$$

where $a(t)$ is the scale factor, p is the fluid pressure, ρ is the fluid density, k is the curvature parameter, Λ is the cosmological constant, and all derivatives are with respect to comoving proper time. From these relations they derive the modified matter conservation

$$\rho + \frac{3\dot{a}}{a} \left(\rho + \frac{p}{c^2(t)} \right) = \frac{3kc^2(t)\dot{c}(t)}{4\pi G a^2}. \quad (43)$$

If we have a time variation of Newton's constant we get

$$\rho + \frac{3\dot{a}}{a} \left(\rho + \frac{p}{c^2(t)} \right) = -\rho \frac{\dot{G}}{G} + \frac{3kc^2(t)\dot{c}(t)}{4\pi G a^2}. \quad (44)$$

The problem is that there is no theory that explains why the speed of light changed. Instead, in de Sitter relativity, we have the speed of light constant only when the spatial and temporal distances are not comparable with de Sitter radius. The speed of light is a function of spacetime length as consequence of de Sitter-Fantappiè group [19], and, once defined a point in the space, it increases by going back in time. At the time of Big Bang the light had infinite speed. This could explain how the universe today seems so uniform without introducing the inflationary cosmology but as natural consequence of algebraic structure of spacetime.

4. Conclusions

De Sitter-Fantappiè relativity is a speculative idea that the fundamental symmetry group of spacetime is that of de Sitter space. The discovery of the accelerating expansion of the universe has led to a revival of interest in de Sitter invariant theories, in conjunction with other speculative proposals for new physics, like doubly special relativity. The Poincaré group generalizes the Galilean group for high velocity. Similarly, the de Sitter-Fantappiè group generalizes Poincaré for long spacetime distances. Cosmological consequences of such extension appear relevant since they resolve many problems of the standard cosmology and the global spacetime structure is univocally individuated by the algebraic structure of the physical laws.

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