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Research Article

The Computing of Intersectant Relations for Its Strength Problem on Damage and Fracture to Materials with Short and Long Crack

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Adopt two types of damage variables, a and D, and make the bidirectional combined coordinate system and the bidirectional curves in the whole process, describing their damage evolutive behaviors on fatigue damage-fracture to elastic-plastic steels; communicate the cross-referencing between their computing models and between describing curves at each stage among varied disciplines; bring forward a viewpoint about the driving force of material damage, that is the damage stress factor at crack forming stage, Provide the computation expressions and computing methods of the strength problem of materials with short crack and long crack at each stage; reveal the geometrical and the physical meaning of force triangle and its edge vector at each stage, Provide the conversion methods between the variables, the equations, the material constants and the dimensional units; Indicate the physical and the geometrical meanings for some key parameters. This will be having practical significance for promoting developing, and applying each discipline.

1. Introduction

As is now well known, we adopt the crack size a as a variable in the fracture mechanics to describe crack growth process undergoing damage for a material and we adopt a damage variable D in the damage mechanics to describe an evolutive process undergoing damage for one. Either the sign a or the sign D are all virtually damage variables so, we could also adopt the damage D to describe the evolutive law of a structure material with crack. References [1–3] had made out such research. In each discipline we have all in-house features and its advantages. If we can communicate and convert corresponding relations for that among the damage variables, and the equations, the material constants, the dimensional units which describe the material behavior for varied discipline and provide some conversion methods, thus we are also able to adopt the same variables D_1 and D_2 or the variable a_1 and a_2 to compute the strength and the life at each stage or even in overall process for structures and materials undergoing fatigue damage [4, 5]. And the conventional materials and damage mechanics are made by inheritance and development and the modern one is made by all better combination and application. Based on this aim THE authors adopt the mathematical derivation and computational analyses used with computer by long-range research, educe a series of the computation expressions and the computing methods. Thus, this may be having practical significance for promoting, developing, and applying of some disciplines.

2. Bidirectional Combined Coordinate System and Bidirectional Curves in the Whole Process

In some of branch disciplines on fatigue-damage fracture, for finding their correlations among variables, curves, equations, and material constants of describing material behaviors at each stage and for connecting their relations to each other, we must put up analysis and developments for a number of problems that are above mentioned. Here, it is by means of bidirectional combined coordinate system that is, adopted in Figure 1 [6] we express the damage evolving process of material behavior at each stage and in the whole course, which consists of six abscissa axes $O'_{\mathrm{I}''}$, $O_{\mathrm{I}'}$, $O_{1\mathrm{I}}$, $O_{2\mathrm{II}}$, $O_{3\mathrm{III}}$, and $O_{4\mathrm{IV}}$ and two bidirectional ordinate axis O_1O_4 and $O'_1O'_4$. Between the axes $O'_{\mathrm{I}''}$ and $O_{1\mathrm{I}}$, it is the calculation domain of the conventional material mechanics; between the axes $O'_{\mathrm{I}''}$ and $O_{\mathrm{I}'}$, it is the calculation domain of the current ultra-high cycle fatigue; among the axes $O_{\mathrm{I}'}$, $O_{1\mathrm{I}}$, and $O_{2\mathrm{II}}$, it is the calculation domain of the damage mechanics and the microfracture mechanics; between the axes $O_{3\mathrm{III}}$ and $O_{4\mathrm{IV}}$, it is calculation domain of the macrofracture mechanics; Between the axes $O_{2\mathrm{II}}$ and $O_{3\mathrm{III}}$, it is all applicable calculation domain for the microfracture mechanics and macrofracture mechanics. The upward direction along the ordinate axis is presented as damage evolving rate dD/dN or crack growth rate da/dN (that can also carve up the damage evolving rate dD_1/dN_1 or short crack growth rate da_1/dN_1 at crack forming stage or the damage evolving rate dD_2/dN_2 or long crack growth rate da_2/dN_2 at crack growth stage), and the downward direction is presented as each stage life 2N. The distance $O'O$ between axis $O'_{\mathrm{I}''}$ and $O_{\mathrm{I}'}$ is shown as the region of the nominal stress S or remote stress σ_o; the distance $O'O_2$ between axis $O'_{\mathrm{I}''}$ and $O_{2\mathrm{II}}$ is shown as the region from uncrack to microcrack initiation; the distance O_2O_3 between axes $O_{2\mathrm{II}}$ and $O_{3\mathrm{III}}$ is shown as the region relative to life $N_{\mathrm{oi}}^{\text{mic-mac}}$ from microcrack growth to macrocrack forming. Consequently, the distance $O'O_3$ is shown as the region relating to life N_{mac} from grains size to microcrack initiation until macrocrack forming; the distance $O'O_4$ is shown as the region relating to the lifelong life 2N from micro-crack initiation until fracture of structure material. The coordinate system combined from upward axis $O'O_4$ and abscissa axes $O_{\mathrm{I}'}$, $O_{1\mathrm{I}}$, and $O_{2\mathrm{II}}$ is presented to be the relationship between the damage evolving rate dD_1/dN_1 (or the short crack growth rate da_1/dN_1) and the damage stress factor amplitude $\Delta H/2$ (or damage strain factor amplitude $\Delta I/2$) at crack forming stage; the coordinate system combined from $O'O_4$ and $O_{3\mathrm{III}}$ ($O_{4\mathrm{IV}}$) at the same direction is presented to be the relationship between macrocrack growth rate and stress intensity factor amplitude $\Delta K/2$, J-integral amplitude $\Delta J/2$, and crack tip displacement amplitude $\Delta\delta_t/2$ (da_2/dN_2-$\Delta K/2$, $\Delta J/2$ and $\Delta\delta_t/2$) at macro-crack growth stage. The coordinate system combined from downward ordinate axis O_4O_1 and abscissa axes $O_{2\mathrm{II}}$, and $O_{3\mathrm{III}}$ is presented as the relationship between the $\Delta H/2$-, $\Delta K/2$-amplitude, and the life 2N (or between the $\Delta\varepsilon_p/2$-, $\Delta\delta_t/2$-amplitude, and the life 2N). The curve abcd is the ultra-high cycle fatigue one to correspond to stress below fatigue limit. On abscissa $O_{3\mathrm{III}}$, point A_1 is corresponding to fatigue strength coefficient σ'_f; point C_1 is corresponding to fatigue ductility coefficient ε'_f; point F is corresponding to ultra-high cycle fatigue strength coefficient σ'_{uhf}. The ABA_1 shows the varying regularities of elastic material behaviors as under high cycle loading at macro-crack forming stage; positive direction ABA_1 shows the relation between dD_1/dN_1 (or da_1/dN_1) and $\Delta H/2$; inverted A_1BA shows the relation between the $\Delta H/2$ and 2N. The curve CBC_1 shows the varying regularities of plastic material behaviors, as is under low-cycle loading at macro-crack forming stage; positive direction CBC_1 shows the relation between da_1/dN_1 and $\Delta I/2$; inverted C_1BC shows the relation between the $\Delta\varepsilon_p/2$-2N. And the curve A_1A_2 at crack growth stage is showed as under high cycle loading: positive direction A_1A_2; shows da_2/dN_2-$\Delta K/2$ ($\Delta J/2$); inverted A_2A_1, shows the relation between the $\Delta K/2$ and $\Delta J/2$-2N. The C_1C_2 shows the positive direction relation between the da_2/dN_2 and $\Delta\delta_t/2$ under low-cycle loading, inverted C_2C_1, shows the relation between $\Delta\delta_t/2$ ($\Delta J/2$) and 2N. And it should point that the AA_1A_2 (curve $11'$) is expressed for the curve under symmetrical cycle loading (i.e., under zero mean stress); the DD_1D_2 (curve $33'$) is expressed for the curve under unsymmetrical cycle loading (i.e., under nonzero mean stress).

3. Relations among Force Triangles and Relations among Edge Vectors of Force Triangle Itself at Each Stage

Due to extent of material undergone damage at each stage is different and due to the variance in the material behaviors after undergoing damage, the rigidity of material is also different to ensue from change, so the force triangles consist of edge vectors at different stages which are also varied relationships among force triangles, and relationships among the edge vectors themselves at each stage, their geometrical and physical meanings are all compiled in Table 1. We can make out from the force triangle in each area from Figure 1 find the mathematic model of driving force is different in each discipline. The driving force seen from force triangle between the axes $O'I''$ and OI' is the damage stress intensity factor under ultra-high cycle fatigue. We should also point out that this is the calculation domain in micro-damage mechanics or in conventional material mechanics

$$\Delta G_{\mathrm{uh}} = \Delta\sigma_{\mathrm{uh}} \cdot a_\mu^{1/n}, \qquad \Delta G'_{\mathrm{uh}} = \Delta\sigma_{\mathrm{uh}} \cdot D_\mu^{1/n}, \tag{1}$$

where the ΔG_{uh} and $\Delta G'_{\mathrm{uh}}$ are, respectively, the microcrock stress intensity factor or the microdamage stress intensity factor under ultra-high cycle fatigue to correspond a crystal grain or microdamage defect $a_\mu(D_\mu)$ which is originated from interior materials. And the mathematic models of driving force between the axes $O_{1\mathrm{I}}$ and $O_{3\mathrm{III}}$ are the stress intensity factor range ΔH_1 of shortcrack or the damage stress intensity factor range $\Delta H'_1$.

$$\Delta H_1 = \Delta\sigma_1 \cdot \sqrt[m_1]{a_1}, \tag{2}$$

or

$$\Delta H'_1 = \Delta\sigma_1 \cdot \sqrt[m_1]{D_1} \tag{3}$$

that is calculation domain of the microfracture mechanics or damage mechanics. And the mathematic models of driving force between the axes $O_{3\mathrm{III}}$ and $O_{4\mathrm{IV}}$ are the stress intensity

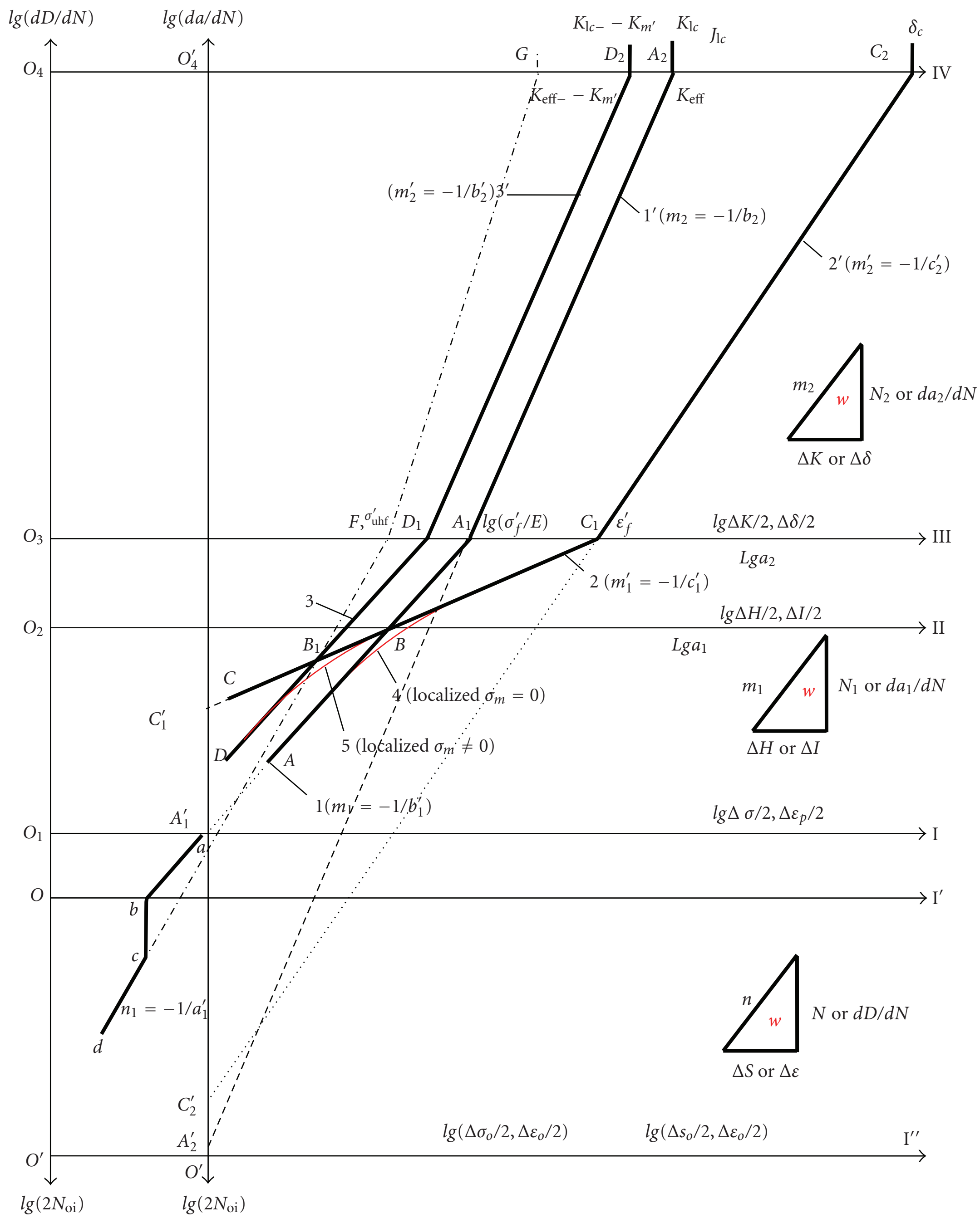

Figure 1: Bidirectional combined coordinate system and bidirectional curves in the whole process.

factor range ΔK_2 of longcrack or the damage stress intensity factor range $\Delta K'_2$ at the second stage

$$\Delta K_2 = \Delta\sigma \cdot \sqrt{\pi a_2}, \tag{4}$$

or

$$\Delta K'_2 = \Delta\sigma \cdot \sqrt{\pi D_2}, \tag{5}$$

that is calculation domain of the macro-fracture mechanics. Relations among force triangles can be expressly made out in Figure 1; the relations among edge vectors of force triangle iteself and their geometrical and physical meaning at each stage are all compiled in Table 1.

4. Computing the Damage Strength for Elastic-Plastic Material under High-Cycle Fatigue

Based upon above-mentioned viewpoints we bring forward the computation expressions and calculation methods of strength for elastic-plastic steels which are the material

Table 1: Relations among the edge vectors of the force triangle iteself and their geometrical and physical meanings at each stage.

	Forward direction		Reverse direction	
Ubiety area of triangle	Geometrical meaning of edge vector	Physical meaning of edge vector	Geometrical meaning of edge vector	Physical meaning of edge vector
Triangle between the axes $O'I''$ and OI'	Level edge	Driving force	Level edge	Driving force
	Vertical edge	Damage rate	Vertical edge	Life of one cycle
	Bevel edge (slope)	Elastoplastic	Bevel vector (slope)	Elastoplastic
	Area of triangle	Power (w)	Area	Work (w) in one cycle
Triangle between the axes O_{1I} and O_{3III}	Level edge	Driving force	Level edge	Driving force
	Vertical edge	Damage rate	Vertical edge	Life of one cycle
	Bevel edge (slope)	Elastoplastic	Bevel vector (slope)	Elastoplastic
	Area of triangle	Power (w)	Area of triangle	Work (w) in one cycle
Triangle between the axes O_{3III} and O_{4IV}	Level edge	Driving force	Level edge	Driving force
	Vertical edge	Damage rate	Vertical edge	Life of one cycle
	Bevel edge (slope)	Elastoplastic and stiffness	Bevel vector (slope)	Elastoplastic and stiffness
	Area of triangle	Power	Area of triangle	Work in one cycle

behaviors undergoing fatigue damage at different stages. And we proportionally analyze for the cross-referencing between their equations, material constants, and dimensional units and also explain conversion methods between them.

4.1. Computing of Strength for Material with Crack

4.1.1. Computing of Strength at the First Stage. About computing the material strength with short crack, many researchers had presented varied computing models and made out valuable contributions [7, 8]. Murakami [9] provides the computation expression as follows:

$$K_{\text{Imax}} \cong 0.65\sigma_0\sqrt{\pi\sqrt{\text{area}}}. \tag{6}$$

And research and analysis of the driving force of force triangle in Figure 1 and according to above explanation and the present author provide again a computational model to describe stress-strain about the tip of short crack that is the stress intensity factor at the first stage as following [10–12]:

$$H_1 = y_1\sigma \cdot \sqrt[m_1]{a_1} \le H_{\text{mac}}\left(\sqrt[m_1]{m} \cdot \text{MPa}\right), \tag{7}$$

where

$$H_{\text{mac}} = \sigma \cdot \sqrt[m_1]{a_{\text{mac}}}\left(\sqrt[m_1]{m} \cdot \text{MPa}\right) \tag{8}$$

H_{mac} = Critical stress intensity factor of short crack that is corresponding to macro-crack size at about the threshold level ΔK_{th} and more than one, that is artificially definite according to the size of structure member.

y_1 is a correction coefficient concerned with shape and size, for example, of a structure member. And it became the damage stress intensity factor at the same stage when we adopt damage variable D_1 to describe [13]

$$H' = y_1\sigma \cdot \sqrt[m_1]{D_1} \le H'_{\text{mac}}, \quad \left(\text{MPa}\sqrt[m_1]{D_1}\right) \text{ or MPa}, \tag{9}$$

where

$$H'_{\text{mac}} = \sigma \cdot \sqrt[m_1]{D_{\text{mac}}} \quad \left(\text{MPa}\sqrt[m_1]{D_1}\right) \text{ or MPa} \tag{10}$$

H'_{mac}: Critical damage stress intensity factor that is equivalent to the H_{mac}. D_{mac}: Critical damage level that is equivalent to a_{mac}; if $a_{\text{mac}} = 0.7$ mm, then $D_{\text{mac}} = 0.7$.

Conversion method between variables a and D and between the dimensional units defines 1 mm equivalent to one damage unit (nondimensional value), 1 m equivalent to 1000 damage unit. If critical $a_{\text{mac}} = 0.7$ mm, then critical damage value of the equivalent is $D_{\text{mac}} = 0.7$ damage unit. And define that $a_0 < a_1 \le a_{\text{mac}}$; $D_0 < D_1 \le D_{\text{mac}}$. The D_0 is an initial damage value corresponding to micro-crack size a_0, and it is advised to take the average value of 10 crystal sizes or to take the maximum size of crystal size. It should also be pointed out that the stresses are all local ones in (2)–(5) and (7)–(10).

4.1.2. Computing of Strength at the Second Stage. The computational model to describe stress strain about the tip of long crack at the second stage had been provided by famous scientists Broek and Hellan as follows [14, 15]:

$$K_{\text{I}} = (K_2) = y_2 \cdot \sigma\sqrt{\pi a_2} < K_{\text{Ic}}(= K_{2c})\left(\sqrt{m} \cdot \text{MPa}\right), \tag{11}$$

$$K_{\text{Ic}} = K_{2c} = \sigma\sqrt{\pi \cdot a_{2c}}, \qquad \left(\sqrt{m} \cdot \text{MPa}\right), \tag{12}$$

where $K_{\text{Ic}}(= K_{2c})$ = Critical stress intensity factor of long crack y_2 is a correctional coefficient concerned with the shape of crack and shape and size of structure member. And it becomes the damage stress intensity factor at the same stage if we adopt damage variable D_2 to describe [16, 17]

$$K'_2 = y_2 \cdot \sigma\sqrt{\pi D_2} < K'_{2c}, \quad \left(\text{MPa}\sqrt[m_1]{D_1}\right) \text{ or MPa}, \tag{13}$$

$$K'_{2c} = \sigma\sqrt{\pi D_{2c}}, \quad \left(\text{MPa}\sqrt{D_2}\right) \text{ or MPa} \tag{14}$$

K'_{2c} is a critical damage stress intensity factor equivalent to the $K_{\text{Ic}}(K_{2c})$. D_{2c} is a critical damage value correspondent to the critical crack size a_{2c}.

Conversion method between variables a_2 and D_2 and between the dimensional units at the second stage is the same as the first stage. It should be pointed out that the dimensional unit of the damage stress intensity factor H' and the K_2' at each stage become identical with the unit of stress because the dimensional unit between the variables a and D is converted.

It should be pointed out that both the values between the stress intensity factor of short-crack and the stress intensity of long crack are different under the condition of the same crack size (e.g., $a_1 = a_2 = 0.1$ mm) because the mechanisms of material damage and because their mathematical models and dimensional units of both driving forces are all different. But both growth rates at the turning point from short crack to long crack should be accordant or near.

5. Computing Example

A pressure vessel adopts the steel 16 MnR to make its strength limit of material $\sigma_b = 545$ MPa, yield limit $\sigma_y = 349$ MPa, the strain-hardening exponent $n = 0.136$, fatigue strength exponent in short crack growth $m_1 = 11.478$, threshold level $\Delta K_{\text{th}} = 6.87\,\text{MPa}\sqrt{m}$, critical stress intensity factor $K_{1c} = 97.3\,\text{MPa}\sqrt{m} = K_{2c} = K_{\text{Ic}} = 97.3\,(\text{MPa}\sqrt{m})$, critical damage stress intensity factor $K_{2c}' = 97.3\,\text{MPa}\sqrt{D_2}$ equivalent to the $K_{\text{Ic}}(K_{2c})$, the critical stress intensity factor $H_{\text{mac}} = 350\sqrt[m_1]{m}$ and the damage stress intensity factor $H_{\text{mac}}' = 350\,(\text{MPa}\sqrt[m_1]{D_1})$ of short crack corresponding to the threshold level ΔK_{th}, and the mean sizes $a_0 = 20 \sim 30\,\mu$m of crystal grains; its working stress is 280 MPa of pressure vessel and local stress is 840 MPa at focal point of stress. Try respectively, to compute the stress intensity factor H and the damage stress intensity factor H' at short crack size $a_0 = 100\,\mu$m undergone after damage for a crystal grain and the stress intensity factor $K_2 = K_{\text{I}}$ and the damage stress intensity factor K_2' at long crack size $a_2 = 2$ mm. Computing approach is as follows.

5.1. Computing the Stress Intensity Factor H and the Damage Stress Intensity Factor H' for Short Crack

5.1.1. Computing the Stress Intensity Factor H for Short Crack. According to (7), select the computing parameter: $a_1 = 100\,\mu\text{m} = 1.0^{-4}$ m; the local stress is 840 MPa at focal point of stress undergone damage; If we take $y_1 = 1.1$,

$$\begin{aligned} H_1 &= y_1\sigma \cdot \sqrt[m_1]{a_1} = 1.1 \times 840 \times \sqrt[11.478]{100\,\mu\text{m}} \\ &= 1.1 \times 840 \times \sqrt[11.478]{1.0^{-4}\,\text{m}} \\ &= 414.41\left(\sqrt[m_1]{m}\right). \end{aligned} \tag{15}$$

This results in

$$H_1 = 414.41\left(\sqrt[m_1]{m}\right) > H_{\text{mac}}' = 350\left(\sqrt[m_1]{m}\,\text{MPa}\right). \tag{16}$$

So this short crack is growth.

5.1.2. Computing of the Damage Stress Intensity Factor H′ for Short Crack. According to (9), when the $a_1 = 100\,\mu\text{m} = 1.0^{-4}$ m, equivalent damage value of short crack is the $D_1 = 100 = 1.0^{-4}$ (damage unit).

So

$$\begin{aligned} H_1 &= y_1 \times \sigma \cdot \sqrt[m_1]{D_1} \\ &= 1.1 \times 840 \times \sqrt[11.478]{100} \\ &= 1.1 \times 840 \times \sqrt[11.478]{1.0^{-4}} \\ &= 414.41\left(\text{MPa}\sqrt[m_1]{D}\right) \text{ or } (\text{MPa}). \end{aligned} \tag{17}$$

This results in

$$H_1 = 414.41\left(\text{MPa}\sqrt[m_1]{D} > H_{\text{mac}}'\right) = 350\,(\text{MPa}). \tag{18}$$

So this short crack is also growth.

5.2. Computing the Stress Intensity Factor $K_2 = K_I$ and the Damage Stress Intensity Factor K_2' for Long Crack

5.2.1. Computing the Stress Intensity Factor K_2 for Long Crack. According to (11), select a parameter, take the correction coefficient $y_2 = 1.05$, $a_2 = 2$ mm, working stress $\sigma = 280$ MPa;

$$\begin{aligned} K_2 = K_{\text{I}} &= y_2 \times \sigma \times \sqrt{\pi \cdot a_2} \\ &= 1.05 \times 280 \times \sqrt{\pi \times 0.002} \\ &= 23.30\,(\text{MPa}\sqrt{m}). \end{aligned} \tag{19}$$

This results in

$$K_2 = 23.30 < K_{2c}' = K_{\text{Ic}} = 97.3\,(\text{MPa}\sqrt{m}). \tag{20}$$

So the pressure vessel is safe.

5.2.2. Computing the Damage Stress Intensity Factor K_2' for Long Crack. According to equation (13), take the computing parameter $y_2 = 1.05$; when take $a_2 = 2$ mm, the equivalent damage value of long crack is $D_2 = 2000 = 2.0^{-3}$ (damage unit).

So

$$\begin{aligned} K_2 = K_{\text{I}} &= y_2 \times \sigma\sqrt{\pi \cdot D_2} \\ &= 1.05 \times 280 \times \sqrt{\pi \times 0.002} \\ &= 23.30\left(\text{MPa}\sqrt{D}\right) \text{ or } (\text{MPa}). \end{aligned} \tag{21}$$

This results in

$$K_2 = 23.30 < K_{2c}' = 97.3\,(\text{MPa}). \tag{22}$$

So the pressure vessel is also safe.

6. Summarization

The bidirectional combined coordinate system, the bidirectional curves in the whole process and their force triangles at each stage are important scientific method and tool to communicate the cross-referencing which is to describe the evolutive process of a material behavior undergoing of the fatigue damage in each of the disciplines, which are able to make available communications and conversions for those complicated correlations among some variables, some equations, curves, and dimensional units, which are clearely able to explain the geometrical and physical meanings for the key parameters. Thus, it is also able to adopt same variables D_1 and D_2 or the variables a_1 and a_2 to compute the strength and the lift at each stage or even in overall process for structures and materials undergoing a fatigue damage. And the conventional material and damage mechanics are able of making inheritance and development, and the modern one are all able to make better combination and application. Thus, that may be having practical significance for promoting developing, and applying some disciplines.

7. Conclusions

(1) *About the Problem of the Relation between the Mathematical Model and the Material Behavior.* Under identical loading, when the structure material is undergoing fatigue damage, the differences between the mathematical models to describe the material behavior are due to the degree of damage undergone at varied stages that makes the stiffness of the material change, which find that expressions in the curves $cbaBA_1A_2$ in evolutive process are turned to take place at the points b, B, and A_1. The slopes of the curves at each stage also are brought to change. The exponents in the equation also became from the $m_1 = -1/b_1$ at crack forming stage to the $m_2 = -1/b_2$ at crack growth stage. The driving forces became from the $H_1(H_1')$ to the $K_2(K_2')$. The compositive material constants became from the A_1 to the A_2.

(2) *About the Problem of Driving Force.* It well known that the driving force of long crack growth in macro fracture mechanics is the stress intensity factor K_{I} but the driving force of microdamage at crack forming stage is defined by the author as the damage stress intensity factor H_1', and the driving force in microfracture mechanics is defined as the stress intensity factor H_1 of short-crack growth.

(3) *About Problem of Dimensional Units.* The stress intensity factor H_1 and the damage stress intensity factor H_1' of short crack at crack forming stage are all to describe the stress strain about the crack tip of micro-crack, the unit of the H_1 is MPa $\cdot \sqrt[m_1]{m}$, and the unit of the H_1' is the MPa $\sqrt[m_1]{D_1}$. Both the units are different, but the unit of the damage stress intensity factor H_1' is the same as the unit of the stress, and the H_1 and the H_1' are all essentially the stress intensity factor and a relation of equivalents. And the stress intensity factor K_2 and the damage stress intensity factor K_2' of long crack at crack growth stage are all to describe the stress strain about the crack tip of long crack, the unit of the K_2 is MPa $\cdot\sqrt{m}$, and the unit of K_2' is the MPa $\cdot\sqrt{D_2}$. Both the units are also different, but the unit of the damage stress intensity factor K_2' is also the same as the unit of the stress, and the K_2 and the K_2' are all essentially the stress intensity factor and a relation between equivalents. In addition, both values and dimensional units between the stress intensity factor of short crack and the stress intensity factor of long crack are also different under the condition of same crack size.

Nomenclature

(1) D = damage variable in the whole process, D_μ, D_1 = microdamage variable equivalent to micro-crack a_μ or short crack a_1 at the crack forming stage (first stage), and D_2 = damage variable at the crack growth stage (the second stage).

(2) a_μ, a_1 = micro-, short-crack size at the crack forming stage (variable of the first stage), a_2 = macro-, long-crack size at the crack growth stage (variable of the second stage), a_{10} = initial size of micro-crack forming (ordaining value), crack size a_{th} = corresponding to the threshold level ΔK_{th}, original size $a_{20} = a_{\mathrm{mac}}$ of macro-crack forming stage (ordaining value), and a_{2c} = critical size of long crack.

(3) $\Delta\sigma$ = nominal stress range, $\Delta\sigma_0$ = remote stress range, $\Delta\sigma/2$ = stress amplitude, $\Delta\varepsilon_p$ = strain range, $\Delta\varepsilon_p/2$ = strain amplitude, and σ_m = mean stress.

(4) ΔG or $\Delta G'$ = micro-crack stress intensity factor range or microdamage stress intensity factor range corresponding to microcrack size a_μ or microdamage D_μ under ultra-high cycle fatigue, H_1 = stress intensity factor of short crack, H_{mac} = critical stress intensity factor of short crack, H_1' = damage stress intensity factor of short crack and $\Delta H_1'$ or ΔH_1 or $\Delta H_1/2$ = stress intensity factor range or stress intensity factor amplitude relative to short-crack a_1; $\Delta H_1'/2$ = damage stress intensity factor range or damage stress intensity factor amplitude relative to damage variable D_1.

ΔI or $\Delta I/2$ = damage strain factor range or damage strain factor amplitude relative to short crack a_1.

(5) a_1' = fatigue strength exponent under ultra-high cycle fatigue, b_1' = fatigue strength exponent under high cycle fatigue, c_1' = fatigue ductility exponent under low cycle fatigue, n_1= fatigue strength exponent in micro-crack growth rate equation under ultra-high cycle fatigue, $n_1 = -1/a_1'$; m_1 = fatigue strength exponent in short-crack growth rate equation under high-cycle fatigue, $m_1 = -1/b_1'$, m_1' = fatigue ductility exponent in short-crack growth rate equation under low cycle fatigue, $m_1' = -1/c_1'$, b_2' = fatigue strength exponent of the macro-crack growth stage under

high-cycle fatigue, c_2' = the fatigue ductility exponent at the macro-crack growth stage under low-cycle fatigue, m_2 = the fatigue strength exponent in crack growth rate equation under high-cycle fatigue, $m_2 = -1/b_2'$, m_2' = the fatigue ductility exponent in crack growth rate equation under low-cycle fatigue, $m_2' = -1/c_2'$.

(6) dD/dN = damage evolutive rate, dD_1/dN_1 = damage evolutive rate at the crack forming stage, dD_2/dN_2 = damage evolutive rate at the macro-crack growth stage, da/dN = crack growth rate, da_1/dN_1 = short crack growth rate at the crack forming stage, and da_2/dN_2 = its rate at the macro-crack growth stage.

(7) N_{oi} = life of correspondance to medial damage variable D_{oi} or short-crack medial size a_{oi} at the first stage, and N_{oj} = life of correspondance to medial damage variable D_{oj} or long-crack medial size a_{oj} at the second stage.

(8) $K_2 = K_{\text{I}}$ = stress intensity factor of long crack, K_2' = damage stress intensity factor of long crack, J-integral of long crack; and crack tip opening displacement of long crack, K_m = mean stress intensity factor, $\Delta K/2$ = stress intensity factor amplitude of correspondance to macro-crack a_2,$\Delta J/2$ = J-integral amplitude corresponding to macro-crack a_2, $\Delta\delta_t/2$ = crack tip opening displacement amplitude corresponding to macro-crack a_2.

(9) K_{1c} = critical stress intensity factor corresponding to macro-crack critical a_{2c}, K_{eff} = effective stress intensity factor to be applicable in Paris's equation, J_c = critical J-integral value corresponding to macro-crack critical a_{2c}, and δ_c = critical crack tip opening displacement corresponding to macro-crack critical a_{2c}.

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