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Research Article

Modelling Adopter Behaviour Based on the Navier Stokes Equation

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Aiming to improve the current models describing the diffusion of innovation, a method based on an analogy with a physical system is presented. In particular, we model the movement of adopters of a particular innovation as a flow and the diffusion of the innovation as heat diffusion, thus making use of mathematical tools such as the Navier-Stokes equation. Building on our previous work, in this study we focus on the movement of adopters in the simulation domain, which we refer to as convection. As an application to modeling the convection phenomenon, we simulate the flow of people in a real building floor, the Tokyo Aquarium. Besides being a necessary part when simulating diffusion of innovations, such a modeling approach may also prove useful in different fields, such as deciding layout of public spaces or evacuation policies.

1. Introduction

The spread of information in general and, in particular, the diffusion of various innovations are strongly influenced by the movement of people. Although the movement of each particular individual is dictated by complex rules, at a macrolevel it is possible to consider the collective movement of large numbers of people (crowds of people) as continuous and model it as a type of flow. Such flows have been previously considered in the literature and the relations between crowd velocities, flow rates, and densities have been studied in crowded places as commuter stations [1].

Flows of people, with characteristics strongly influenced by the structure of the particular domain the flow takes place in (e.g., office building, public places) are important for a number of reasons. From an economical point of view, they spawn economic activity

in the region, as various entrepreneurs compete for high-visibility spots to sell their merchandise. From a social point of view, they are the main places where information is disseminated. From a safety point of view, they are potentially dangerous places where accidents or panic can result in mass injury and loss of life. Because of this, flows of crowds have been analyzed using both continuous and discrete formulations [2–6].

Our purpose is to elucidate the spatial spread of innovations by means of a physical analogy with the phenomenon of heat diffusion. This builds on our previous work, where we have modeled the diffusion of innovation [7] making use of various data such as population density or the economic situation. In the innovation diffusion phenomenon, we define three main ways by which an innovation can propagate among the members of a population. This is achieved via an analogy with the phenomenon of heat propagation through some medium, which can take place in three different modes: heat transfer (when heat is transferred through a solid, a stationary liquid, or a vaporous medium), heat convection (when heat is conveyed by the movement of particles in a liquid or a vaporous medium), and heat radiation (when heat is transferred between two disconnected media).

In this paper, we concentrate on modeling the movement of adopters in a continuous way, using mathematical tools such as the Navier-Stokes equation. After introducing the theoretical aspects of our model, we perform a real simulation by modeling the flow of people in the Tokyo Aquarium.

2. Definition

2.1. Adoption Ratio

In order to calculate the number of accumulated adopters by employing the relevant equations, the value of each variable in those equations must be continuous in the domain. However, this is usually not true for the number of potential adopters, as it depends on the population in the respective region of the domain. The number of adopters should therefore be substituted with a continuous average value. The level of saturation with respect to the diffusion of an innovation, after a given period of time, will vary between the nodes of the domain mesh (segmented domain). This is often the result of differences in innovation adoption between a region with a large population and a region with a small population. The diffusion equation is not capable of capturing this aspect of diffusion, with the exception of the defined boundary conditions. That is, the saturation values of different nodes over the computational domain equalize over time, according the diffusion equation. To overcome this problem, we introduce the concept of adoption ratio, defined as follows:

$$N = \frac{M}{G}, \tag{2.1}$$

where N and M denote the adoption ratio and the number of the accumulated adopters, respectively. The adoption ratio is distributed over space. After enough time has passed, the adoption ratio becomes $N = 1$ over the entire computational domain. This adoption ratio is intended to be similar to the distribution of temperature over a medium in the transfer of heat. In natural sciences, temperature is considered a macroscale (or statistical) value. When a microscale region is observed, a small set of molecules would be considered. In this context, the temperature becomes statistically unstable (unreflective of the larger

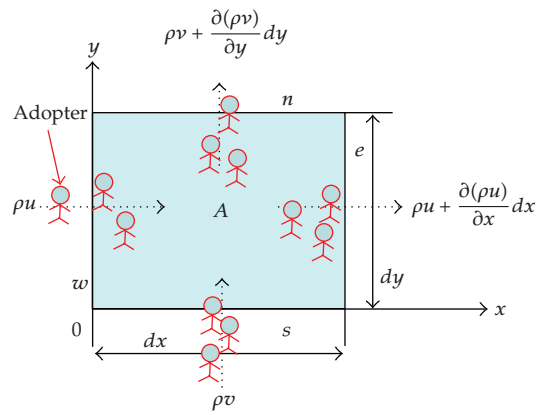

Figure 1: Flow of people in the infinitesimal element.

medium) and has little meaning. In addition, it would be effectively impossible to observe the motion of numerous molecules, independently, in tandem using available measurement equipment. In social science, the adoption ratio is also a macroscale (or statistical) value. In the microdomain, in which only a few potential adopters are considered, this value similarly becomes statistically unstable and thus has little meaning. As a result of the associated increase in computational costs, as observed with the agent models [8, 9], the computation necessary to capture the movement of numerous independent adopters in real time is effectively impossible.

2.2. Density of Potential Adopters

It is assumed that the set of potential adopters, G_i, lives within an area A_i. The density of the number of potential adopter is therefore defined as follows:

$$\rho_i = \frac{G_i}{A_i} \quad (i = 1, 2, \ldots, n). \tag{2.2}$$

This parameter is defined as a constant and is distributed over the computational domain according to the domain segmentation (index i). The index of parameter ρ will coincide with the indices of G and A.

2.3. Pressure

In areas such as crowded public spaces, direct physical interaction between people is generally unavoidable. In large crowds, there is a potential for injury and even loss of life, resulting from the dynamics of the crowd's behavior. Using the standard forward-backward autoregressive modeling approach for spectral analysis of a measured signal, predictions of pressures generated by very high densities of pedestrians have been formulated [10]. In this study, pressure is defined in a similar way. Basically, it is assumed that two persons A and B found in close contact will fill uncomfortable and thus try to move away from each other.

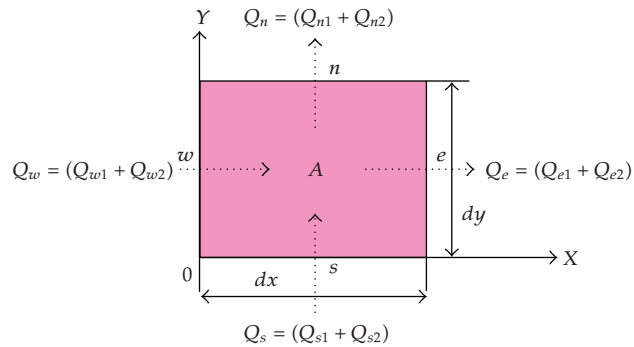

Figure 2: Adopter flow in infinitesimal element. (Energy of innovation diffusion is defined as the number of accumulated adopters.)

2.4. Viscosity

In the same way as we defined the pressure as a repulsive force, where people try to avoid and move away from close contacts, we also define an attractive force, where groups of people try to keep moving together. One possible reason for such behavior is friends, family members, or tour groups trying to keep together. This also prevents groups of people from mixing. In this study, this effect, similar to the viscosity in a fluid, is defined as viscosity.

3. Theory

3.1. Innovation Convection Based on Navier Stokes Equation

Figure 1 shows an infinitesimal element of width dx and the height dy in the flow of people. The speed of movement with respect to the x-axis and the y-axis are denoted u and v, respectively. In the x axis, the change with respect to time in the number of people found in the infinitesimal element must equal the difference between the number of people coming in and the number of people going out. This fact is described in the equation below:

$$\rho u dy - \left[\rho u + \frac{\partial(\rho u)}{\partial x} dx\right] dy = -\frac{\partial(\rho u)}{\partial x} dx dy. \tag{3.1}$$

Similarly, for the y direction we have

$$\rho v dx - \left[\rho v + \frac{\partial(\rho v)}{\partial y} dy\right] dx = -\frac{\partial(\rho v)}{\partial y} dx dy. \tag{3.2}$$

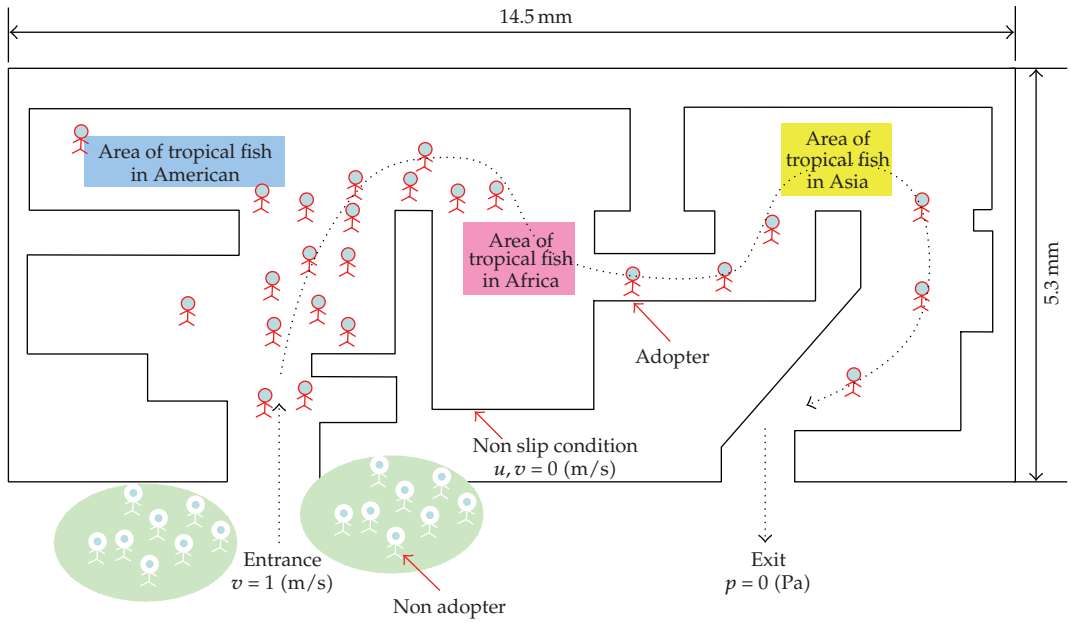

Figure 3: Floor Map of Tokyo Tower Aquarium [14].

The number of peoples $(\rho dxdy)$ in the infinitesimal element increases by an amount of $\partial(\rho dxdy)/\partial t$. The equation is as follows:

$$-\frac{\partial(\rho u)}{\partial x}dxdy - \frac{\partial(\rho v)}{\partial y}dxdy = \frac{\partial(\rho dxdy)}{\partial t},$$
$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial x} = 0. \tag{3.3}$$

Newton's second law is applied to the infinitesimal element as shown in Figure 1. The forces acting on the infinitesimal element are denoted by F_x and F_y with respect to the x direction and the y direction, respectively. From the equilibrium of forces, the following equations are obtained:

$$\rho dxdy\frac{du}{dt} = F_x,$$
$$\rho dxdy\frac{dv}{dt} = F_y. \tag{3.4}$$

The change of the velocity occurs both the time and the space. The velocity du with respect to the time dt is as follows:

$$du = \frac{\partial u}{\partial t}dt + \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy. \tag{3.5}$$

Equation (3.5) is transformed as follows:

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}. \tag{3.6}$$

Using (3.6), and so forth, (3.4) are as follows:

$$\begin{aligned} \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)dxdy &= F_x, \\ \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right)dxdy &= F_y. \end{aligned} \tag{3.7}$$

The friction F is assumed to be consist of the pressure term P and the viscosity term S. These equations are as follows:

$$\begin{aligned} F_x &= P_x + S_x, \\ F_y &= P_y + S_y \end{aligned} \tag{3.8}$$

The pressure (p_x, p_y) is obtained as follows:

$$P_x = pdy - \left(p + \frac{\partial p}{\partial x}dx\right)dy = -\frac{\partial p}{\partial x}dxdy, \tag{3.9}$$

$$P_y = pdx - \left(p + \frac{\partial p}{\partial y}dy\right)dx = -\frac{\partial p}{\partial y}dxdy. \tag{3.10}$$

The viscosity (S_x, S_y) is obtained as follows:

$$S_x = \mu\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial x}\right)dxdy + \mu\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)dxdy, \tag{3.11}$$

$$S_y = \mu\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)dxdy + \mu\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial y} + \frac{\partial v}{\partial y}\right)dxdy. \tag{3.12}$$

Using (3.9)–(3.12) and (3.7), (3.8) are as follows:

$$\begin{aligned} \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) &= -\frac{\partial p}{\partial x} + \mu\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial x}\right) + \mu\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \\ \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) &= -\frac{\partial p}{\partial y} + \mu\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) + \mu\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial y} + \frac{\partial v}{\partial y}\right). \end{aligned} \tag{3.13}$$

3.2. Innovation Convection Based on Energy Equation

In the previous section the flow of people in an infinitesimal element is described based on the Navier Stokes equation. These people are both adopters and nonadopters of the

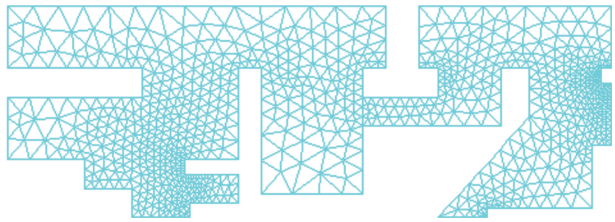

Figure 4: Mesh of Tokyo Tower Aquarium.

innovation being studied. In this study, the number of adopters found in the flow is called the innovation energy [11, 12]. As shown in Figure 2, the inflow of adopters from the left side in the infinitesimal element represents the variable Q_w. The variable Q_w consists of the innovation convection Q_{w1} and the innovation transfer Q_{w2}. This equation is as follows:

$$Q_w = Q_{w1} + Q_{w2}. \tag{3.14}$$

The variable Q_{w1} on the left side in Figure 2 is as follows:

$$Q_{w1} = \rho c u N dy, \tag{3.15}$$

where the variable "c" shows the ratio of the adopter capacity. This variable shows the number of adopters needed to increase the adopter ratio (N) by 1.0% [11]. The number of adopter Q_{w2} is as follows:

$$Q_{w2} = -\lambda \frac{\partial N}{\partial x} dy, \tag{3.16}$$

where the variable λ is called innovation diffusivity [11]. This parameter reflects the number of interactions between potential adopters and existing adopters. Similarly, the number of outflow adopters Q_e on the right side is as follows:

$$\begin{aligned} Q_e &= Q_{e1} + Q_{e2} \\ &= \left[\rho c u N + \frac{\partial}{\partial x}(\rho c u N) dx\right] dy + \left[-\lambda \frac{\partial N}{\partial x} + \frac{\partial}{\partial x}\left(-\lambda \frac{\partial N}{\partial x}\right) dx\right] dy. \end{aligned} \tag{3.17}$$

Similarly, the number of inflow adopters Q_s on the bottom side is as follows:

$$Q_s = Q_{s1} + Q_{s2} = \left[\rho c v N - \lambda \frac{\partial N}{\partial y}\right] dx \tag{3.18}$$

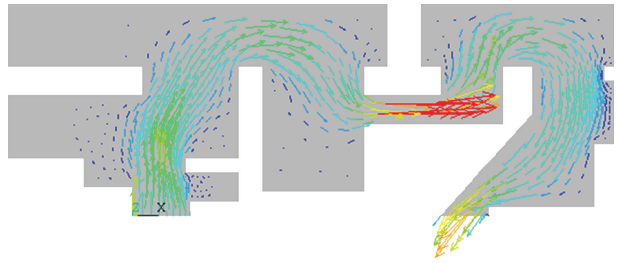

Figure 5: Adopter flow in Tokyo Tower Aquarium (calculated by (3.13)).

while the number of outflow adopters Q_n on the upper side is as follows:

$$\begin{aligned} Q_n &= Q_{n1} + Q_{n2} \\ &= \left[\rho c v N + \frac{\partial}{\partial y}(\rho c v N) dy\right] dx + \left[-\lambda \frac{\partial N}{\partial y} + \frac{\partial}{\partial y}\left(-\lambda \frac{\partial N}{\partial y}\right) dy\right] dx. \end{aligned} \tag{3.19}$$

In the steady state, the number of inflow adopters should be equal to the number of outflow adopters. Therefore, it is as follows:

$$Q_w - Q_e + Q_s - Q_n = 0. \tag{3.20}$$

Using (3.15)–(3.19), (3.20) is as follows:

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = \frac{\lambda}{\rho c}\left(\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2}\right). \tag{3.21}$$

The left side of (3.21) and the right side become the innovation convective term and the innovation transfer term, respectively. In the unsteady state with respect to time, the time-derivative term is added in (3.21):

$$\frac{\partial N}{\partial t} + u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = \frac{\lambda}{\rho c}\left(\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2}\right). \tag{3.22}$$

In case the flow of adopters does not occur in the infinitesimal element ($u = 0$ and $v = 0$), the innovation diffusion equation is obtained [11, 12]. The number of adopters in the infinitesimal element can also increase via social factors (e.g., imitation), so a Bass model term [13] is added in (3.22) as follows:

$$\frac{\partial N}{\partial t} + u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = \frac{\lambda}{\rho c}\left(\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2}\right) + U(1 - N) + VN(1 - N), \tag{3.23}$$

where U and V represent the innovation and imitation coefficients [13].

4. Modeling Crowd Movement

Modeling the movement of crowds is an important aspect of the previously presented innovation diffusion model. In the same way as heat can be transported by a flow field, a particular innovation can spread due to the movement of adopters. However, there are different applications for such models as well. For example, modeling the movements of people in crowded public spaces may provide insight into ways of streamlining them or to design or test efficient evacuation procedure. In this section we perform such a simulation in which we model the movement of people into the crowded space of the Tokyo Tower Aquarium [14] by making use of the Navier-stokes equations: (3.13).

4.1. Computing Model and Conditions

The computational domain, shown in Figure 3, was created according to the building schematics. The total area of the floor is 76 m^2. People move from the entrance (bottom left) to the exit (bottom right).

At the entrance, the velocity is assumed to be 1.0 (m/s) and that people continuously enter into the floor and that they will continue their movement until leaving the domain through the exit. The calculation condition on the exit is set as $P = 0$, that is, there is nothing blocking the exit. It is further assumed that there is no movement in the immediate vicinity of the walls (the boundary condition on the wall is set to $u = 0$). The viscosity effect is defined as $\nu = 1$ (m^2/s). The system is solved using the Finite Element Method, with the domain being discretized using a triangular mesh (the number of nodes and of elements is 951 and 1652, resp.). The mesh is shown in Figure 4.

4.2. Resulting Flow Field

Figure 5 shows the flow of people, according to the Navier-Stokes equations (3.13), after a steady state has been obtained. The arrows show the direction of the flow while the color indicates the walking velocity (red indicates fast movements while blue represents slow movement).

5. Conclusions

In this paper the Navier-Stokes equation and an analogy with heat diffusion are used to model the diffusion of innovations. In particular, we have focused on the phenomenon of innovation convection, the movement of adopters throughout the domain. This builds on our previous work in this field, where we have only considered domains in which adopters where fixed in space.

After introducing the theoretical aspects, we have shown a numerical example consisting of a finite element analysis of the flow of people in a public space (the Tokyo Aquarium). The result of such a simulation can be used in two ways. First, one can consider these people as potential adopters and model innovation diffusion on top of this flow. In this way the innovation will not only spread in the regular (Bass-model like) way, as usually considered in the literature, but it would also be influenced by the movement of the adopters throughout the domain. As time passes, the innovations are transmitted in the same way as

heat (energy) is conveyed by a water flow. In future work, we aim to combine the diffusion models proposed in out previous work with the innovation convection model explored here to perform more complex innovation diffusion simulations.

Second, without relation to innovation diffusion, such results can be used to find important spots (e.g., bottlenecks) in the design of the building and to find ways to streamline the flow of people through it. Such designs could reduce congestion and result in safer and more efficient public spaces.

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