Research Article

On Hyperideals in Left Almost Semihypergroups

Kostaq Hila and Jani Dine

Department of Mathematics & Computer Science, Faculty of Natural Sciences, University of Gjirokastra, Gjirokastra 6001, Albania

Correspondence should be addressed to Kostaq Hila, khila@uogj.edu.al

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This paper deals with a class of algebraic hyperstructures called left almost semihypergroups (LA-semihypergroups), which are a generalization of LA-semigroups and semihypergroups. We introduce the notion of LA-semihypergroup, the related notions of hyperideal, bi-hyperideal, and some properties of them are investigated. It is a useful nonassociative algebraic hyperstructure, midway between a hypergroupoid and a commutative hypersemigroup, with wide applications in the theory of flocks, and so forth. We define the topological space and study the topological structure of LA-semihypergroups using hyperideal theory. The topological spaces formation guarantee for the preservation of finite intersection and arbitrary union between the set of hyperideals and the open subsets of resultant topologies.

1. Introduction and Preliminaries

The applications of mathematics in other disciplines, for example in informatics, play a key role, and they represent, in the last decades, one of the purposes of the study of the experts of hyperstructures theory all over the world. Hyperstructure theory was introduced in 1934 by a French mathematician Marty [1], at the 8th Congress of Scandinavian Mathematicians, where he defined hypergroups based on the notion of hyperoperation, began to analyze their properties, and applied them to groups. In the following decades and nowadays, a number of different hyperstructures are widely studied from the theoretical point of view and for their applications to many subjects of pure and applied mathematics and computer science by many mathematicians. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. Some principal notions about hyperstructures and semihypergroups theory can be found in [1–7].

The Theory of ideals, in its modern form, is a contemporary development of mathematical knowledge to which mathematicians of today may justly point with pride. Ideal theory is important not only for the intrinsic interest and purity of its logical structure but because it is a necessary tool in many branches of mathematics and its applications such
as in informatics, physics, and others. As an example of applications of the concept of an ideal in informatics, let us mention that ideals of algebraic structures have been used recently to design efficient classification systems, see [8–12].

The study of LA-semigroup as a generalization of commutative semigroup was initiated in 1972 by Kazim and Naseeruddin [13]. They have introduced the concept of an LA-semigroup and have investigated some basic but important characteristics of this structure. They have generalized some useful results of semigroup theory. Since then, many papers on LA-semigroups appeared showing the importance of the concept and its applications [13–23]. In this paper, we generalize this notion introducing the notion of LA-semihypergroup which is a generalization of LA-semigroup and semihypergroup, proposing so a new kind of hyperstructure for further studying. It is a useful nonassociative algebraic hyperstructure, midway between a hypergroupoid and a commutative hypersemigroup, with wide applications in the theory of flocks etc. Although the hyperstructure is nonassociative and noncommutative, nevertheless, it possesses many interesting properties which we usually find in associative and commutative algebraic hyperstructures. A several properties of hyperideals of LA-semihypergroup are investigated. In this note, we define the topological space and study the topological structure of LA-semihypergroups using hyperideal theory. The topological spaces formation guarantee for the preservation of finite intersection and arbitrary union between the set of hyperideals and the open subsets of resultant topologies.

Recall first the basic terms and definitions from the hyperstructure theory.

**Definition 1.1.** A map $\circ : H \times H \to \mathcal{P}^*(H)$ is called hyperoperation or join operation on the set $H$, where $H$ is a nonempty set and $\mathcal{P}^*(H) = \mathcal{P}(H) \setminus \{\emptyset\}$ denotes the set of all nonempty subsets of $H$.

**Definition 1.2.** A hyperstructure is called the pair $(H, \circ)$, where $\circ$ is a hyperoperation on the set $H$.

**Definition 1.3.** A hyperstructure $(H, \circ)$ is called a semihypergroup if for all $x, y, z \in H$, $(x \circ y) \circ z = x \circ (y \circ z)$, which means that

$$
\bigcup_{u \in x \circ y} u \circ z = \bigcup_{v \in y \circ z} x \circ v.
$$

If $x \in H$ and $A, B$ are nonempty subsets of $H$, then

$$
A \circ B = \bigcup_{a \in A, b \in B} a \circ b, \quad A \circ x = A \circ \{x\}, \quad x \circ B = \{x\} \circ B.
$$

**Definition 1.4.** A nonempty subset $B$ of a semihypergroup $H$ is called a sub-semihypergroup of $H$ if $B \circ B \subseteq B$, and $H$ is called in this case super-semihypergroup of $B$.

**Definition 1.5.** Let $(H, \circ)$ be a semihypergroup. Then $H$ is called a hypergroup if it satisfies the reproduction axiom, for all $a \in H$, $a \circ H = H \circ a = H$.

**Definition 1.6.** A hypergroupoid $(H, \circ)$ is called an LA-semihypergroup if, for all $x, y, z \in H$, $(x \circ y) \circ z = (z \circ y) \circ x$.
Every LA-semihypergroup \((H, \circ)\) satisfies the medial law, that is, for all \(x, y, z, w \in H\),

\[(x \circ y) \circ (z \circ w) = (x \circ z) \circ (y \circ w).\] (1.4)

In every LA-semihypergroup with left identity, the following law holds:

\[(x \circ y) \circ (z \circ w) = (w \circ y) \circ (z \circ x),\] (1.5)

for all \(x, y, z, w \in H\).

An element \(e\) in an LA-semihypergroup \(H\) is called identity if \(x \circ e = e \circ x = \{x\}\), for all \(x \in H\). An element \(0\) in a semihypergroup \(H\) is called zero element if \(x \circ 0 = 0 \circ x = \{0\}\), for all \(x \in H\). A subset \(I\) of an LA-semihypergroup \(H\) is called a right (left) hyperideal if \(I \circ H \subseteq I(H \circ I \subseteq I)\) and is called a hyperideal if it is two-sided hyperideal, and if \(I\) is a left hyperideal of \(H\), then \(I \circ I = I^2\) becomes a hyperideal of \(H\). By a bi-hyperideal of an LA-semihypergroup \(H\), we mean a sub-LA-semihypergroup \(B\) of \(H\) such that \((B \circ H) \circ B \subseteq B\). It is easy to note that each right hyperideal is a bi-hyperideal. If \(H\) has a left identity, then it is not hard to show that \(B^2\) is a bi-hyperideal of \(H\) and \(B^2 \subseteq H \circ B^2 = B^2 \circ H\). If \(E(B_H)\) denotes the set of all idempotents subsets of \(H\) with left identity \(e\), then \(E(B_H)\) forms a hypersemilattice structure, also if \(C = C^2\), then \((C \circ H) \circ C \in E(B_H)\). The intersection of any set of bi-hyperideals of an LA-semihypergroup \(H\) is either empty or a bi-hyperideal of \(H\). Also the intersection of prime bi-hyperideals of an LA-semihypergroup \(H\) is a semiprime bi-hyperideal of \(H\).

2. Main Results

**Proposition 2.1.** Let \(H\) be an LA-semihypergroup with left identity, \(T\) a left hyperideal, and \(B\) a bi-hyperideal of \(H\). Then \(B \circ T\) and \(T^2 \circ B\) are bi-hyperideals of \(H\).

**Proof.** Using the medial law (1.4), we get

\[
((B \circ T) \circ H) \circ (B \circ T) = ((B \circ T) \circ B) \circ (H \circ T) \subseteq ((B \circ H) \circ B) \circ T \subseteq B \circ T,
\] (2.1)

also

\[
(B \circ T) \circ (B \circ T) = (B \circ B) \circ (T \circ T) \subseteq B \circ T.
\] (2.2)

Hence, \(B \circ T\) is a bi-hyperideal of \(H\). we obtain

\[
\left( (T^2 \circ B) \circ H \right) \circ (T^2 \circ B) = \left( (T^2 \circ H) \circ (B \circ H) \right) \circ (T^2 \circ B) \subseteq \left( T^2 \circ (B \circ H) \right) \circ (T^2 \circ B) = (T^2 \circ T^2) \circ ((B \circ H) \circ B) \subseteq T^2 \circ B,
\] (2.3)

also

\[
(T^2 \circ B) \circ (T^2 \circ B) = (T^2 \circ T^2) \circ (B \circ B) \subseteq T^2 \circ B.
\] (2.4)

Hence, \(T^2 \circ B\) is a bi-hyperideal of \(H\). \(\square\)
**Proposition 2.2.** Let \( H \) be an LA-semihypergroup with left identity and \( B_1, B_2 \) two bi-hyperideals of \( H \). Then \( B_1 \circ B_2 \) is a bi-hyperideal of \( H \).

**Proof.** Using (1.4), we get

\[
\begin{align*}
((B_1 \circ B_2) \circ H) \circ (B_1 \circ B_2) &= ((B_1 \circ B_2) \circ (H \circ H)) \circ (B_1 \circ B_2) \\
&= ((B_1 \circ H) \circ (B_2 \circ H)) \circ (B_1 \circ B_2) \\
&= ((B_1 \circ H) \circ B_1) \circ ((B_2 \circ H) \circ B_2) \subseteq B_1 \circ B_2. 
\end{align*}
\]

By the above, if \( B_1 \) and \( B_2 \) are nonempty, then \( B_1 \circ B_2 \) and \( B_2 \circ B_1 \) are connected bi-hyperideals. Proposition 2.1 leads us to an easy generalization, that is, if \( B_1, B_2, B_3, \ldots, B_n \) are bi-hyperideals of an LA-semihypergroup \( H \) with left identity, then

\[
(\cdots((B_1 \circ B_2) \circ B_3) \cdots) \circ B_n, \quad (\cdots((B_1^2 \circ B_2^2) \circ B_3^2) \cdots) \circ B_n^2
\]

are bi-hyperideals of \( H \), consequently the set \( C(H_B) \) of bi-hyperideals forms an LA-semihypergroup.

If \( H \) is an LA-semihypergroup with left identity \( e \), then \( \langle a \rangle_L = H \circ a, \langle a \rangle_R = a \circ H \) and \( \langle a \rangle_H = (H \circ a) \circ H \) are bi-hyperideals of \( H \). It can be easily shown that \( \langle a \circ b \rangle_L = \langle a \rangle_L \circ \langle b \rangle_L, \langle a \circ b \rangle_R = \langle a \rangle_R \circ \langle b \rangle_R, \) and \( \langle a \circ b \rangle_R = \langle b \rangle_R \circ \langle a \rangle_L \). Hence, this implies that \( \langle a \rangle_R \circ \langle b \rangle_L = \langle a \rangle_L \circ \langle b \rangle_R \). Also, \( \langle a \rangle_L \circ \langle b \rangle_R = \langle b \rangle_L \circ \langle a \rangle_L \). Hence, \( \langle a \rangle_L = \langle a \rangle_R \) if \( a \) is an idempotent, consequently \( \langle a \circ a \rangle_L = \langle a \circ a \rangle_R \). It is easy to show that \( \langle a \rangle_R \circ a^2 = a^2 \circ \langle a \rangle_L \).

**Lemma 2.3.** Let \( H \) be an LA-semihypergroup with left identity, and let \( B \) be an idempotent bi-hyperideal of \( H \). Then \( B \) is a hyperideal of \( H \).

**Proof.** By the definition of LA-semihypergroup (1.3), we have

\[
B \circ H = (B \circ B) \circ H = (H \circ B) \circ B = \left( H \circ B^2 \right) \circ B = \left( B^2 \circ H \right) \circ B = (B \circ H) \circ B,
\]

and every right hyperideal in \( H \) with left identity is left. \( \square \)

**Lemma 2.4.** Let \( H \) be an LA-semihypergroup with left identity \( e \), and let \( B \) be a proper bi-hyperideal of \( H \). Then \( e \notin B \).

**Proof.** Let us suppose that \( e \in B \). Since \( hob \subseteq (eoh)ob \subseteq B \), using (1.3), we have \( h \in (eoe)oh = (h \circ e)oe \subseteq (S \circ B) \circ B \subseteq B \). It is impossible. So, \( e \notin B \). \( \square \)

It can be easily noted that \( \{x \in H : (x \circ a) \circ x = e \} \not\subseteq B \).

**Proposition 2.5.** Let \( H \) be an LA-semihypergroup with left identity, and let \( A, B \) be bi-hyperideals of \( H \). Then the following statements are equivalent:

1. every bi-hyperideal is idempotent,
2. \( A \cap B = A \circ B \),
3. the hyperideals of \( H \) form a hypersemilattice \((L_H, \wedge)\), where \( A \wedge B = A \circ B \).
Similarly, associativity follows. Hence, \((L_H, \wedge)\) is a hypersemilattice.

A bi-hyperideal \(B\) of an LA-semihypergroup \(H\) is called a prime bi-hyperideal if \(B_1 \circ B_2 \subseteq B\) implies either \(B_1 \subseteq B\) or \(B_2 \subseteq B\) for every bi-hyperideal \(B_1\) and \(B_2\) of \(H\). The set of bi-hyperideals of \(H\) is totally ordered under the set inclusion if for all bi-hyperideals \(I, J\) either \(I \subseteq J\) or \(J \subseteq I\).

**Theorem 2.6.** Let \(H\) be an LA-semihypergroup with left identity. Every bi-hyperideal of \(H\) is prime if and only if it is idempotent and the set of the bi-hyperideals of \(H\) is totally ordered under the set inclusion.

Proof. Let us assume that every bi-hyperideal of \(H\) is prime. Since \(B^2\) is a hyperideal and so is prime which implies that \(B \subseteq B \circ B\), hence \(B\) is idempotent. Since \(B_1 \cap B_2\) is a bi-hyperideal of \(H\) (where \(B_1\) and \(B_2\) are bi-hyperideals of \(H\)) and so is prime. Now by Lemma 2.3, either \(B_1 \subseteq B_1 \cap B_2\) or \(B_2 \subseteq B_1 \cap B_2\) which further implies that either \(B_1 \subseteq B_2\) or \(B_2 \subseteq B_1\). Hence, the set of bi-hyperideals of \(H\) is totally ordered under set inclusion.

Conversely, let us assume that every bi-hyperideal of \(H\) is idempotent and the set of bi-hyperideals of \(H\) is totally ordered under set inclusion. Let \(B_1, B_2\) and \(B\) be the bi-hyperideals of \(H\) with \(B_1 \circ B_2 \subseteq B\) and without loss of generality assume that \(B_1 \subseteq B_2\). Since \(B_1\) is an idempotent, so \(B_1 \circ B_1 \subseteq B_1 \circ B_2 \subseteq B\) implies that \(B_1 \subseteq B\) and, hence, every bi-hyperideal of \(H\) is prime.

A bi-hyperideal \(B\) of an LA-semihypergroup \(H\) is called strongly irreducible bi-hyperideal if \(B_1 \cap B_2 \subseteq B\) implies either \(B_1 \subseteq B\) or \(B_2 \subseteq B\) for every bi-hyperideal \(B_1\) and \(B_2\) of \(H\).

**Theorem 2.7.** Let \(H\) be an LA-semihypergroup with zero. Let \(D\) be the set of all bi-hyperideals of \(H\), and \(\Omega\) the set of all strongly irreducible proper bi-hyperideals of \(H\), then \(\Gamma(\Omega) = \{O_B : B \in D\}\) forms a topology on the set \(\Omega\), where \(O_B = \{J \in \Omega : B \not\subseteq J\}\) and \(\phi : \text{Bi-hyperideal}(H) \to \Gamma(\Omega)\) preserves finite intersection and arbitrary union between the set of bi-hyperideals of \(H\) and open subsets of \(\Omega\).

Proof. Since \(\{0\}\) is a bi-hyperideal of \(H\) and 0 belongs to every bi-hyperideal of \(H\), then \(O_B = \{J \in \Omega : \{0\} \not\subseteq J\} = \{\}\), also \(O_H = \{J \in \Omega : H \not\subseteq J\} = \Omega\) which is the first axiom for the topology. Let \(\{O_{B\alpha} : \alpha \in I\} \subseteq \Gamma(\Omega)\), then \(\bigcup O_{B\alpha} = \{J \in \Omega : B_{\alpha} \not\subseteq J\}\) for some \(\alpha \in I\) = \(\{J \in \Omega, \langle \bigcup B_{\alpha} \rangle \not\subseteq J\}\) is a bi-hyperideal of \(H\) generated by \(\langle \bigcup B_{\alpha} \rangle\). Let \(O_{B_1}\) and \(O_{B_2}\) be the bi-hyperideals of \(\Omega\), if \(J \in O_{B_1} \cap O_{B_2}\), then \(J \in \Omega\) and \(B_1 \not\subseteq B_2\) and \(B_2 \not\subseteq B_1\). Let us suppose \(B_1 \cap B_2 \subseteq J\), this implies that either \(B_1 \subseteq J\) or \(B_2 \subseteq J\). It is impossible. Hence, \(B_1 \cap B_2 \not\subseteq J\) which further implies that \(J \in O_{B_1 \cap B_2}\). Thus \(O_{B_1} \cap O_{B_2} \subseteq O_{B_1 \cap B_2}\). Now if \(J \in O_{B_1 \cap B_2}\), then \(J \in \Omega\) and \(B_1 \not\subseteq B_2\) and \(B_2 \not\subseteq B_1\). Thus \(J \in O_{B_1}\) and \(J \in O_{B_2}\) and \(J \in O_{B_1 \cap B_2}\) which implies that \(O_{B_1} \cap O_{B_2} \subseteq O_{B_1 \cap B_2}\). Hence \(\Gamma(\Omega)\) is the topology on \(\Omega\). Define \(\phi : \text{Bi-hyperideal}(H) \to \Gamma(\Omega)\) by \(\phi(B) = O_B\), then it is easy to note that \(\phi\) preserves finite intersection and arbitrary union.

A hyperideal \(P\) of an LA-semihypergroup \(H\) is called prime if \(A \circ B \subseteq P\) implies that either \(A \subseteq P\) or \(B \subseteq P\) for all hyperideals \(A\) and \(B\) in \(H\).

Let \(P_H\) denotes the set of proper prime hyperideals of an LA-semihypergroup \(H\) absorbing 0. For a hyperideal \(I\) of \(H\), we define the sets \(\Theta_I = \{J \in P_H : I \not\subseteq J\}\) and \(\Gamma(P_H) = \{\Theta_I, I\} is a hyperideal of \(H\).\)
Theorem 2.8. Let $H$ be an LA-semihypergroup with zero. The set $\Gamma(P_H)$ constitutes a topology on the set $P_H$.

Proof. Let $\Theta_1, \Theta_2 \in \Gamma(P_H)$, if $J \in \Theta_1 \cap \Theta_2$, then $J \in P_H$ and $I_1 \subseteq J$ and $I_2 \subseteq J$. Let $I_1 \cap I_2 \subseteq J$ which implies that either $I_1 \subseteq J$ or $I_2 \subseteq J$, which is impossible. Hence, $J \in \Theta_1 \cap \Theta_2$. Similarly $\Theta_{I_1 \cap I_2} \subseteq \Theta_{I_1} \cap \Theta_{I_2}$. The remaining proof follows from Theorem 2.7.

The assignment $I \rightarrow \Theta_I$ preserves finite intersection and arbitrary union between the hyperideal $(H)$ and their corresponding open subsets of $\Theta_I$.

Let $P$ be a left hyperideal of an LA-semihypergroup $H$. $P$ is called quasiprime if for left hyperideals $A, B$ of $H$ such that $A \cap B \subseteq P$, we have $A \subseteq P$ or $B \subseteq P$.

Theorem 2.9. Let $H$ be an LA-semihypergroup with left identity $e$. Then a left hyperideal $P$ of $H$ is quasiprime if and only if $(H \circ a) \circ b \subseteq P$ implies that either $a \in P$ or $b \in P$.

Proof. Let $P$ be a left hyperideal of $H$. Let us assume that $(H \circ a) \circ b \subseteq P$, then

$$H \circ ((H \circ a) \circ b) \subseteq H \circ P \subseteq P,$$

that is,

$$H \circ ((H \circ a) \circ b) = (H \circ a) \circ (H \circ b).$$

Hence, either $a \in P$ or $b \in P$.

Conversely, let us assume that $A \circ B \subseteq P$, where $A$ and $B$ are left hyperideal of $H$ such that $A \subseteq P$. Then there exists $x \in A$ such that $x \notin P$. Now, by the hypothesis, we have $(H \circ x) \circ y \subseteq (H \circ A) \circ B \subseteq A \circ B \subseteq P$ for all $y \in B$. Since $x \notin P$, so by hypothesis, $y \in P$ for all $y \in B$, we obtain $B \subseteq P$. This shows that $P$ is quasiprime.

An LA-semihypergroup $H$ is called an antirectangular if $a \in (b \circ a) \circ b$, for all $a, b \in H$. It is easy to see that $H = H \circ H$. In the following results for an antirectangular LA-semihypergroup $H$, $e \notin H$.

Proposition 2.10. Let $H$ be an LA-semihypergroup. If $A, B$ are hyperideals of $H$, then $A \circ B$ is a hyperideal.

Proof. Using (1.4), we have

$$(A \circ B) \circ H = (A \circ B) \circ (H \circ H) = (A \circ H) \circ (B \circ H) \subseteq A \circ B,$$

also

$$H \circ (A \circ B) = (H \circ H) \circ (A \circ B) = (H \circ A) \circ (H \circ B) \subseteq A \circ B,$$

which shows that $A \circ B$ is a hyperideal.

Consequently, if $I_1, I_2, \ldots, I_n$ are hyperideals of $H$, then

$$(\cdots \circ ((I_1 \circ I_2) \circ I_3) \cdots \circ I_n), \quad (\cdots \circ (I_1^2 \circ I_2^2) \circ I_3^2) \cdots \circ I_n^2)$$

(2.12)
are hyperideals of $H$ and the set $\mathcal{I}(I)$ of hyperideals of $H$ form an antirectangular LA-semihypergroup.

**Lemma 2.11.** Let $H$ be an antirectangular LA-semihypergroup. Any subset of $H$ is left hyperideal if and only if it is right.

*Proof.* Let $I$ be a right hyperideal of $H$, then using (1.3), we get $h \circ i \subseteq ((k \circ h) \circ k) \circ i \subseteq (i \circ k) \circ (k \circ h) \subseteq I$.

Conversely, let us suppose that $I$ is a left hyperideal of $H$, then using (1.3), we have $i \circ h \subseteq ((t \circ i) \circ t) \circ h \subseteq (h \circ t) \circ (t \circ i) \subseteq I$. \hfill \Box

It is fact that $H \circ I = I \circ H$. From the above lemma, we remark that every quasiprime hyperideal becomes prime in an antirectangular LA-semihypergroup.

**Lemma 2.12.** Let $H$ be an antirectangular LA-semihypergroup. If $I$ is a hyperideal of $H$, then $S(a) = \{x \in H : a \in (x \circ a) \circ x, \text{ for } a \in I\} \subseteq I$.

*Proof.* Let $y \in S(a)$, then $y \in (y \circ a) \circ y \subseteq (H \circ I) \circ H \subseteq I$. Hence $H(a) \subseteq I$. Also, $S(a) = \{x \in H : x \in (x \circ a) \circ x, \text{ for } a \in I\} \subseteq I$. \hfill \Box

An hyperideal $I$ of an LA-semihypergroup $H$ is called an idempotent if $I \circ I = I$. An LA-semihypergroup $H$ is said to be fully idempotent if every hyperideal of $H$ is idempotent.

**Proposition 2.13.** Let $H$ be an antirectangular LA-semihypergroup, and, $A, B$ be hyperideals of $H$. Then the following statements are equivalent:

1. $H$ is fully idempotent,
2. $A \cap B = A \circ B$,
3. the hyperideals of $H$ form a hypersemilattice $(L_H, \wedge)$ where $A \wedge B = A \circ B$.

The proof follows from Proposition 2.5.

The set of hyperideals of $H$ is totally ordered under set inclusion if for all hyperideals $I, J$ either $I \subseteq J$ or $J \subseteq I$ and denoted by hyperideal($H$).

**Theorem 2.14.** Let $H$ be an antirectangular LA-semihypergroup. Then every hyperideal of $H$ is prime if and only if it is idempotent and hyperideal ($H$) is totally ordered under set inclusion.

*Proof.* The proof follows from Theorem 2.6. \hfill \Box

In conclusion, let us mention that it would be interesting to investigate whether it is possible to apply hyperideals of hyperstructures to the construction of classification systems similar to those introduced in [8–12].

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