Research Article

On the Magnetogravitational Instability of a Ferromagnetic Dust Cloud in the Presence of Nonuniform Rotation

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The effect of a nonuniform magnetic field on the gravitational instability of a nonuniformly rotating infinitely extending axisymmetric cylinder in a homogenous ferromagnetic medium has been studied. The propagation of the wave is allowed along radial direction. A general dispersion relation, using the normal mode analysis method on the perturbation equations of the problem, is obtained. It is found that *Bel* and *Schatzman criterion* determines the gravitational instability of this general problem. Thus, it appears that the effect of non-uniform magnetic field on the gravitational instability as discussed by (Dhiman and Dadwal, 2010) is marginalized by the magnetic polarizability of ferrofluid.

1. Introduction

The gravitational instability (GI) or the *Jeans instability* causes the collapse of interstellar gas clouds and subsequent star formation. It occurs when the internal gas pressure is not strong enough to prevent gravitational collapse of a region filled with matter. GI plays a dominant role in determining the star formation properties of galaxies. The problem of instability of self-gravitating large gas clouds was first considered by Jeans [1] and underlined the importance of self-gravitating instabilities in astrophysics because of their crucial role in understanding collapse, formation, and evolution of interstellar molecular clouds, star formation, galactic structure and its evolution, and so forth. For a latest and broader review on the subject of gravitational instability, one may refer to [1–8] and references therein.

Theoretical studies have shown that the magnetic field plays a vital role in self-gravitating star-forming regions and in the evolution of interstellar clouds into self-gravitating star forming regions. The problem of magnetogravitational instability of interstellar rotating medium is of considerable importance in connection with the protostar and star formation in magnetic dust clouds. In astrophysics, the problems that are considered generally assume magnetic field/rotation to be uniform: however, this idealization of

the uniform character in theoretical investigations is valid only for laboratory purposes (*cf.* Larson [8] and references within), because in the interstellar interior and atmosphere, the magnetic field/rotation may be variable and may together alter the nature of the instability. Limited efforts have been put to investigate the GI by considering the magnetic field or rotation to be variable. Bel and Schatzman [9] studied the effect of nonuniform rotation on the onset of gravitational instability and obtained a modified expression for the *Jeans criterion*, which is now known as the *Bel and Schatzman criterion*. Recently, Dhiman and Dadwal [10, 11] also studied the simultaneous effect of nonuniform magnetic field and rotation on the gravitational instability of gaseous medium and obtained some general qualitative results.

In astronomy, the interstellar medium (ISM) is the matter that exists in the space between the star systems in a galaxy. This matter includes gas in ionic, atomic, and molecular form, dust, and cosmic rays. It fills interstellar space and blends smoothly into the surrounding intergalactic space. The dust, which is composed of small solid particles by segregation in the interstellar clouds, is a very important component of the ISM. In the recent past, there have been dramatic changes in the conception of the interstellar medium, and recent observational and numerical works have suggested that interstellar medium (ISM) plays

an important role in the star formation. Spitzer [12] has shown that solid particles in space including dust grains are electically charged, relative to surrounding plasma. Further, the efficiency and timescale of stellar birth in Galactic molecular clouds strongly depend on the properties of interstellar medium (ISM).

Ferrofluids are colloidal liquids made of nanoscale ferromagnetic, or ferrimagnetic, particles suspended in a carrier fluid (usually an organic solvent or water). A ferrofluid flow in the presence of an applied magnetic field is accompanied by an intertwinement of hydrodynamic and magnetic interactions (Rosenwieg [13]). A ferrofluid is stable and does not solidify even in the presence of a high magnetic field and is commonly characterized by its strong tendency to magnetize in the direction of the magnetic field. Jones and Spitzer [14] provided a model for the existence of gas-dust interstellar mediums with a highly pronounced property of magnetic polarizability. This can be assumed due to a super paramagnetic dispersion of the fine ferromagnetic grains suspended in a gaseous cloud of molecular hydrogen. Mamun and Shukla [15] observed the usual Jeans instability in a self-gravitating dark interstellar molecular cloud containing ferromagnetic dust grains and baryonic gas clouds and supported the existence of ferromagnetic dust particles in a magnetically supported dark interstellar self-gravitating interstellar molecular cloud.

The present study is primarily motivated by the investigations of Mamun and Shukla [15] regarding the instability of a self-gravitating dark interstellar molecular cloud containing ferromagnetic dust grains. Our aim here is to investigate the gravitational instability of an infinite axisymmetric cylinder of a homogenous nonuniformly rotating medium containing the ferromagnetic dust particles in the presence of nonuniform magnetic field. The mathematical analysis followed in this paper is precisely the same as that of [9, 10]. Since the system of linearized perturbation equation governing the problem contains variable coefficients, the sufficient condition for GI is derived using the local instability approach as adopted in [9].

2. Mathematical Model and Basic Equations

Consider an infinite homogeneous, interstellar self-gravitating molecular cloud containing ferromagnetic dust particles. The system is permeated with the simultaneous action of a nonuniform rotation and a nonuniform magnetic field. In a rectangular coordinate system, let $\vec{u} = (u_x, u_y, u_z)$ be the velocity, $\vec{M} = (M_x, M_y, M_z)$ be the ferrofluid magnetization under the magnetic field $\vec{H} = (H_x, H_y, H_z)$. For the treatment of this physical configuration for gravitational instability, the conventional set of ferrohydrodynamic equations are given by (cf. [16, 17])

$$\frac{d\vec{u}}{dt} = -\text{grad } p + \rho \text{ grad } \phi + \mu_0 (\vec{M} \cdot \nabla) \vec{H}, \quad (1)$$

$$\frac{\partial \rho}{\partial t} + (\vec{u} \cdot \text{grad}) \rho + \rho (\nabla \cdot \vec{u}) = 0, \quad (2)$$

$$\nabla^2 \phi = -4\pi G(\rho - \rho_0). \quad (3)$$

The ferrofluid magnetization $\vec{M}$ satisfies Shliomis' [16, 17] equation of magnetization, which is given by

$$\frac{d\vec{M}}{dt} = \frac{1}{2} (\nabla \times \vec{u}) \times \vec{M} - \alpha (\vec{M} - \vec{M}_0) - \beta \vec{M} \times (\vec{M} \times \vec{H}). \quad (4)$$

In the above equations p, ϕ, μ_0, ρ , and G , respectively, denote the pressure, gravitational potential, the inverse Brownian time constant for particle diffusion in the ferrofluid, the magnetic permeability, density, the gravitational constant, $\alpha = 3\eta V/k_b T$ is the Brownian time of rotational particle diffusion and $\beta = 1/6\eta\varphi$, where η is the dynamic viscosity of the carrier fluid and $\varphi = nV$ is the volume fraction of magnetic grains in the liquid. Here, n is the number density and V the volume of a single particle. Also, ρ_0 and $\vec{M}_0 = (0, 0, M_0)$ are the *equilibrium* values of density and magnetization and M_0 the equilibrium magnetization of ferrofluid is related to the equilibrium magnetic field H_0 by

$$M_0 = n\bar{M} \left(\coth \psi - \frac{1}{\psi} \right) \frac{H_0}{\|H_0\|}, \quad (5)$$

where $\bar{M}$ is the magnetic moment of single ferromagnetic particle and the nondimensional quantity ψ is given by $\psi = \mu_0 (\bar{M} \|H_0\| / k_b T_b)$, where k_b is the Boltzman constant and T_b is temperature of the ferrofluid. Further, the Poisson's equation (3) is considered so as to avoid the “*Jeans Swindle*” (cf. Spiegel and Thiffeault [7]).

In Chu formulation of electrodynamics (see Penfield and Haus [18]), the magnetic field $\vec{H}$, magnetization $\vec{M}$, and magnetic induction $\vec{B}$ are related by

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}). \quad (6)$$

We know that the magnetic field $\vec{H}$ satisfies the following Maxwell's equations:

$$\nabla \times \vec{H} = 0, \quad (7)$$

$$\nabla \cdot \vec{B} = 0. \quad (8)$$

Equation (6), using (8) yields

$$\nabla \cdot \vec{H} = -\nabla \cdot \vec{M}. \quad (9)$$

In the present analysis, we shall consider an infinite axisymmetric cylinder of homogeneous, infinitely conducting self-gravitating ferromagnetic dust cloud under the simultaneous effect of a nonuniform rotation and a nonuniform magnetic field, therefore, transforming the above fundamental equations in cylindrical coordinates (r, θ, z) . Denoting $\vec{u} = (u_r, u_\theta, u_z)$ and $\vec{M} = (M_r, M_\theta, M_z)$ by the components of velocity $\vec{u}$ and ferrofluid magnetization $\vec{M}$ along the radial r , the transverse θ , and the axial z directions, respectively. The cylinder is assumed to be rotating about its axis (z -axis) with nonuniform angular velocity ω . The propagation of wave is taken along the radial direction “ r ”

of the cylinder, hence $\partial/\partial r$ is the only nonzero component of the gradient.

Following Dhiman and Dadwal [10], the basic equations (1)–(4), (7) and (9) under these assumptions take the following forms in cylindrical polar coordinates

$$\begin{aligned}
 & \rho \left(\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} - \frac{u_\theta^2}{r} \right) \\
 &= \rho \frac{\partial \phi}{\partial r} - \frac{\partial p}{\partial r} + \mu_0 \left[M_r \frac{\partial H_r}{\partial r} - \frac{M_\theta H_\theta}{r} \right], \\
 & \rho \left(\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial}{\partial r} u_\theta + \frac{u_r u_\theta}{r} \right) \\
 &= \frac{\mu_0}{2} \left[M_r \frac{\partial H_\theta}{\partial r} + H_r \frac{\partial M_\theta}{\partial r} + M_\theta \frac{\partial H_r}{\partial r} \right. \\
 &\quad \left. - H_\theta \frac{\partial M_r}{\partial r} + \frac{2M_\theta H_r}{r} \right], \\
 & \rho \left(\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} \right) \\
 &= \frac{\mu_0}{2} \left[M_r \frac{\partial H_z}{\partial r} + H_r \frac{\partial M_z}{\partial r} + M_z \frac{\partial H_r}{\partial r} - H_z \frac{\partial M_r}{\partial r} \right. \\
 &\quad \left. + \left(\frac{M_z H_r - H_z M_r}{r} \right) \right], \\
 & \frac{\partial M_r}{\partial t} + u_r \frac{\partial M_r}{\partial r} + \frac{1}{2} \left(M_\theta \left(\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \right) + M_z \frac{\partial u_z}{\partial r} \right) \\
 &\quad + \alpha(M_r - M_0) + \beta(M_\theta [M_r H_\theta - H_r M_\theta] \\
 &\quad \quad - M_z [H_r M_z - M_r H_z]) = 0, \\
 & \frac{\partial M_\theta}{\partial t} + u_r \frac{\partial M_\theta}{\partial r} - \frac{1}{2} \left(M_r \left(\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \right) \right) \\
 &\quad + \alpha(M_\theta - M_0) + \beta(M_z [M_\theta H_z - H_\theta M_z] \\
 &\quad \quad - M_r [M_r H_\theta - H_r M_\theta]) = 0, \\
 & \frac{\partial M_z}{\partial t} + u_r \frac{\partial M_z}{\partial r} - \frac{1}{2} \left(M_r \frac{\partial u_z}{\partial r} \right) + \alpha(M_z - M_0) \\
 &\quad + \beta(M_r [H_r M_z - M_r H_z] \\
 &\quad \quad - M_\theta [M_\theta H_z - H_\theta M_z]) = 0, \\
 & \frac{\partial H_z}{\partial r} = 0, \\
 & \left(\frac{\partial H_\theta}{\partial r} + \frac{H_\theta}{r} \right) = 0, \\
 & \left(\frac{\partial H_r}{\partial r} + \frac{H_r}{r} \right) = - \left(\frac{\partial M_r}{\partial r} + \frac{M_r}{r} \right), \\
 & \frac{\partial \rho}{\partial t} + (\vec{u} \cdot \text{grad})\rho + \rho(\nabla \cdot \vec{u}) = 0, \\
 & \nabla^2 \phi = -4\pi G(\rho - \rho_0).
 \end{aligned} \tag{10}$$

In these equations, the operators ∇ and ∇^2 now are respective operators in cylindrical coordinates.

3. Equilibrium State and Perturbation Equations

Following the physical models of Mamun and Shukla [15] and Kumar et al. [19], the magnetization is taken along r and z directions.

The equilibrium state under discussion is clearly characterized as follows:

$$\begin{aligned}
 \vec{u} &= (0, r\omega, 0); & \vec{H} &= (0, 0, H_z); & \vec{M} &= (M_r, 0, M_z) \\
 p &= p_0; & \phi &= \phi_0; & \rho &= \rho_0
 \end{aligned} \tag{11}$$

Using (11) in (10) and using the fact that propagation of wave is along r direction and thus $\partial/\partial r$ is the only nonzero component, we obtain the following basic solution:

$$\frac{\partial p_0}{\partial r} = \rho_0 \left(\frac{\partial \phi_0}{\partial r} + r\omega^2 \right) \tag{12}$$

$$M_r \frac{\partial H_z}{\partial r} - H_z \left(\frac{\partial M_r}{\partial r} + \frac{M_r}{r} \right) = 0 \tag{13}$$

$$\alpha(M_r - M_0) + \beta M_z M_r H_z = 0 \tag{14}$$

$$-\frac{1}{2} M_r \left(\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \right) + \alpha M_0 = 0 \tag{15}$$

$$\alpha(M_z - M_0) - \beta M_r^2 H_z = 0 \tag{16}$$

$$\frac{\partial H_z}{\partial r} = 0 \tag{17}$$

$$\left(\frac{\partial M_r}{\partial r} + \frac{M_r}{r} \right) = 0 \tag{18}$$

$$\frac{1}{r} \left(\frac{\partial}{\partial r} \left(r \frac{\partial \phi_0}{\partial r} \right) \right) = 0 \tag{19}$$

Equation (17) implies that

$$H_z = \text{Constant} \tag{20}$$

Since pressure and density are uniform initially, therefore it is clear from (12) that the gravitational potential is balanced by centrifugal force.

Let us allow the small perturbation in the initial state described by (11) as following;

$$\begin{aligned}
 \vec{u}' &= (u'_r, r\omega + u'_\theta, u'_z); & \vec{H}' &= (h'_r, h'_\theta, H_z + h'_z); \\
 p' &= p_0 + \delta p & \vec{M}' &= (M_r + m'_r, m'_\theta, M_z + m'_z); \\
 \phi' &= \phi_0 + \delta \phi; & \rho' &= \rho_0 + \delta \rho.
 \end{aligned} \tag{21}$$

Using (21) in (10), ignoring the terms of second and higher orders in the perturbations, and using (12)–(20), we have the following linearized perturbed equations:

$$\rho_0 \left(\frac{\partial u_r}{\partial t} - 2\omega u_\theta \right) = \rho_0 \frac{\partial \delta \phi}{\partial r} - \frac{\partial \delta p}{\partial r} + \mu_0 \left(M_r \frac{\partial h_r}{\partial r} \right), \quad (22)$$

$$\rho_0 \left(\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial}{\partial r}(r\omega) + u_r \omega \right) = \frac{\mu_0}{2} \left(M_r \frac{\partial h_\theta}{\partial r} - h_\theta \frac{\partial M_r}{\partial r} \right), \quad (23)$$

$$\rho_0 \frac{\partial u_z}{\partial t} = \frac{\mu_0}{2} \left(h_r \frac{\partial M_z}{\partial r} + M_z \frac{1}{r} \frac{\partial}{\partial r}(rh_r) - H_z \frac{1}{r} \frac{\partial}{\partial r}(rm_r) \right), \quad (24)$$

$$\begin{aligned} \frac{\partial m_r}{\partial t} + u_r \frac{\partial M_r}{\partial r} + \frac{1}{2} \left(M_z \frac{\partial u_z}{\partial r} + m_\theta \left(\frac{\partial}{\partial r}(r\omega) + \omega \right) \right) + \alpha m_r \\ + \beta (-M_z^2 h_r + H_z M_z m_r + M_z M_r h_z + H_z M_r m_z) = 0, \end{aligned} \quad (25)$$

$$\begin{aligned} \frac{\partial m_\theta}{\partial t} - \frac{1}{2} \left(m_r \left(\frac{\partial}{\partial r}(r\omega) + \omega \right) + M_r \frac{1}{r} \frac{\partial}{\partial r}(ru_\theta) \right) + \alpha m_\theta \\ + \beta (H_z M_z m_\theta - M_z^2 h_\theta - M_r^2 h_\theta) = 0, \end{aligned} \quad (26)$$

$$\begin{aligned} \frac{\partial m_z}{\partial t} + u_r \frac{\partial M_z}{\partial r} - \frac{1}{2} M_r \frac{\partial u_z}{\partial r} + \alpha m_z \\ + \beta (M_r M_z h_r - 2M_r H_z m_r - M_r^2 h_z) = 0, \end{aligned} \quad (27)$$

$$\frac{\partial h_z}{\partial r} = 0, \quad (28)$$

$$\left(\frac{\partial h_\theta}{\partial r} + \frac{h_\theta}{r} \right) = 0, \quad (29)$$

$$\frac{1}{r} \frac{\partial}{\partial r}(rm_r) = -\frac{1}{r} \frac{\partial}{\partial r}(rh_r), \quad (30)$$

$$\frac{\partial}{\partial t} \delta \rho + \rho_0 \left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} \right) = 0, \quad (31)$$

$$\frac{1}{r} \left(\frac{\partial}{\partial r} \left(r \frac{\partial \delta \phi}{\partial r} \right) \right) = -4\pi G \delta \rho, \quad (32)$$

where (u'_r, u'_θ, u'_z) , (h'_r, h'_θ, h'_z) , (m'_r, m'_θ, m'_z) , δp , $\delta \rho$, and $\delta \phi$ are the respective perturbations from basic state in velocity, magnetic field, magnetization, pressure, density, and gravitational potential vector. In the above equations, the dashes have been dropped for convenience in writing.

Since we have considered that the fluctuations in pressure and density take place adiabatically, therefore

$$\delta p = c^2 \delta \rho. \quad (33)$$

Using this equation of state (33) for adiabatic medium, (22) reduces to

$$\rho_0 \left(\frac{\partial u_r}{\partial t} - 2\omega u_\theta \right) = \rho_0 \frac{\partial \delta \phi}{\partial r} - c^2 \frac{\partial \delta \rho}{\partial r} + \mu_0 \left(M_r \frac{\partial h_r}{\partial r} \right). \quad (34)$$

4. Gravitational Instability

In order to investigate the stability of the forgoing stationary state, we shall consider the dependence of the perturbation on r and t of the form

$$\psi^*(r) \exp(\sigma t), \quad (35)$$

where σ is frequency of the perturbation.

For this type of dependence of perturbation on r and t , we have

$$\frac{\partial}{\partial t} \equiv \sigma, \quad \frac{\partial}{\partial r} f(r) = \frac{d}{dr} f(r). \quad (36)$$

Using the above dependence, the perturbations equations (34) and (23)–(32) assume the following forms:

$$\sigma u_r - 2\omega u_\theta - \frac{d\delta \phi}{dr} + \frac{c^2}{\rho_0} \frac{d\delta \rho}{dr} - \frac{\mu_0}{\rho_0} \left(M_r \frac{dh_r}{dr} \right) = 0, \quad (37)$$

$$\sigma u_\theta + u_r \frac{d}{dr}(r\omega) + u_r \omega - \frac{1}{2} \frac{\mu_0}{\rho_0} \left(M_r \frac{dh_\theta}{dr} - h_\theta \frac{dM_r}{dr} \right) = 0, \quad (38)$$

$$\sigma u_z - \frac{1}{2} \frac{\mu_0}{\rho_0} \left(h_r \frac{dM_z}{dr} + M_z \frac{1}{r} \frac{d}{dr}(rh_r) - H_z \frac{1}{r} \frac{d}{dr}(rm_r) \right) = 0, \quad (39)$$

$$\begin{aligned} (\sigma + \alpha) m_r + u_r \frac{dM_r}{dr} + \frac{1}{2} \left(M_z \frac{du_z}{dr} + m_\theta \left(\frac{d}{dr}(r\omega) + \omega \right) \right) \\ + \beta (-M_z^2 h_r + H_z M_z m_r + M_z M_r h_z + H_z M_r m_z) = 0, \end{aligned} \quad (40)$$

$$\begin{aligned} (\sigma + \alpha) m_\theta - \frac{1}{2} \left(m_r \left(\frac{d}{dr}(r\omega) + \omega \right) + M_r \frac{1}{r} \frac{d}{dr}(ru_\theta) \right) \\ + \beta (H_z M_z m_\theta - M_z^2 h_\theta - M_r^2 h_\theta) = 0, \end{aligned} \quad (41)$$

$$\begin{aligned} (\sigma + \alpha) m_z + u_r \frac{dM_z}{dr} - \frac{1}{2} M_r \frac{du_z}{dr} \\ + \beta (M_r M_z h_r - 2M_r H_z m_r - M_r^2 h_z) = 0, \end{aligned} \quad (42)$$

$$\frac{dh_z}{dr} = 0, \quad (43)$$

$$\frac{1}{r} \frac{d}{dr}(rh_\theta) = 0, \quad (44)$$

$$\frac{1}{r} \frac{d}{dr}(rm_r) = -\frac{1}{r} \frac{d}{dr}(rh_r), \quad (45)$$

$$\sigma \delta \rho + \frac{\rho_0}{r} \frac{d}{dr}(ru_r) = 0, \quad (46)$$

$$\frac{1}{r} \left(\frac{d}{dr} \left(r \frac{d\delta \phi}{dr} \right) \right) = -4\pi G \delta \rho. \quad (47)$$

Since, the above equations involve the variable coefficients, we shall therefore investigate the local stability of the

above system in the neighborhood of $r = r_0$. For this, let us assume that the perturbations have a periodic form in the neighborhood of $r = r_0$, as

$$\bar{f} \exp(-ikr), \quad (48)$$

where k is the wave number. For this type of dependence, we have

$$\frac{d}{dr} \equiv -ik. \quad (49)$$

Equations (43)–(45) upon using (49) yield

$$\bar{h}_z = 0, \quad \bar{h}_\theta = 0, \quad \bar{h}_r = -\bar{m}_r. \quad (50)$$

Now, substituting (49) in (37)–(42), (46), (47) and using (50), we obtain a system of algebraic equations (with coefficients, in the vicinity of $r = r_0$) for amplitudes, marked with bars, which can be put in the matrix notation as following;

$$\begin{bmatrix} \frac{1}{\sigma}(\sigma^2 + Q) & -2\omega & 0 & -ik\frac{\mu_0}{\rho_0}M_r & 0 & 0 \\ P & \sigma & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma & \frac{\mu_0}{2\rho_0}\left[\frac{dM_z}{dr} - ikB_z\right] & 0 & 0 \\ \frac{dM_z}{dr} & 0 & -\frac{ikM_z}{2} & \sigma + N & \frac{P}{2} & \beta H_z M_z \\ 0 & -\frac{ikM_r}{2} & 0 & \frac{P}{2} & \sigma + R & 0 \\ \frac{dM_z}{dr} & 0 & -\frac{ikM_r}{2} & T & 0 & \sigma + \alpha \end{bmatrix} \begin{bmatrix} \bar{u}_r \\ \bar{u}_\theta \\ \bar{u}_z \\ \bar{m}_r \\ \bar{m}_\theta \\ \bar{m}_z \end{bmatrix} = 0. \quad (51)$$

Here, $B_z = (M_z + H_z)$, $Q = k^2 c^2 - 4\pi G\rho_0$, $N = \alpha + \beta(M_z^2 + H_z M_z)$, $R = (\alpha + \beta M_z H_z)$, and $T = (M_r M_z - 2M_r H_z)$.

For the nontrivial solution of the system (51), the determinant of the matrix of coefficients should vanish. Thus, on expanding the coefficient matrix and equating the real part equal to zero, we get the following dispersion relation:

$$\sigma^6 + B_1\sigma^5 + B_2\sigma^4 + B_3\sigma^3 + B_4\sigma^2 + B_5\sigma + B_6 = 0, \quad (52)$$

where,

$$B_1 = (\alpha + N + R),$$

$$B_2 = \left(8P\omega + 4Q - P^2 + 4NR + 4\alpha N + 4\alpha R + k^2 \frac{\mu_0}{\rho_0} B_z M_z - 4\beta T H_z M_r \right),$$

$$B_3 = \left(\frac{\mu_0}{\rho_0} 4NQ + 4QR + 4\alpha Q - \alpha P^2 + 4NR + 8\omega NP + 8\omega PR + 8\alpha\omega P + k^2 \frac{\mu_0}{\rho_0} R B_z M_z + \alpha k^2 \frac{\mu_0}{\rho_0} R B_z M_z + k^2 \beta \frac{\mu_0}{\rho_0} R B_z M_z M_r^2 - 4\beta R T M_r H_z \right),$$

$$B_4 = \left(-P^2 Q - 2\omega P^3 + 4\rho_0 NQR + 4\alpha NQ + 4\alpha\rho_0 QR + 8\omega NPR + 8\alpha\omega NP + 8\alpha\omega PR - 4\beta Q T M_r H_z - 8\omega\beta P T M_r H_z + k^2 \frac{\mu_0}{\rho_0} P^2 M_r^2 + k^2 \frac{\mu_0}{\rho_0} Q B_z M_z + \alpha k^2 \frac{\mu_0}{\rho_0} R B_z M_z + 2\omega k^2 \frac{\mu_0}{\rho_0} P B_z M_z + \beta k^2 \frac{\mu_0}{\rho_0} R H_z M_r^2 \right),$$

$$\begin{aligned} B_5 = & \left(-2\alpha\omega P^3 - \alpha P^2 Q + 4\alpha R N Q + 8\alpha\omega N P R - 4\beta Q R T M_r H_z - 8\omega\beta P R T M_r H_z + k^2 \frac{\mu_0}{\rho_0} Q R B_z M_z + \alpha k^2 \frac{\mu_0}{\rho_0} Q B_z M_z + \alpha k^2 \frac{\mu_0}{\rho_0} P^2 M_r^2 + \beta k^2 \frac{\mu_0}{\rho_0} Q B_z H_z M_r^2 + 2\omega k^2 \frac{\mu_0}{\rho_0} P R B_z M_z + 2\alpha\omega k^2 \frac{\mu_0}{\rho_0} P B_z M_z - 2\omega\beta k^2 \frac{\mu_0}{\rho_0} P B_z H_z M_r^2 \right), \\ B_6 = & \frac{\mu_0 k^2}{4\rho_0} R B_z (\alpha M_z + \beta H_z M_r^2) (Q + P). \end{aligned} \quad (53)$$

Equation (52) is of sixth degree in σ with all the coefficients real and the coefficients of σ^6 and σ^5 are clearly positive while the coefficients of σ^4 , σ^3 , σ^2 , σ , and the constant term may be negative depending on B_2 , B_3 , B_4 , B_5 , and B_6 . As per the criteria of Guillemin [20] for the signs of the roots, we get that if B_6 is negative, then the constant term of (52) is negative, which is sufficient to give the required condition of instability. Therefore, we obtain the following condition for the onset of gravitational instability for the present problem:

$$k^2 c^2 + 2\omega \frac{d}{dr}(\omega r^2) < (4\pi G\rho_0), \quad (54)$$

which is the same result as obtained by Bel and Schatzman [9].

5. Conclusions

In the present analysis, we have investigated the effect of ferromagnetization in the presence of nonuniform magnetic field on the gravitational instability of a homogenous, ferromagnetic dust cloud in the presence of nonuniform rotation. The inequality (54) implies that the *Bel and Schatzman criterion* determines the gravitational instability of this general problem, thus we can conclude that the ferromagnetism in the presence of magnetic field has no effect on the gravitational instability on the self-gravitating cloud. Comparing this result with result obtained by Dhiman and Dadwal [10] for the gravitational instability of a homogeneous gaseous medium in the presence of nonuniform magnetic field for nonferromagnetic case, it appears that the effect of magnetic field on the criterion for instability has been marginalized by the magnetic polarizability of ferrofluid. Further, from inequality (54) for $\omega = 0$, we can deduce that gravitational instability is governed by the *Jeans criterion*, in the absence of nonuniform rotation.

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