Research Article

Magnetogasdynamic Shock Waves in a Rotating Gas with Exponentially Varying Density

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Nonsimilar solutions are obtained for one-dimensional adiabatic flow behind a magnetogasdynamic cylindrical shock wave propagating in a rotating or nonrotating perfect gas in presence of a constant azimuthal magnetic field. The density of the gas is assumed to be varying and obeying an exponential law. In order to obtain the solutions, the angular velocity of the ambient medium is assumed to be decreasing exponentially as the distance from the axis increases. The shock wave moves with variable velocity and the total energy of the wave is nonconstant. The effects of variation of Alfvén-Mach number and time are obtained. Also, a comparison between the solutions in the cases of rotating and non-rotating media with or without magnetic field is made.

1. Introduction

Hayes [1], Laumbach and Probstein [2], DebRay [3], Verma and Vishwakarma [4, 5], Vishwakarma [6], Vishwakarma and Nath [7], and Vishwakarma et al. [8] have discussed the propagation of shock waves in a medium where density varies exponentially and obtained similarity or nonsimilarity solutions. These authors have not taken into account the effects of rotation of the medium.

The formation of self-similar problems and examples describing the adiabatic motion of nonrotating gas models of stars is considered by Sedov [9], Zel'dovich and Raizer [10], Lee and Chen [11], and Summers [12]. Rotation of the stars significantly affects the process taking place in their outer layers. Therefore questions connected with the explosions in rotating gas atmospheres are of definite astrophysical interest. Chaturani [13] studied the propagation of

cylindrical shock waves through a gas having solid body rotation and obtained the solution by similarity method adopted by Sakurai [14]. Nath et al. [15] obtained the similarity solutions for the flow behind spherical shock waves propagating in a nonuniform and rotating interplanetary atmosphere with increasing energy. Vishwakarma et al. [16] obtained the similarity solution for the magnetogasdynamics cylindrical shock waves propagation in a rotating medium in the presence of a constant azimuthal magnetic field by assuming that the density of the medium ahead of the shock wave is constant. Levin and Skopina [17] studied the propagation of detonation wave in rotational gas flows by taking into account the variable azimuthal and axial fluid velocities, and the components of the vorticity vector. Nath [18] obtained the similarity solution for the magnetogasdynamic shock wave generated by a moving piston in a rotational axisymmetric isothermal flow of perfect gas with variable density according to power law. Also, Nath [19] obtained the nonsimilarity solutions for the propagation of a strong cylindrical shock waves in a rotational axisymmetric dusty gas with exponentially varying density.

In the present work, we investigated the effects of the presence of an ambient azimuthal magnetic field and the rotation of the ambient medium on the flow field behind a magnetogasdynamic cylindrical shock wave. Nonsimilarity solutions for the flow field behind the shock wave are obtained. The density in the medium ahead of the shock is assumed to obey an exponential law. The angular velocity of rotation of the ambient medium is also assumed to be obeying exponential law and to be decreasing as the distance from the axis increases. It is expected that such an angular velocity may occur in the atmospheres of rotating planets and stars. The medium is assumed to be a perfect gas and the initial magnetic field to be constant.

Variation of the flow variables behind the shock for different values of Alfvén-Mach number and time is obtained. Also, a comparison between the solutions in the cases of rotating and nonrotating media is made for both the magnetic and nonmagnetic cases.

2. Fundamental Equations and Boundary Conditions

The fundamental equations governing the unsteady adiabatic cylindrically symmetric flow of an electrically conducting gas, which is rotating about the axis of symmetry in the presence of an azimuthal magnetic field may, in Eulerian coordinates, can be expressed as [13, 16, 20]

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{\rho u}{r} = 0, \quad (2.1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \left(\frac{\partial p}{\partial r} + \mu h \frac{\partial h}{\partial r} + \frac{\mu h^2}{r} \right) - \frac{v^2}{r} = 0, \quad (2.2)$$

$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial r} + h \frac{\partial u}{\partial r} = 0, \quad (2.3)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + \frac{uv}{r} = 0, \quad (2.4)$$

$$\frac{\partial U}{\partial t} + u \frac{\partial U}{\partial r} - \frac{p}{\rho^2} \left(\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) = 0, \quad (2.5)$$

where r and t are independent space and time coordinates, ρ the density, p the pressure, h the azimuthal magnetic field, U the internal energy per unit mass, u and v are the radial and azimuthal components of the fluid velocity, and μ is the magnetic permeability. The electrical conductivity of the gas is assumed to be infinite and the effects of viscosity and heat conduction are not considered.

Also,

$$v = Ar, \quad (2.6)$$

where " A " is the angular velocity of the medium at radial distance r from the axis of symmetry. The magnetic field equation (2.3) contains all the relevant information needed from Maxwell's equations and Ohm's law; the diffusion term is omitted from it by virtue of the assumed infinite electrical conductivity. The assumption of infinite electrical conductivity of the gas is physically realistic in the case of astrophysical phenomena where the magnetic Reynolds number is very high (or infinite) due to astrophysical scale. The magnetic Reynolds number is a dimensionless parameter defined by $R_m = \bar{U} \bar{L} / \eta_m$, where $\bar{U}, \bar{L}$ are the characteristic velocity and characteristic length of the flow field, and $\eta_m = 1 / \mu \sigma$ is the magnetic diffusivity (magnetic viscosity), σ being the electrical conductivity of the medium [21, page 169]. In this case, the Reynolds number R_e of the flow is also very high, as R_e is defined by $R_e = \bar{U} \bar{L} / \nu$, where ν is the kinematic coefficient of viscosity. It is well known that the effects of viscosity and heat conduction are negligibly small for the high Reynolds number flow except in the boundary layer region near the solid boundary or in any other region of large variations in velocity and temperature such as the inside of a shock wave [22, page 139]. Therefore omission of the effects of viscosity and heat conduction on the flow field may be justified in the present study.

The above system of equations should be supplemented with an equation of state. A perfect gas behaviour of the medium is assumed, so that

$$p = \Gamma \rho T, \quad U = \frac{p}{\rho(\gamma - 1)}, \quad (2.7)$$

where Γ is the gas constant and γ is the ratio of specific heats at constant pressure and volume.

We assume that a cylindrical shock is propagating outwordally in a perfect gas with infinite electrical conductivity and variable density in presence of a constant azimuthal magnetic field.

The ambient density of the medium is assumed to obey the exponential law, namely,

$$\rho_a = \rho_0 e^{\delta R}, \quad (2.8)$$

where R is the shock radius and ρ_0, δ are suitable constants.

In order to obtain the solution, it is assumed that the ambient angular velocity of the medium A_a varies as

$$A_a = A_0 e^{\alpha R}, \quad (2.9)$$

where A_0 and α are constants.

The jump conditions at the shock wave are given by the principles of conservation of mass, momentum, magnetic field, and energy across the shock [20, 23], namely,

$$\begin{aligned}
 \rho_a V &= \rho_n (V - u_n), \\
 h_a V &= h_n (V - u_n), \\
 p_a + \frac{1}{2} \mu h_a^2 + \rho_a V^2 &= p_n + \frac{1}{2} \mu h_n^2 + \rho_n (V - u_n)^2, \\
 U_a + \frac{p_a}{\rho_a} + \frac{\mu h_a^2}{\rho_a} + \frac{1}{2} V^2 &= U_n + \frac{p_n}{\rho_n} + \frac{\mu h_n^2}{\rho_n} + \frac{1}{2} (V - u_n)^2, \\
 v_a &= v_n,
 \end{aligned} \tag{2.10}$$

where the subscripts “a” and “n” denote the conditions immediately ahead and behind of the shock front, and $V (= dR/dt)$ denotes the velocity of the shock front.

If the shock is a strong one, then the jump conditions (2.10) become

$$\begin{aligned}
 u_n &= (1 - \beta) V, \\
 \rho_n &= \frac{\rho_a}{\beta}, \\
 h_n &= \frac{h_a}{\beta}, \\
 p_n &= \left[(1 - \beta) + \frac{1}{2M_A^2} \left(1 - \frac{1}{\beta^2} \right) \right] \rho_a V^2, \\
 v_n &= (A_0 e^{\alpha R}) R,
 \end{aligned} \tag{2.11}$$

where $M_A = (\rho_a V^2 / \mu h_a^2)^{1/2}$ is the Alfvén-Mach number. The quantity $\beta (0 < \beta < 1)$ is obtained by the relation

$$\beta^2 - \beta \left(\frac{\gamma(M_A^{-2} + 1) - 1}{(\gamma + 1)} \right) + \frac{(\gamma - 2)M_A^{-2}}{(\gamma + 1)} = 0. \tag{2.12}$$

Let the solution of (2.1)–(2.5) be of the form [4, 6, 19]

$$\begin{aligned}
 u &= \frac{1}{t} U(\eta), \\
 \rho &= t^{\Omega} D(\eta), \\
 p &= t^{\Omega-2} P(\eta),
 \end{aligned}$$

$$\begin{aligned} v &= \frac{1}{t} K(\eta), \\ \sqrt{\mu} h &= t^{(\Omega-2)/2} H(\eta), \end{aligned} \quad (2.13)$$

where

$$\eta = t e^{\lambda r} \quad (2.14)$$

while Ω and λ are constants. The variable η assumes constant value η_0 at the shock surface. Hence

$$V = -\frac{1}{\lambda t}, \quad (2.15)$$

which represents an outgoing shock surface, if $\lambda < 0$.

The solution of (2.1)–(2.5) in the form (2.13) to (2.15) are compatible with the shock conditions, if

$$\Omega = 2, \quad \lambda = \alpha = -\frac{\delta}{2}. \quad (2.16)$$

Since necessarily $\lambda < 0$, relation (2.16) shows that $\delta > 0$, meaning thereby that the shock surface expands outwardly in an exponentially increasing medium.

From (2.15) and (2.16), we obtain

$$R = \frac{2}{\delta} \log\left(\frac{t}{t_0}\right), \quad (2.17)$$

where t_0 is the duration of the almost instantaneous explosion.

3. Solution to the Equations

The flow variables in the flow field behind the shock front will be obtained by solving the (2.1) to (2.5). From (2.13), (2.15), and (2.16), we obtain

$$\frac{\partial u}{\partial t} = \lambda u V - V \frac{\partial u}{\partial r}, \quad (3.1)$$

$$\frac{\partial \rho}{\partial t} = -2\rho\lambda V - V \frac{\partial \rho}{\partial r}, \quad (3.2)$$

$$\frac{\partial p}{\partial t} = -V \frac{\partial p}{\partial r}, \quad (3.3)$$

$$\frac{\partial h}{\partial t} = -V \frac{\partial h}{\partial r}, \quad (3.4)$$

$$\frac{\partial v}{\partial t} = \lambda V v - V \frac{\partial v}{\partial r}. \quad (3.5)$$

Using (3.1) to (3.5) and the transformation,

$$r' = \frac{r}{R}, \quad u' = \frac{u}{V}, \quad v' = \frac{v}{V}, \quad \rho' = \frac{\rho}{\rho_n}, \quad h' = \frac{h}{h_n}, \quad p' = \frac{p}{p_n} \quad (3.6)$$

in the fundamental equations (2.1) to (2.5), we obtain

$$\frac{d\rho'}{dr'} = \frac{\rho'}{(1-u')} \left[\frac{du'}{dr'} + 2 \log \frac{t}{t_0} + \frac{u'}{r'} \right], \quad (3.7)$$

$$\begin{aligned} \frac{dp'}{dr'} &= \frac{\rho'}{[(1-\beta)\beta + (M_A^{-2}/2)(\beta - (1/\beta))]} \\ &\times \left[\left\{ (1-u') - \frac{M_A^{-2}h'}{\beta\rho'(1-u')} \right\} \frac{du'}{dr'} - u' \left(\log \frac{t}{t_0} \right) - \frac{M_A^{-2}h'^2}{\beta\rho'r'} + \frac{v'^2}{r'} \right], \end{aligned} \quad (3.8)$$

$$\frac{dh'}{dr'} = \frac{h'}{(1-u')} \frac{du'}{dr'}, \quad (3.9)$$

$$\frac{dv'}{dr'} = \frac{1}{(1-u')} \left[\left(\log \frac{t}{t_0} \right) v' + \frac{u'v'}{r'} \right], \quad (3.10)$$

$$\frac{du'}{dr'} = \frac{(1-u') \left[\rho' u' (\log(t/t_0)) + (M_A^{-2}h'^2/\beta r') - (v'^2 \rho'/r') \right]}{\left[\rho' (1-u')^2 - (M_A^{-2}h'/\beta) - \gamma p' \beta \{ (1-\beta) + (M_A^{-2}/2)(1 - (1/\beta^2)) \} \right]}. \quad (3.11)$$

Also, the total energy of the disturbance is given by

$$E = 2\pi \int_{\bar{r}}^R \rho \left[U + \frac{1}{2} (u^2 + v^2) + \frac{\mu h^2}{2\rho} \right] r dr, \quad (3.12)$$

where $\bar{r}$ is the position of inner boundary of the disturbance. Using (2.7), (3.4), and (2.11), (3.12) becomes

$$E = \frac{8\pi\mu}{\delta^2\eta_0^2} R^2 \int_{\bar{r}}^1 \left[\left\{ (1-u) + \frac{M_A^{-2}}{2} \left(1 - \frac{1}{\beta^2} \right) \right\} \frac{p'}{(\gamma-1)} + \frac{(1/2)\rho'(u^2 + v^2)}{\beta} + \frac{M_A^{-2}h'^2}{2\beta} \right] r' dr'. \quad (3.13)$$

Hence, the total energy of the shock wave is nonconstant and varies as R^2 . The increase of total energy may be achieved by the pressure exerted on the fluid by the inner expanding surface (a contact surface or a piston). A situation very much of the same kind may prevail during the formation of a cylindrical spark channel from exploding wires. In addition, in the usual cases of spark break down, time-dependent energy input is a more realistic assumption than instantaneous energy input [24, 25].

Table 1: The position of inner expanding surface $\bar{r}'$ for $M_A^{-2} = 0, 0.05, 0.1$ and $t/t_0 = 2, 5$.

M_A^{-2}	t/t_0	Position of inner expanding surface ($\bar{r}'$)	
		Rotating gas	Non-rotating gas
0	2	0.13100	0.21000
0	5	0.20000	0.37000
0.05	2	0.33678	0.37041
0.05	5	0.28146	0.39486
0.1	2	0.36408	0.39330
0.1	5	0.31025	0.41210

In terms of dimensionless variables r', u', ρ', p', h' , and v' the shock conditions (2.11) take the form

$$r' = 1, \quad u' = (1 - \beta), \quad \rho' = 1, \quad p' = 1, \quad h' = 1, \quad v' = (A_0 \eta_0) \left(\log \frac{t}{t_0} \right). \quad (3.14)$$

Equations (3.7) to (3.12) along with the boundary conditions (3.14) give the solution of our problem. The solution so obtained is a nonsimilar one, since the motion behind the shock can be determined only when a definite value for time is prescribed.

4. Results and Discussion

Distribution of the flow variables behind the shock front are obtained from (3.7) to (3.11). For the purpose of numerical integration, the values of the constant parameters are taken to be [16, 22, 23, 26, 27] $\gamma = 1.4$; $M_A^{-2} = 0, 0.05, 0.1$; $t/t_0 = 2, 5$; the value $M_A^{-2} = 0$ corresponds to the nonmagnetic case. Starting from the shock front the numerical integration is carried out until the singularity of the solution

$$p' \gamma \beta \left\{ (1 - \beta) + \frac{M_A^{-2}}{2} \left(1 - \frac{1}{\beta^2} \right) \right\} + \frac{M_A^{-2} h'}{\beta} - \rho' (1 - u')^2 = 0, \quad (4.1)$$

is reached. This marks the inner boundary of the disturbance and at this surface the value of $r' (= \bar{r}')$ remains constant. The results are shown in Figures 1(a) to 2. These figures show that the rotation has significant effects on the flow variables. Value of $\bar{r}'$ (the reduced position of the inner expanding surface) are shown in Table 1 for different cases.

From Table 1, it is obvious that $\bar{r}'$ increases by an increase in M_A^{-2} in both the rotating and nonrotating cases but it increases with the increase of t/t_0 in nonrotating case and decreases in rotating case except for $M_A^{-2} = 0$.

Figures 1(a) and 1(b) show the distributions of reduced density and the reduced pressure, respectively. The density and pressure (except for $M_A^{-2} = 0$, in the case of pressure) both decrease from the shock front and approach to zero near the inner expanding surface. In the nonmagnetic case, that is, for $M_A^{-2} = 0$, the pressure increases and becomes constant after attaining a maximum value.

Figure 1(c) shows the distribution of the reduced radial velocity u' in the flow field behind the shock front. The reduced radial velocity increases as we move from the shock

Figure 1: Variation of reduced flow variables in the region behind the shock front for $\gamma = 1.4$. (1) $M_A^{-2} = 0$, $t/t_0 = 2$; (2) $M_A^{-2} = 0.05$, $t/t_0 = 2$; (3) $M_A^{-2} = 0.1$, $t/t_0 = 2$; (4) $M_A^{-2} = 0$, $t/t_0 = 5$; (5) $M_A^{-2} = 0.05$, $t/t_0 = 5$; (6) $M_A^{-2} = 0.1$, $t/t_0 = 5$.

front to the inner expanding surface, in general, whereas it becomes constant after attaining a maximum value in the nonmagnetic case.

Figure 1(d) shows that the reduced azimuthal velocity v' has a similar behaviour as the reduced density ρ' .

Figure 2: Variation of reduced magnetic field h' in the region behind the shock front for $\gamma = 1.4$. (1) $M_A^{-2} = 0.05$, $t/t_0 = 2$; (2) $M_A^{-2} = 0.1$, $t/t_0 = 2$; (3) $M_A^{-2} = 0.05$, $t/t_0 = 5$; (4) $M_A^{-2} = 0.1$, $t/t_0 = 5$.

Figure 2 shows that the reduced magnetic field h' increases slowly behind the shock front and tends to infinity near the inner expanding surface in both the rotating and nonrotating cases.

From Figures 1(a) to 1(d) and 2, it is found that the effects of an increase in the ambient magnetic field (i.e. the effects of increasing M_A^{-2}) in both the rotating and nonrotating cases are

- (i) a reduction of the radial velocity and a growth in the azimuthal velocity, in general,
- (ii) to increase the pressure except for $M_A^{-2} = 0$,
- (iii) to increase the density,
- (iv) to decrease the distance of the inner expanding surface from the shock front (see Table 1).

Effects of an increase in the time (t/t_0) are

- (i) to decrease the density in both the rotating and nonrotating cases, at a point in the flow field behind the shock,
- (ii) to increase the azimuthal velocity,
- (iii) to decrease the radial velocity and the azimuthal magnetic field in rotating case and to increase both in nonrotating case,
- (iv) to decrease the pressure in rotating case and to increase that in nonrotating case,
- (v) to increase the distance of the inner expanding surface from the shock front in rotating case except for $M_A^{-2} = 0$ and to decrease that in nonrotating case.

From Figures 1(a) to 1(c), 2, and Table 1, it is found that the effects of rotation of the gas are

- (i) to decrease the radial velocity and the azimuthal magnetic field and to increase the density at any point in the flow field behind the shock,
- (ii) to increase the pressure in the magnetic case and to decrease it in the nonmagnetic case,
- (i) to increase the distance between the shock front and the inner expanding surface (see Table 1).

Present nonsimilar model may be used to describe some of the overall features of a “driven” shock wave produced by a flare energy release E (see (3.13)), that is time dependent. The energy E increases with time and the solutions then correspond to a blast wave produced by intense, prolonged flare activity in a rotating star when the wave is driven by fresh erupting plasma for some time, and its energy tends to increase as it propagates from the star into a cold atmosphere whose density varies exponentially with altitude. The atmospheric scale height of a star is generally small compared to its radius so that the solutions still describe a stellar explosion.

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