Research Article

A Delay-Dependent Approach to Stability of Uncertain Discrete-Time State-Delayed Systems with Generalized Overflow Nonlinearities

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This paper addresses the problem of global asymptotic stability of a class of uncertain discrete-time state-delayed systems employing generalized overflow nonlinearities. The systems under investigation involve parameter uncertainties that are assumed to be deterministic and norm bounded. A new computationally tractable delay-dependent criterion for global asymptotic stability of such systems is presented. A numerical example is given to illustrate the effectiveness of the proposed method.

1. Introduction

In the implementation of linear discrete systems, signals are usually represented and processed in a finite wordlength format which frequently generates several kinds of nonlinearities, such as overflow and quantization. Such nonlinearities may lead to instability in the designed system. Therefore, the study of stability problem for discrete-time systems with finite wordlength nonlinearities is important not only for its theoretical interest but also for application to practical system design. Many publications [1–22] relating to the issue of the global asymptotic stability of discrete-time systems with overflow nonlinearities have appeared.

Parameter uncertainties are often introduced in many physical systems as a consequence of variations in system parameters, modeling errors or some ignored factors. Such uncertainties may result in the deterioration of system performance and instability of the system.

Time delay is another source of instability for discrete-time systems. They are frequently introduced in many physical, industrial, and engineering systems due to finite capabilities of information processing and signal transmission among various parts of the system. During the past few decades, there has emerged a considerable interest on the stability analysis problems for delayed systems [17–21, 23–31]. According to the dependence of delay, the available stability criteria for delayed systems can be broadly classified into two types: delay independent and delay dependent. Increasing attention is being paid to delay-dependent stability criteria for delayed systems since they can often provide less conservative results than delay-independent criteria [17, 25, 27].

The problem of establishing delay-dependent criteria for the global asymptotic stability of discrete-time uncertain state-delayed systems with overflow nonlinearities is an important and challenging task. So far, very little attention has been paid for the investigation of this problem [17, 21].

In this paper, we consider the problem of global asymptotic stability of a class of discrete-time uncertain state-delayed systems employing generalized overflow nonlinearities. The system under investigation involves parameter uncertainties that are assumed to be norm-bounded. The paper is organized as follows. Section 2 presents a description of the system under consideration. New computationally tractable delay-dependent criteria for global asymptotic stability of uncertain discrete-time state-delayed systems employing generalized overflow nonlinearities are proposed in Section 3. In Section 4, a comparison of the proposed...
method with [18] is made. It is shown that the result presented in [18] is recovered from the presented approach as a special case. A numerical example illustrating the applicability of the proposed criterion is given in Section 5.

2. System Description

The class of nonlinear discrete-time uncertain state-delayed systems under consideration is given by

$$x(k + 1) = f(y(k))$$

$$= \left[ f_1(y_1(k)) \quad f_2(y_2(k)) \quad \cdots \quad f_n(y_n(k)) \right]^T,$$

$$y(k) = (A + \Delta A)x(k) + (A_d + \Delta A_d)x(k - d(k))$$

$$= \left[ y_1(k) \quad y_2(k) \quad \cdots \quad y_n(k) \right]^T,$$  \hspace{1cm} (1a)

\hspace{1cm} \hspace{1cm} (1b)

where $x(k) \in \mathbb{R}^n$ is the state vector, $A, A_d \in \mathbb{R}^{n \times n}$ are the known constant matrices and $\Delta A, \Delta A_d \in \mathbb{R}^{n \times n}$ are assumed to be of the usual norm-bounded type as follows:

$$\begin{bmatrix} \Delta A & \Delta A_d \end{bmatrix} = BF \begin{bmatrix} C_0 & C_1 \end{bmatrix},$$  \hspace{1cm} (2a)

where $B \in \mathbb{R}^{m \times l}$, $C_0, C_1 \in \mathbb{R}^{m \times n}$ are known matrices representing the structure of uncertainty and $F \in \mathbb{R}^{l \times n}$ is an unknown matrix which satisfies

$$F^T F \preceq I.$$  \hspace{1cm} (2b)

It may be mentioned that the uncertainty structure of (2a) and (2b) has been widely adopted in robust control and filtering for uncertain systems [32–34]. \{\varphi(k), k = -d_2, -d_2 + 1, \ldots, 0\} $\in \mathbb{R}^n$ is the initial state value at time $k$ and $f(\cdot)$ represents the generalized overflow nonlinearities.

The generalized overflow characteristic is given by (see Figure 1):

$$L \leq f_i(y_i(k)) \leq 1, \quad y_i(k) > 1, \quad i = 1, 2, \ldots, n,$$

$$f_i(y_i(k)) = y_i(k), \quad -1 \leq y_i(k) \leq 1, \quad i = 1, 2, \ldots, n,$$

$$-1 \leq f_i(y_i(k)) \leq -L, \quad y_i(k) < -1,$$  \hspace{1cm} (3a)

\hspace{1cm} \hspace{1cm} (3b)

where

$$-1 \leq L \leq 1.$$  \hspace{1cm} (3b)

With appropriate choice of $L$, (3a) and (3b) represent the usual types of overflow arithmetics employed in practice, such as saturation ($L = 1$), zeroing ($L = 0$), two’s complement ($L = -1$), and triangular ($L = -1$). The time-varying delay $d(k)$, known as range-like or interval-like time-varying delay [28, 30, 31], satisfying

$$d_1 \leq d(k) \leq d_2,$$  \hspace{1cm} (4)

where $d_1$ and $d_2$ are known positive integers representing the lower and upper delay bounds, respectively. Such delays may be used to characterize the realistic situation in many practical applications [28, 31, 35, 36].

The purpose of this paper is to develop delay-dependent robust stability criteria for the system (1a) and (1b)–(3a) and (3b) for any interval like time-varying delay $d(k)$ satisfying (4).

3. Main Results

In this section, delay-dependent criteria for the global asymptotic stability of system (1a) and (1b)–(4) are established.

The following lemmas are needed in the proof of our main results.

**Lemma 1** (see [8]). An $n \times n$ positive definite symmetric matrix $H = [h_{ij}]$ satisfies

$$y^T(k) H y(k) - f^T(y(k)) H f(y(k)) \geq 0,$$  \hspace{1cm} (5)

if and only if

$$(1 + L)h_{ii} \geq \sum_{j=1, j \neq i}^n |h_{ij}|, \quad i = 1, 2, \ldots, n.$$  \hspace{1cm} (6)

**Lemma 2** (see [32–34, 37]). Let $A, F, M$ be real matrices of appropriate dimensions with $M = M^T$, then

$$M + \Sigma F \Gamma + \Gamma^T F^T \Sigma^T \leq 0,$$  \hspace{1cm} (7)

for all $F^T F \leq I$, if and only if there exists a scalar $\varepsilon > 0$ such that

$$M + \varepsilon^{-1} \Sigma \Sigma^T + \varepsilon \Gamma^T \Gamma \leq 0.$$  \hspace{1cm} (8)

Next, we present the main results of the paper.
Theorem 3. For given positive integers $d_1$ and $d_2$ with $d_2 \geq d_1$, the system described by (1a) and (1b)–(4) is globally asymptotically stable if there exists appropriately dimensioned matrices $P = P^T > 0$, $Q_i = Q_i^T > 0$ ($i = 1, 2, 3$, $Z_4 = Z_i^T > 0$) ($i = 1, 2$), $H = H^T = [h_{ij}] > 0$, $X = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix} \succeq 0$, $Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \succeq 0$, $N = \begin{bmatrix} N_{12} \\ N_2 \end{bmatrix}$, $M = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix}$, $S = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}$ and a positive scalar $\varepsilon$ such that

\[(1 + L)h_{ii} \geq \sum_{j=1, j \neq i}^{n} \left| h_{ij} \right|, \quad i = 1, 2, \ldots, n, \quad (9)\]

and the following (10)–(13) hold,

\[
\begin{bmatrix}
\xi_{11} & \xi_{12} & S_1 & -M_1 & -Z & A^T H & 0 \\
\xi_{12}^T & \xi_{22} & S_2 & -M_2 & 0 & A_3^T H & 0 \\
S_1^T & S_2^T & -Q_1 & 0 & 0 & 0 & 0 \\
-M_1^T & -M_2^T & 0 & -Q_2 & 0 & 0 & 0 \\
-Z & 0 & 0 & 0 & P - H + Z & 0 & 0 \\
HA & HA_d & 0 & 0 & 0 & -H & HB \\
0 & 0 & 0 & 0 & 0 & B^T H - \varepsilon I
\end{bmatrix} < 0,
\]

\[
\psi_1 = \begin{bmatrix} X & N \\ N^T & Z_1 \end{bmatrix} \succeq 0, \quad (10)
\]

\[
\psi_2 = \begin{bmatrix} Y & S \\ S^T & Z_2 \end{bmatrix} \succeq 0, \quad (11)
\]

\[
\psi_3 = \begin{bmatrix} X + Y & M \\ M^T & Z_1 + Z_2 \end{bmatrix} \succeq 0, \quad (12)
\]

where

\[
\begin{aligned}
Z &= d_2 Z_1 + (d_2 - d_1) Z_2 \\
\xi_{11} &= -P + Q_1 + Q_2 + (d_2 - d_1 + 1) Q_3 + Z + N_1 + N_1^T \\
&\quad + d_2 X_{11} + (d_2 - d_1) Y_{11} + \varepsilon C_d^T C_0, \\
\xi_{12} &= -N_1 + N_2^T + M_1 - S_1 + d_2 X_{12} + (d_2 - d_1) Y_{12} \\
&\quad + \varepsilon C_d^T C_1, \\
\xi_{22} &= -N_2 - N_2^T + M_2 + M_2^T - S_2 - S_2^T - Q_3 \\
&\quad + d_2 X_{22} + (d_2 - d_1) Y_{22} + \varepsilon C_d^T C_1.
\end{aligned}
\]

Proof. Let

\[
\eta(k) = x(k + 1) - x(k), \quad (15)
\]

\[
= f(y(k)) - x(k). \quad (16)
\]

Consider a quadratic Lyapunov function [23]

\[
\begin{aligned}
\nu(x(k)) &= v_1(x(k)) + v_2(x(k)) + v_3(x(k)) \\
&\quad + v_4(x(k)), \\
v_1(x(k)) &= x^T(k) P x(k), \\
v_2(x(k)) &= \sum_{\theta = -d_2 + 1}^{l = -d_2 - 1} \eta^T(k + l) Z_1 \eta(k + l), \\
v_3(x(k)) &= \sum_{l = -d_2}^{l = -d_2} x^T(k + l) Q_1 x(k + l) \\
&\quad + \sum_{l = -d_2}^{l = -d_2} x^T(k + l) Q_2 x(k + l), \\
v_4(x(k)) &= \sum_{\theta = -d_2}^{\theta = -d_2} x^T(k + l) Q_3 x(k + l),
\end{aligned}
\]

Defining

\[
\Delta \nu(x(k)) = \nu(x(k + 1)) - \nu(x(k)), \quad (18)
\]

yields

\[
\begin{aligned}
\Delta \nu_1(x(k)) &= x^T(k + 1) P x(k + 1) - x^T(k) P x(k) \\
&= f^T(y(k)) P f(y(k)) - x^T(k) P x(k), \\
\Delta \nu_2(x(k)) &= d_2 \eta^T(k) Z_1 \eta(k) - \sum_{\theta = -d_2}^{\theta = -d_2} \eta^T(k + \theta) Z_1 \eta(k + \theta) \\
&\quad + (d_2 - d_1) \eta^T(k) Z_2 \eta(k) \\
&\quad - \sum_{\theta = -d_2}^{\theta = -d_2} \eta^T(k + \theta) Z_2 \eta(k + \theta) \\
&= \eta^T(k) Z \eta(k) - \sum_{\theta = -d_2}^{\theta = -d_2} \eta^T(k + \theta) Z_1 \eta(k + \theta) \\
&\quad + (d_2 - d_1) \eta^T(k) Z_2 \eta(k) \\
&\quad - \sum_{\theta = -d_2}^{\theta = -d_2} \eta^T(k + \theta) Z_2 \eta(k + \theta) \\
&= \eta^T(k) Z \eta(k) - \sum_{\theta = -d_2}^{\theta = -d_2} \eta^T(k + \theta) (Z_1 + Z_2) \eta(k + \theta),
\end{aligned}
\]

\[
\Delta \nu_3(x(k)) = x^T(k)(Q_1 + Q_2)x(k) - x^T(k - d_1)Q_1x(k - d_1) \\
&\quad - x^T(k - d_2)Q_2x(k - d_2),
\]
\[ \Delta \nu_4(x(k)) = (d_2 - d_1 + 1)x^T(k)Q_3x(k) \]
\[ - \sum_{\theta = -d_2}^{d_2} x^T(k + \theta)Q_3x(k + \theta) \]
\[ \leq (d_2 - d_1 + 1)x^T(k)Q_3x(k) \]
\[ - x^T(k - d(k))Q_3x(k - d(k)). \]

Using (15), we obtain the following null products [23]
\[ 0 = 2\zeta^T_1(k)[N_1^T N_2^T]^T \]
\[ \times \left[ x(k) - x(k - d(k)) - \sum_{l = -d(k)}^{-1} \eta(k + l) \right], \]
\[ 0 = 2\zeta^T_1(k)[M_1^T M_2^T]^T \]
\[ \times \left[ x(k - d(k)) - x(k - d_2) - \sum_{l = -d_2}^{-d(k) - 1} \eta(k + l) \right], \]
\[ 0 = 2\zeta^T_1(k)[S_1^T S_2^T]^T \]
\[ \times \left[ x(k - d_1) - x(k - d(k)) - \sum_{l = -d(k)}^{-d_1 - 1} \eta(k + l) \right], \]
where
\[ \zeta_1(k) = \left[ x^T(k) \ x^T(k - d(k)) \right]^T. \] (21)

For any appropriately dimensioned matrices \( X = X^T \geq 0 \)
and \( Y = Y^T \geq 0 \), we have the following relations:
\[ 0 = d_2 \zeta^T_1(k)X\zeta_1(k) - \sum_{l = k - d(k)}^{k - d_2} \zeta^T_1(k)X\zeta_1(k) \]
\[ - \sum_{l = k - d(k)}^{k - d_2} \zeta^T_1(k)X\zeta_1(k), \]
\[ 0 = (d_2 - d_1)\zeta^T_1(k)Y\zeta_1(k) - \sum_{l = k - d(k)}^{k - d_1 - 1} \zeta^T_1(k)Y\zeta_1(k) \]
\[ - \sum_{l = k - d(k)}^{k - d_2} \zeta^T_1(k)Y\zeta_1(k). \] (22)

Using (18)–(22), we obtain
\[ \Delta \nu(x(k)) \leq \zeta^T_1(k)\pi\zeta_2(k) + \sum_{l = k - d(k)}^{k - d_1 - 1} \zeta^T_1(k, l)\psi_1\zeta_3(k, l) \]
\[ - \sum_{l = k - d(k)}^{k - d_2} \zeta^T_1(k, l)\psi_2\zeta_3(k, l) \]
\[ - \sum_{l = k - d(k)}^{k - d_2} \zeta^T_1(k, l)\psi_3\zeta_3(k, l) - \beta, \] (23a)
where
\[ \zeta_2(k) = \left[ x^T(k) \ x^T(k - d(k)) \ x^T(k - d_1) \ x^T(k - d_2) \ f^T(y(k)) \right]^T, \]
\[ \zeta_3(k, l) = \left[ \zeta^T_1(k) \ \eta^T(l) \right]^T. \] (23i)

In view of Lemma 1, (9) implies that the quantity \( \beta \) (see (23b)) is nonnegative. From (23a), it is clear that \( \Delta \nu(x(k)) \leq 0 \) if \( \pi < 0 \) and (11)–(13) hold true and \( \Delta \nu(x(k)) = 0 \) only when \( \zeta_2(k) = 0 \) and \( \zeta_3(k, l) = 0 \). Thus the conditions \( \pi < 0 \) and (11)–(13) are sufficient conditions for the global asymptotic stability of system (1a) and (1b)–(4).

By the well-known Schur's complement the condition \( \pi < 0 \) is equivalent to the following:
\[ \begin{bmatrix} \pi_{11} + \bar{A}_1^T \bar{H} \bar{A}_1 & \pi_{12} + \bar{A}_1^T \bar{H} \bar{A}_2 & S_1 & -M_1 & -Z \\ \pi_{12} + \bar{A}_2^T \bar{H} \bar{A}_1 & \pi_{22} + \bar{A}_2^T \bar{H} \bar{A}_2 & S_2 & -M_2 & 0 \\ S_1^T & S_2^T & -Q_1 & 0 & 0 \\ -M_1^T & -M_2^T & 0 & -Q_2 & 0 \\ -Z & 0 & 0 & P - H + Z & 0 \end{bmatrix} < 0. \] (24)

Now, using (2a), condition (24) can be expressed in the following form:
\[ M + BF + C^T P BT < 0, \] (25a)
where

\[
M = \begin{bmatrix}
\pi_{11} & \pi_{12} & S_1 & -M_1 & -Z & A^TH \\
\pi_{12} & \pi_{22} & S_2 & -M_2 & 0 & A_2^TH \\
S_1^T & S_2^T & -Q_1 & 0 & 0 & 0 \\
-Z & 0 & 0 & 0 & -P + Z & 0 \\
HA & HA_4 & 0 & 0 & 0 & -H \\
\end{bmatrix},
\]

(25b)

\[
B^T = \begin{bmatrix}
0 & 0 & 0 & 0 & B^TH \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\]

(25c)

\[
C = \begin{bmatrix}
C_0 & C_1 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\]

(25d)

By Lemma 2, (25a), (25b), (25c), and (25d) is equivalent to the following:

\[
M + \varepsilon^{-1}BB^T + \varepsilon CTC < 0,
\]

(26)

where \( \varepsilon > 0 \). The equivalence of (26) and (10) follows trivially from Schur’s complement. This completes the proof of Theorem 3.

**Remark 4.** Theorem 3 provides a stability condition which depends on both the lower delay bound \( d_1 \) and upper delay bound \( d_2 \). For a given \( d_1(d_2) \), \( d_2(d_1) \) can be obtained by iteratively solving the inequalities (10)–(13) with respect to \( d_2(d_1) \).

**Remark 5.** Pertaining to the case of \( L = -1 \), the matrix \( H \) in Theorem 3 assumes the form of a positive definite diagonal matrix and consequently, the conditions in Theorem 3 become in true LMI settings. Thus, for \( L = -1 \), the conditions in Theorem 3 can be easily tested using MATLAB LMI toolbox [37, 38].

**Remark 6.** Note that, in case of \( L \neq -1 \), the conditions of Theorem 3 are not in true LMI settings. In this case, to determine the global asymptotic stability of system (1a) and (1b)–(4) via Theorem 3, one needs to solve the conditions (10)–(13) for \( \varepsilon > 0 \), \( P = P^T > 0, Q_i = Q_i^T > 0 (i = 1, 2, 3), Z_i = Z_i^T > 0 (i = 1, 2), H = H^T > 0, X \geq 0, Y \geq 0, N, M \) and \( S \) using MATLAB LMI toolbox [37, 38] and also check if there exists a solution \( H = H^T > 0 \) meeting (9). This method essentially involves repeated searching of \( \varepsilon > 0 \), \( P = P^T > 0, Q_i = Q_i^T > 0 (i = 1, 2, 3), Z_i = Z_i^T > 0 (i = 1, 2), H = H^T > 0, X \geq 0, Y \geq 0, N, M \) and \( S \) satisfying (10)–(13) until a solution \( H = H^T > 0 \) meeting (9) is found. If (9)–(13) provide a feasible solution, then the system (1a) and (1b)–(4) is globally asymptotically stable; otherwise, no conclusion regarding the global asymptotic stability of the system under consideration can be drawn.

In the following, as an extension of the present approach, we develop a criterion which is true LMI-based and computationally simpler than Theorem 3.

Let the matrix \( H = [h_{ij}] \in \mathbb{R}^{n \times n} \) be represented by

\[
h_{ii} = g_i + \sum_{j=1, j \neq i}^n (\alpha_{ij} + \beta_{ij}), \quad i = 1, 2, \ldots, n,
\]

(27a)

\[
h_{ij} = h_{ji} = \left(\frac{1+L}{2}\right)(\alpha_{ij} - \beta_{ij}), \quad i, j = 1, 2, \ldots, n (i \neq j),
\]

(27b)

\[
\alpha_{ij} = \alpha_{ji} > 0, \quad \beta_{ij} = \beta_{ji} > 0, \quad i, j = 1, 2, \ldots, n (i \neq j),
\]

(27c)

\[
g_i > 0, \quad i = 1, 2, \ldots, n,
\]

(27d)

where it is understood that, for \( n = 1 \), \( H \) corresponds to a scalar \( y > 0 \). Thus, corresponding to \( n = 3 \), the matrix \( H \) follows the form

\[
H = \begin{bmatrix}
g_1 + \alpha_{12} + \beta_{12} + \alpha_{13} + \beta_{13} & \left(\frac{1+L}{2}\right) & \left(\frac{1+L}{2}\right) \\
\left(\frac{1+L}{2}\right) & \left(\frac{1+L}{2}\right) & \left(\frac{1+L}{2}\right) \\
\left(\frac{1+L}{2}\right) & \left(\frac{1+L}{2}\right) & \left(\frac{1+L}{2}\right) \\
\end{bmatrix},
\]

(28)

asymptotically stable if there exists appropriately dimensioned matrices \( P = P^T > 0, Q_i = Q_i^T > 0 (i = 1, 2, 3), Z_i = Z_i^T > 0 (i = 1, 2), X \geq 0, Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \geq 0, N = \begin{bmatrix} N_1 & N_2 \\ N_2 & N_3 \end{bmatrix}, M = \begin{bmatrix} M_1 & M_2 \\ M_2 & M_3 \end{bmatrix}, S = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} \) and scalars \( \varepsilon > 0, g_i > 0 (i = 1, 2, \ldots, n), \alpha_{ij} = \alpha_{ji} > 0, \beta_{ij} = \beta_{ji} > 0 (i, j = 1, 2, \ldots, n (i \neq j)) \) satisfying (10)–(13) where \( H \) is characterized by (27a), (27b), (27c), and (27d).

**Remark 8.** For \( L = -1 \), the matrix \( H \) in Theorem 7 (or Theorem 3) reduces to a positive definite diagonal matrix and consequently, the conditions in Theorem 7 (or
Theorem 3) become in true LMI settings. However, for $L \neq -1$, Theorem 7 provides true LMI conditions for global asymptotic stability, without a need for searching $H$ meeting (9) as in Theorem 3 (see Remark 6), which is beneficial in terms of numerical complexity. It may be observed that, for $L \neq -1$, the matrix inequalities in Theorem 7 become linear in the variables $P$, $Q_i$ $(i = 1, 2, 3)$, $Z_i$ $(i = 1, 2)$, $X$, $Y$, $M$, $S$, $C_i$ $(i = 1, 2, \ldots, n)$, $\alpha_{ij}$, $\beta_{ij}$ $(i, j = 1, 2, \ldots, n (i \neq j))$ and, thus, are computationally tractable. Since the matrix $H$ described by (27a), (27b), (27c), and (27d) has a built-in feature of satisfying (9), one needs not bother about (9) while using Theorem 7.

In the case of constant delay, the lower and the upper delay bounds in (4) becomes identical (i.e., $d_1 = d_2 = d$) and Theorem 7 leads to the following corollary.

**Corollary 9.** The system (1a) and (1b)–(4) with $d(k) = d$ is globally asymptotically stable if there exists appropriately dimensioned matrices $P = P^T > 0$, $Q_i = Q_i^T > 0$ $(i = 1, 2, 3)$, $Z_i = Z_i^T > 0$ $(i = 1, 2)$, $X = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix} \geq 0$, $Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \geq 0$, $N = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix}$, $M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$, $S = \begin{bmatrix} S_1 & S_2 \\ S_3 & S_4 \end{bmatrix}$, scalars $\varepsilon > 0$, $\xi_i > 0$ $(i = 1, 2, \ldots, n)$, $\alpha_{ij} = \alpha_{ji} > 0$, $\beta_{ij} = \beta_{ji} > 0$ $(i, j = 1, 2, \ldots, n (i \neq j))$ satisfying (11)–(13) and

\[
\begin{bmatrix}
\hat{\xi}_{11} & \hat{\xi}_{12} \\
\hat{\xi}_{12}^T & \hat{\xi}_{22}
\end{bmatrix} =
\begin{bmatrix}
S_1 & -M_1 \\
S_1^T & S_2
\end{bmatrix}
- Q_1 0 0 0 0 0 0 0 < 0,
\]

\[
\begin{bmatrix}
\hat{\xi}_{22} & \hat{\xi}_{21} \\
\hat{\xi}_{21}^T & \hat{\xi}_{11}
\end{bmatrix} =
\begin{bmatrix}
-M_2 & -M_1 \\
-M_2^T & S_2
\end{bmatrix}
- Q_2 0 0 0 0 0 0 0 < 0,
\]

\[
\begin{bmatrix}
\hat{\xi}_{11} & \hat{\xi}_{12} \\
\hat{\xi}_{12}^T & \hat{\xi}_{22}
\end{bmatrix} =
\begin{bmatrix}
-P + Q + \varepsilon C_0^T C_0 & \varepsilon C_0^T C_1 \\
\varepsilon C_1^T C_0 & -Q + \varepsilon C_1^T C_1
\end{bmatrix}
0 A^T H 0
\]

\[
\begin{bmatrix}
P + Q + \varepsilon C_0^T C_0 & \varepsilon C_0^T C_1 \\
\varepsilon C_1^T C_0 & -Q + \varepsilon C_1^T C_1
\end{bmatrix}
0 0 0 P - H 0 0 0 < 0,
\]

\[
\begin{bmatrix}
HA & HA_d 0 & -H \\
0 & 0 & 0 & B^T H - \varepsilon I
\end{bmatrix}
< 0,
\]

\[
\begin{bmatrix}
\hat{\xi}_{11} & \hat{\xi}_{12} \\
\hat{\xi}_{12}^T & \hat{\xi}_{22}
\end{bmatrix} =
\begin{bmatrix}
-X_11 & X_12 \\
-X_12^T & X_22
\end{bmatrix}
+ d X_{11} + \varepsilon C_{01}^T C_0,
\]

\[
\begin{bmatrix}
\hat{\xi}_{12} & \hat{\xi}_{21} \\
\hat{\xi}_{21}^T & \hat{\xi}_{11}
\end{bmatrix} =
\begin{bmatrix}
-X_21 & X_22 \\
-X_22^T & X_11
\end{bmatrix}
+ d X_{21} + \varepsilon C_{11}^T C_1,
\]

\[
\begin{bmatrix}
\hat{\xi}_{11} & \hat{\xi}_{12} \\
\hat{\xi}_{12}^T & \hat{\xi}_{22}
\end{bmatrix} =
\begin{bmatrix}
-X_11 & X_12 \\
-X_12^T & X_22
\end{bmatrix}
+ d X_{11} + \varepsilon C_{01}^T C_0,
\]

and $H$ is given by (27a), (27b), (27c), and (27d).

**4. Comparison with A Previous Work [18]**

A delay-independent criterion for the global asymptotic stability of a class of uncertain discrete-time systems involving multiple state delays and generalized overflow nonlinearities is reported in [18]. In this section, for the case of single delay, it will be shown how the delay-independent stability criterion [18, Theorem 1] is recovered from Corollary 9 as a special case.

Following the proof of [18, Theorem 1], one can easily see that the system (1a) and (1b)–(4) with $d(k) = d$ is globally asymptotically stable if there exists a positive scalar $\varepsilon$ and $n \times n$ positive definite symmetric matrices $P$, $Q$, and $H$ such that

\[
\begin{bmatrix}
-P + Q + \varepsilon C_0^T C_0 & \varepsilon C_0^T C_1 \\
\varepsilon C_1^T C_0 & -Q + \varepsilon C_1^T C_1
\end{bmatrix}
0 0 0 P - H 0 0 0 < 0,
\]

\[
\begin{bmatrix}
HA & HA_d 0 & -H \\
0 & 0 & 0 & B^T H - \varepsilon I
\end{bmatrix}
< 0,
\]

where $H$ is defined by (27a), (27b), (27c), and (27d).

By choosing the parameters $X_{12} = Y_{12} = N = M = S = 0$, $Q_1 = \rho_1 I$, $Q_2 = \rho_2 I$, $Q_3 = Q$, $Z_1 = \rho_3 I/d$, $Z_2 = \rho_4 I/d$, $X_{11} = \rho_5 I/d$, $X_{22} = \rho_6 I/d$, $Y_{11} = \rho_7 I/d$, $Y_{22} = \rho_8 I/d$, for some suitably small positive scalars $\rho_i (i = 1, 2, \ldots, 8)$, the conditions in Corollary 9 reduces to (31). Thus, for the case of single delay, the delay-independent stability criterion [18, Theorem 1] is recovered from Corollary 9 as a special case.

**5. An Illustrative Example**

Consider a system represented by (1a) and (1b)–(4) with

\[
L = -1, \quad A = \begin{bmatrix} 0.8 & 0 \\ 0.05 & 0.9 \end{bmatrix}, \quad A_d = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.02 \end{bmatrix},
\]

\[
B = \begin{bmatrix} 0 & 0.1 \end{bmatrix}^T, \quad C_0 = \begin{bmatrix} 0.01 & 0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0 & 0.01 \end{bmatrix}.
\]

Let us select $d_1 = 2$ and $d_2 = 10$. With the help of Matlab LMI toolbox [37, 38], it is found that (10)–(13) are feasible for the following values of unknown parameters.

\[
P = \begin{bmatrix} 834.51 & 5.7678 \\ 5.7678 & 569.3174 \end{bmatrix},
\]

\[
Q_1 = \begin{bmatrix} 23.1690 & -2.2272 \\ -2.2272 & 4.3780 \end{bmatrix},
\]

\[
Q_2 = \begin{bmatrix} 23.4337 & -2.2530 \\ -2.2530 & 4.4249 \end{bmatrix},
\]

\[
Q_3 = \begin{bmatrix} 9.8871 & -0.7648 \\ -0.7648 & 4.4261 \end{bmatrix}.
\]
nonlinearities has been established. As shown in Section 4, with time-varying delay subject to generalized overflow A new computationally tractable delay-dependent stability

Therefore, according to Theorem 3, the system under consideration is globally asymptotically stable. Further, by selecting $d_1 = 2$ and iteratively solving (10)–(13) with respect to $d_2$, it is seen that the system (1a) and (1b)–(4), (32) is also globally asymptotically stable for $2 \leq d(k) \leq 23$.

6. Conclusions
A new computationally tractable delay-dependent stability criterion for a class of uncertain discrete-time systems with time-varying delay subject to generalized overflow nonlinearities has been established. As shown in Section 4, pertaining to the systems involving single delay, the delay-independent stability criterion [18, Theorem 1] has been recovered from Corollary 9 as a special case. The effectiveness of the results presented has been illustrated with a numerical example. The results discussed in this paper can easily be extended to a class of nonlinear uncertain discrete-time systems involving multiple state delays.

Abbreviations
$\mathbb{R}^{p \times q}$: Set of $p \times q$ real matrices
$\mathbb{R}^p$: Set of $p \times 1$ real vectors
0: Null matrix or null vector of appropriate dimension
I: Identity matrix of appropriate dimension
$G^T$: Transpose of the matrix (or vector) $G$
$G > 0$: $G$ is positive definite symmetric matrix
$G \geq 0$: $G$ is positive semidefinite symmetric matrix
$G < 0$: $G$ is negative definite symmetric matrix.

References


