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Research Article

Effects of Viscous Dissipation on the Thermal Boundary Layer of Pseudoplastic Power-Law Non-Newtonian Fluids Using Discretization Method and the Boubaker Polynomials Expansion Scheme

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Heat transfer of pseudoplastic power-law non-Newtonian fluids aligned with a semi-infinite plate is studied. Unlike in most classical works, the effects of viscous dissipation which is coupled with the temperature-dependent thermal diffusivity are considered in the energy equation. The discretization method is used to convert the governing partial differential equations into a set of nonlinear ordinary differential equations. The solutions are presented numerically by using the shooting technique coupled with the Newtonian method and the Boubaker polynomials expansion scheme. The effects of power-law index and the Zheng number on the dynamics are analyzed. The associated heat-transfer characteristics are also tabulated in some domains of the said parameters.

1. Introduction

Since 1960, a considerable attention has been devoted to predict the drag force behavior and energy transport characteristics of the non-Newtonian fluid flows. The main reason for this is probably that fluids (such as molten plastics, pulps, slurries, emulsions), which do not obey the assumption of Newtonian fluids that the tress tensor is directly proportional to the deformation tensor, are found in various engineering applications. A variety of constitutive equations have been proposed to describe the flow and heat transfer non-Newtonian characteristics, among them the empirical Ostwaald-de Waele model, which is known as the so-called power-law model, gained much acceptance. For an incompressible power law non-Newtonian fluid, its shear stress is characterized as $\tau_{xy} = K|\partial U/\partial Y|^{n-1}\partial U/\partial Y$ and the kinematics' viscosity is $\nu = \gamma|\partial U/\partial Y|^{n-1}$ (where $\gamma = K/\rho$, ρ is the density and K is the consistency index). The case $n = 1$ corresponds to a Newtonian fluid. $0 < n < 1$ is descriptive of pseudoplastic fluids while $n > 1$ describes dilatant's fluids.

Schowalter [1] and Acrivos et al. [2] successfully applied the power-law model to the boundary layer problems. The boundary-layer equations were formulated, and the conditions for the existence of similarity solutions were established. Following the pioneering works of Schowalter and Acrivos et al. [1, 2], Wang [3, 4], and Hady [5] analyzed the mixed convection heat transfer from a vertical or horizontal plate of non-Newtonian fluids with uniform surface heat flux. Kumari et al. [6] considered the free-convection boundary-layer flow of a non-Newtonian fluid along a vertical wavy surface. Howell et al. [7] and Rao et al. [8] examined the momentum and heat transfer of the power-law fluids on a continuous moving surface. Hassanien et al. [9] investigated the heat transfer in power-law flow

over a nonisothermal stretching sheet. Luna et al. [10] considered the case of conjugated heat transfer of a power law laminar fluid in circular ducts. Pinarbasi and Imal [11] extended their work to the viscous heat effects on the linear stability of Poiseuille flow of an inelastic fluid. Chen [12] analyzed the effect of viscous dissipation on heat transfer in non-Newtonian power law liquids over an unsteady stretching sheet. For the non-Newtonian power-law fluids, the hydrodynamic problem of the MHD boundary layer flow over a continuously moving surface has been dealt with by several authors (e.g., Andersson et al. [13], Cortell [14], and M. A. A. Mahmoud and M. A. E. Mahmoud [15]). The effect of magnetic field is found to decrease the velocity distribution and thus to increase the skin-friction coefficient.

It seems that, in all the works cited above, the power law kinematic viscosity was applied only on the momentum equations and the thermal conductivity is still treated as constant. This is inconsistent with the known fact that the change of viscosity should affect both the momentum and heat equation of the power law flows. Pop et al. [16, 17] proposed a model which states that the thermal conductivity of non-Newtonian fluids was power-law-dependent on the velocity gradient. Based on this consumption, Gorla et al. and Ece and Buyuk [18, 19] performed a boundary-layer analysis for the free convection flow over a vertical flat plate embedded in a porous medium saturated by a power-law non-Newtonian fluid and gave the similarity solution to the classical boundary layer equations of the power-law wall plume problem.

Motivated by works mentioned above, this paper takes the effects of power law kinematic viscosity into account in the boundary layer heat transfer equations. In terms of the analogy between viscosity diffusion and thermal diffusion, we assumed that the thermal conductivity will be chosen as $k(T) = \lambda|\partial T/\partial Y|^{n-1}$ and the thermal diffusivity α be defined as $\alpha = \omega|\partial T/\partial Y|^{n-1}$ with $\omega = \lambda/\rho c_P$ as a positive constant (c_P is the specific heat at constant pressure) [20, 21]. We first formulate a thermal boundary layer equation for the power-law non-Newtonian fluids with a new variable thermal conductivity and provide similarity solutions.

2. Formalization of the Nonlinear Boundary Value Problem

Consider a semi-infinite plate at uniform wall temperature aligned with a power law fluid of constant speed U_∞. In the presence of viscous dissipation, the laminar boundary layer equations are written as:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{1}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \frac{1}{\rho}\frac{\partial \tau_{xy}}{\partial Y}, \tag{2}$$

$$U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{\partial}{\partial Y}\left(\alpha\frac{\partial T}{\partial Y}\right) + \frac{\nu}{c_p}\left(\frac{\partial U}{\partial Y}\right)^2, \tag{3}$$

where U and V are the velocity components along X and Y directions, respectively. The boundary conditions are

$$U|_{Y=0} = 0, \qquad V|_{Y=0} = 0, \qquad U|_{Y=+\infty} = U_\infty, \tag{4}$$

$$T|_{Y=0} = T_w, \qquad T|_{Y=+\infty} = T_\infty. \tag{5}$$

The following dimensionless variables are introduced:

$$x = \frac{X}{L}, \qquad y = \left[\frac{\rho U_\infty^{2-n} L^n}{K}\right]^{1/n+1}\frac{Y}{L},$$
$$u = \frac{U}{U_\infty}, \qquad v = \left[\frac{\rho U_\infty^{2-n} L^n}{K}\right]^{1/n+1}\frac{V}{U_\infty}, \tag{6}$$

$$\theta = \frac{T - T_w}{T_\infty - T_w},$$
$$N_{\mathrm{Zh}} = \frac{U_\infty^{n-1}\gamma}{|T_\infty - T_w|^{n-1}\omega}, \qquad Z^* = \frac{U_\infty^2}{c_p(T_\infty - T_w)}, \tag{7}$$

N_{Zh} is defined as the Zheng number.

The boundary layer equations then become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{8}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right),$$
$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{N_{\mathrm{Zh}}}\frac{\partial}{\partial y}\left(\left|\frac{\partial \theta}{\partial y}\right|^{n-1}\frac{\partial \theta}{\partial y}\right) + Z^*\left|\frac{\partial u}{\partial y}\right|^{n-1}\left(\frac{\partial u}{\partial y}\right)^2, \tag{9}$$

with the boundary conditions:

$$u\Big|_{y=0} = 0, \qquad v\Big|_{y=0} = 0, \qquad u\Big|_{y=+\infty} = 1, \tag{10}$$

$$\theta\Big|_{y=0} = 0, \qquad \theta\Big|_{y=+\infty} = 1. \tag{11}$$

Equations (8)–(11) are super-nonlinear. Since the problem is very complex to solve analytically, a discretized model may be a considerable approach.

The stream function $\psi(x, y)$, similarity variable η, and dimensionless temperature function $w(\eta)$ are defined as:

$$\psi = Ax^{\tilde{\alpha}} f(\eta), \quad \theta(x, y) = w(\eta),$$
$$\eta = Bx^{\tilde{\beta}} y, \tag{12}$$

where A, B, $\tilde{\alpha}$, and $\tilde{\beta}$ are constant to be determined, and $f(\eta)$ denotes the dimensionless stream function. The u and v velocity component are

$$u = \frac{\partial \psi}{\partial y} = ABx^{\tilde{\alpha}+\tilde{\beta}} f'(\eta),$$
$$v = \frac{\partial \psi}{\partial x} = -A\tilde{\alpha}x^{\tilde{\alpha}-1}[f(\eta) - \eta f'(\eta)]. \tag{13}$$

Substituting u and v into (8)–(11) combining with

$$\tilde{\beta} = -\tilde{\alpha}, \qquad AB = 1,$$
$$\tilde{\alpha} = \frac{1}{n+1}, \qquad B = (n+1)^{(-1/(n+1))}, \tag{14}$$

we arrive at the nonlinear boundary value problems of the form as:

$$\left(|f''(\eta)|^{n-1} f''(\eta)\right)' + f(\eta) f''(\eta) = 0, \tag{15}$$

$$f(0) = 0, \qquad f'(0) = 0, \qquad f'(\eta)\big|_{\eta=+\infty} = 1, \tag{16}$$

$$f(\eta) w'(\eta) + \frac{1}{N_{Zh}}\left(|w'(\eta)|^{n-1} w'(\eta)\right)',$$
$$+ Z^* |f''(\eta)|^{n-1} (f''(\eta))^2 = 0, \tag{17}$$

$$w(0) = 0, \qquad w(\eta)\big|_{\eta=+\infty} = 1. \tag{18}$$

3. Discretization Method Solution

In order to obtain numerical solutions, we transfer the problem (15)–(18) to a system of four first-order equations by denoting the f, f', f'', w, w' using variables f, u, v, w, y, respectively:

$$f' = u, \tag{19}$$

$$u' = v, \tag{20}$$

$$v' = -\frac{1}{n} f v^{2-n}, \tag{21}$$

$$w' = y, \tag{22}$$

$$y' = -\frac{Z^* \cdot N_{Zh}}{n} y^{1-n} v^{n+1} - \frac{N_{Zh}}{n} f y^{2-n}. \tag{23}$$

The corresponding boundary conditions are

$$f(0) = 0, \qquad u(0) = 0, \qquad w(0) = 0 \tag{24}$$

$$u(+\infty) = 1, \qquad w(+\infty) = 1. \tag{25}$$

We introduce the parameters t and s as

$$v(0) = t, \qquad y(0) = s. \tag{26}$$

Then, the problem is to find parameters t and s such that the solution to (19)–(24), (26) satisfies the boundary conditions (25).

The solution to the initial-value problems (19)–(24), (26) can be denoted as $f(\eta, t, s)$, $u(\eta, t, s)$, $v(\eta, t, s)$, $w(\eta, t, s)$, and $y(\eta, t, s)$. Thus the following equations are to be solved:

$$\phi(t, s) = u(+\infty, t, s) - 1 = 0, \tag{27}$$

$$\varphi(t, s) = w(+\infty, t, s) - 1 = 0. \tag{28}$$

The Newtonian method and Runge-Kutta scheme are convenient to solve the nonlinear problem above.

4. BPES Solution

The Boubaker polynomials expansion scheme (*BPES*) [22–37] solutions starts from assigning the following expression:

$$w(\eta) = \frac{1}{2M} \sum_{j=1}^{M} \xi_j \times B_{4j}\left(\varpi_j \varepsilon(\eta)\right),$$
$$\varepsilon(\eta) = (\eta + 1) e^{-\eta}, \tag{29}$$

with B_{4j}: $4j$-order Boubaker polynomials, ϖ_j: B_{4j} minimal positive roots, M: a prefixed integer, ϖ_j: B_{4j} minimal positive roots, and $\xi_j|_{j=1\ldots M}$: unknown pondering real coefficients.

Using expression (29), the boundary conditions:

$$w(\eta)\big|_{\eta=0} = 0,$$
$$\lim_{\eta \to +\infty} w(\eta) = 1 \tag{30}$$

are verified automatically with reference to the Boubaker polynomials properties:

$$\sum_{q=1}^{N} B_{4q}(x)\bigg|_{x=0} = -2N \neq 0,$$
$$\sum_{q=1}^{N} B_{4q}(x)\bigg|_{x=\alpha_q} = 0, \tag{31}$$

$$\sum_{q=1}^{N} \frac{dB_{4q}(x)}{dx}\bigg|_{x=0} = 0,$$
$$\sum_{q=1}^{N} \frac{dB_{4q}(x)}{dx}\bigg|_{x=\alpha_q} = \sum_{q=1}^{N} H_q,$$
$$\sum_{q=1}^{N} \frac{d^2B_{4q}(x)}{dx^2}\bigg|_{x=0} = \frac{8}{3}\left(N\left(N^2 - 1\right)\right),$$
$$\sum_{q=1}^{N} \frac{d^2B_{4q}(x)}{dx^2}\bigg|_{x=\alpha_q} = \sum_{q=1}^{N} G_q, \tag{32}$$

with

$$H_n = B'_{4n}(\alpha_n) = \left(\frac{4\alpha_n\left[2 - \alpha_n^2\right] \times \sum_{q=1}^{n} B_{4q}^2(\alpha_n)}{B_{4(n+1)}(\alpha_n)} + 4\alpha_n^3\right), \tag{33}$$

$$G_q = \frac{d^2B_{4q}(x)}{dx^2}\bigg|_{x=\alpha_q}$$
$$= \frac{3\alpha_q\left(4q\alpha_q^2 + 12q - 2\right)H_q - 8q\left(24q^2\alpha_q^2 + 8q^2 - 3q + 4\right)}{\left(\alpha_q^2 - 1\right)\left(12q\alpha_q^2 + 4q - 2\right)}. \tag{34}$$

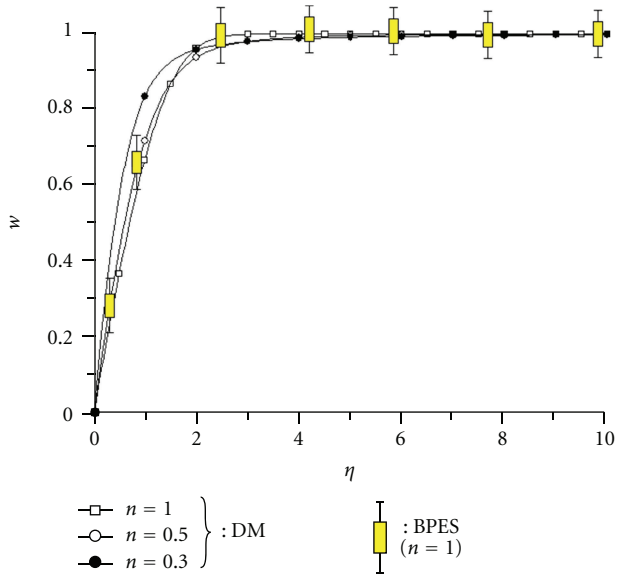

Figure 1: Dimensionless temperature for $Z^* = 0.5$, $N_{\mathrm{Zh}} = 2.0$.

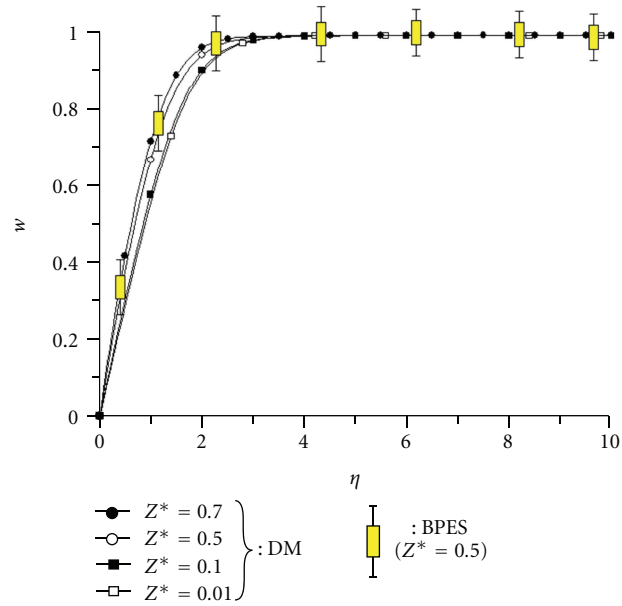

Figure 2: Dimensionless temperature for $n = 0.8$, $N_{\mathrm{Zh}} = 2.0$.

The final solution is derived by introducing the expression (29) in the system (15)–(18) and calculating the coefficients $\xi_j^{\text{optim.}}|_{j=1\ldots M}$ which minimize the functional determinant Δ:

$$\Delta = \left| \left(\left| \sum_{j=1}^{M} \xi_j^{\text{optim.}} \times \Phi_j^{(2)} \right| \right)^{n-1} \left(\sum_{j=1}^{M} \xi_j^{\text{optim.}} \times \Phi_j^{(2)} \right) - \left(\sum_{j=1}^{M} \xi_j^{\text{optim.}} \times \Phi_j^{(0)} \right) \left(\sum_{j=1}^{M} \xi_j^{\text{optim.}} \times \Phi_j^{(1)} \right) \right|, \tag{35}$$

where $\Phi_j^{(0)}$, $\Phi_j^{(1)}$, and $\Phi_j^{(2)}$ are calculated using the properties expressed by (32)–(34).

The solution is consequently:

$$w_{\text{sol.}}(\eta) = \frac{1}{2M} \sum_{j=1}^{M} \xi_j^{\text{optim.}} \times B_{4j}\left(\varpi_j \varepsilon(\eta)\right), \qquad \varepsilon(\eta) = (\eta + 1)e^{-\eta}. \tag{36}$$

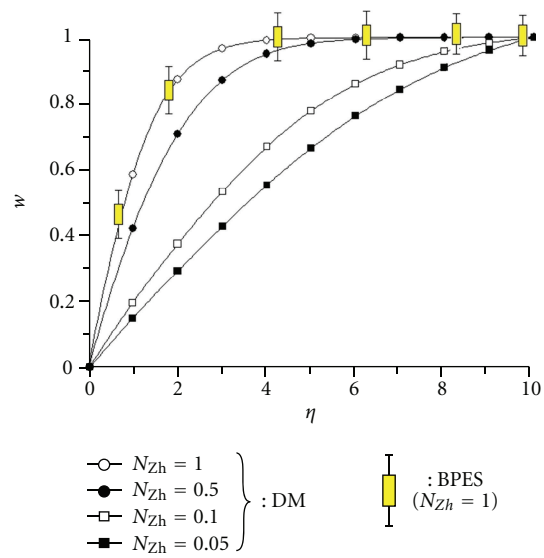

Figure 3: Dimensionless temperature for $n = 0.8$, $Z^* = 1.0$.

5. Numerical Results

The heat transfer of pseudoplastic power law fluids aligned with a semi-infinite plate is investigated in the presence of viscous dissipation. Numerical solutions are achieved using the discretization shooting method and the Boubaker polynomials expansion echeme (BPES). Figures 1–4 show the dimensionless temperature profiles for different power law index n, the Zheng number N_{Zh} and Z^*. It is observed that the length scale of thermal diffusion is significantly different for the values of parameters. On the whole, the thermal diffusion ratio increases with the increase of N_{Zh} and Z^*.

Figure 1 depicts the effect of power-law index n on the temperature profiles using the two methods (Section 3, Section 4). It is a known fact that increasing values of n implies difference in temperature. The same is reiterated by Figure 1: the changing factor n arises some perturbation in temperature profiles and lead to the varying w for each η.

Figure 2 reveals the effect made by the viscous dissipation on the temperature. These profiles are well behaved and very little change occurs in the shapes of profiles with the varying parameter Z^*. The different values of Z^* contribute little to the thickness of the thermal boundary layer.

Figures 3-4 show temperature profiles for fixed Z^* and n with different values of N_{Zh}. For each $N_{\mathrm{Zh}}(N_{\mathrm{Zh}} > 0)$, the thermal diffusion ratio increases with the increasing in N_{Zh} as recorded earlier by Zheng et al. [38] and Ece and Buyuk [19]. It is evident that small values of N_{Zh} result in thinning

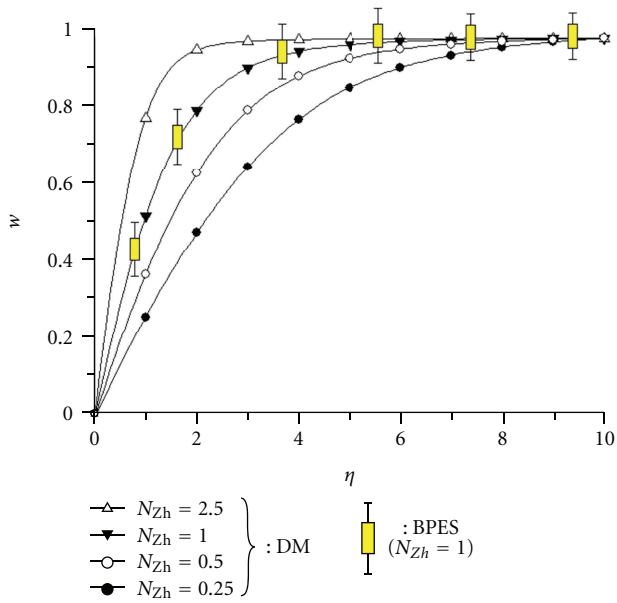

FIGURE 4: Dimensionless temperature for = 0.5, Z^* = 0.5.

of the thermal boundary layer. This is similar to the effects of Prandtl number Pron heat transfer.

6. Conclusions

An investigation on heat transfer of pseudoplastic non-Newtonian fluids aligned with a semi-infinite plate is made. The boundary layer equations for both momentum and energy boundary layers have been developed. Numerical solutions are presented and the associated transfer behavior for different power-law index, and the Zheng number N_{Zh} and Z^* are discussed. Two significant physical phenomena are observed, that is, large values of N_{Zh} result in thicker thermal boundary layers, and, the different values of Z^*contribute little to the thickness of the thermal boundary layer.

Nomenclature

c_P: Fluid specific heat ($\mathrm{J\,kg^{-1}\,C^{-1}}$)
f: Stream function (dimensionless)
K: Consistency index
k: Thermal conductivity ($\mathrm{W\,kg^{-1}\,K^{-1}}$)
N_{Zh}: Zheng number (dimensionless)
n: Power-law index (dimensionless)
T: Temperature (K)
U, V: Velocity components along X and Y directions ($\mathrm{m\,s^{-1}}$)
X, Y: Cartesian coordinates along the plate and normal to it (m).

Greek symbols

α: Fluid thermal diffusivity ($\mathrm{m^2\,s^{-1}}$)
η: Similarity variable (dimensionless)
θ: Normalized temperature (dimensionless)
$\bar{\omega}$: Temperature function (dimensionless)
ψ: Stream function (dimensionless)
ν: Kinematic viscosity ($\mathrm{m\,s^{-1}}$)
ρ: Density ($\mathrm{Kg\,m^{-3}}$)
ω: Constant (dimensionless).

Subscripts

w, ∞: Conditions at the surface and in the free stream, respectively.

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