

## Research Article

# Analysis of Interfering Fully Developed, Colinear Deepwater Waves

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The sea surface is normally irregular as a result of dissimilar waves generated in different areas. To describe such a sea state, various methods have been proposed, but there is no general consensus as to the best characterizing parameters of the interwaves. Three simple methods are proposed here to calculate a characteristic interwave period, length, and height for fully developed, colinear deepwater waves. The results of this study indicate that the interwave period and length are equal or very close to the period and length of the dominant component wave, irrespective of the periods of the subordinate waves. In cases where the dominant wave period is double or more than double the periods of the subordinate waves, the wave period, length and height are within 4% of the dominant wave parameters, so that such interfering, irregular waves have virtually the same characteristics as monochromatic waves. Secondary, individual interwaves propagate at the velocity of the component wave with the shortest period, that is, slower than the primary interwaves which have the same celerity as the dominant component wave.

## 1. Introduction

Waves are generated by wind in different areas and may be dissimilar in period, length, height, and celerity. Where such waves converge, interference takes place so that their crests are reinforced or diminished depending on whether crest-crest or crest-trough interference occurs. For ocean and coastal engineers, it is important to understand such interactions, because the design of structures naturally has to consider the peak conditions arising from them. Wave interference also influences sediment transport as well as coastal and coral reef erosion.

Complex wave fields arise because of different stages of wave development and water depths (both of which result in significant changes in the individual wave profiles), as well as the fact that waves may come from different directions. Modelling all the possible combinations resulting from these differences would be a very demanding task, so that this paper focuses instead on the relatively simple situation of interfering colinear waves with Airy characteristics. Such sinusoidal waves are typical of deepwater conditions where the wind has blown for a sufficient period of time over

a long enough fetch for them to become fully developed. This would be the case for synoptic-scale (100–1000 km) pressure systems developed over the deep ocean, where there is no interference from bathymetric and land effects and where waves generated upwind have longer wavelengths than downwind waves, thus overtaking the latter. For example, a 13 s wave would require a wind speed and duration of  $20 \text{ m s}^{-1}$  and 32 hours, respectively, over a minimum fetch of about 600 km, whereas an 8 s wave would be generated by a  $12.5 \text{ m s}^{-1}$  wind blowing for at least 24 hours over a fetch of about 300 km [1]. Because the wavelength and height of fully developed waves are fixed by their respective periods (see (2) and (3)), changing the latter ( $T$ ) automatically adapts the corresponding lengths and amplitudes.

Currently two approaches are used to characterize the undulating surface formed by interfering waves, hereafter referred to as the interwave surface or simply interwaves. Spectral methods (e.g., [2, 3]) are based on the Fourier transform of the sea surface [4], whereas wave train analysis uses direct measurements of the sea surface elevation at set intervals (typically every second) at a specific locality

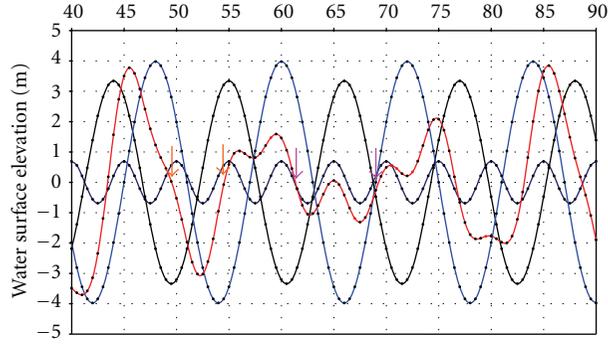


FIGURE 1: Definition of prominent troughs on  $t/\eta$  diagram. The width-depth value  $A$  for the trough between the orange arrows is  $3.0218(54.5 - 49.5) = 15.109$ , which is more than 50% of  $\bar{A} = 0.5(29.6735 - 10.3164) = 9.6786$ . The two troughs between the pink arrows have  $A$ -values less than 9.6786, but in combination their  $A$ -value of  $1.3138(69.5 - 61.5) = 10.5104$  exceeds this cutoff, so that they are counted as one trough. Blue line: dominant component wave; black lines: subordinary component waves; red line: resultant interwaves.

or measuring station [5]. The main drawback of this method is that it cannot distinguish the wave direction [4]. Nevertheless, if criteria can be developed to identify individual waves in a record of colinear waves, the way they interact can be used for prediction and design purposes.

The approach followed here was to generate Airy waves of different periods on the accompanying Excel spreadsheet (INTERWAVES) and to examine the ways in which they interact. This reveals useful relationships between the characterizing interwave parameters.

## 2. Simulation of Interfering Colinear Fully Developed Deepwater Waves

In the discussion below, the subscript 1 refers to the dominant wave (i.e., the component wave with the longest period), and the subscripts 2 and 3 to the subordinate waves, 2 indicating a component wave with a longer period than 3. The subscript  $i$  refers to interwaves.

In deep water, interfering colinear Airy waves can be modeled by the standard equation [4]

$$\eta_i = \eta_1 + \eta_2 + \eta_3 = \frac{H_1}{2} \cos\left(\frac{2\pi X}{L_1} - \frac{2\pi t}{T_1}\right) + \frac{H_2}{2} \cos\left(\frac{2\pi X}{L_2} - \frac{2\pi t}{T_2}\right) + \frac{H_3}{2} \cos\left(\frac{2\pi X}{L_3} - \frac{2\pi t}{T_3}\right), \quad (1)$$

where  $\eta_i$  is the surface elevation of the interference wave,  $\eta_1 + \eta_2 + \eta_3$  are the elevations of the respective wave components,  $X$  is the distance from the measuring station in the direction of wave propagation,  $L_1, L_2, L_3$ , and  $T_1, T_2, T_3$  are the lengths and periods of the components waves, respectively, and  $t$  is the time in seconds. In Figures 1 and 2, the blue and black lines represent the component wave shapes and the red lines the resultant interwaves.

In the spreadsheet setup, two-component and three-component interwaves were simulated using different combinations of  $T_1, T_2$ , and  $T_3$ . For each period, the wavelength  $L$  was obtained from [6]

$$L = \frac{gT^2}{2\pi}, \quad (2)$$

and the deepwater fully developed wave height  $H$  from [7, 8]:

$$H = \frac{gT^2}{18\pi^2}. \quad (3)$$

Equation (3) is based on data collected since 1967 during the Joint North Sea Wave Project (JONSWAP) as graphically summarized in [9].

The spreadsheet was designed to calculate the water surface elevation at intervals each representing 1/240th of the time required for 10 dominant wavelengths to pass the observation point, and also at 240 points over a distance of 10 dominant wavelengths. This was found to be sufficient in establishing interference patterns and relationships among the various parameters. Two graphs were plotted, one showing the water surface elevation  $\eta$  over time  $t$  at any specific point, and the other showing  $\eta$  against distance  $X$  in the direction of wave propagation at any specific moment in time. The different wave periods and heights were determined using the  $t/\eta$  graph, whereas the  $X/\eta$  graph was employed for the wavelengths. At the graph origin ( $x = 0; t = 0$ ), full crest interference among the different wave components was modeled, but the spreadsheet was designed so that different wave phases could also be simulated at this point if required. The different combinations of wave periods used are shown in the headings of Table 1. For every combination of waves, the different interwave parameters described below were calculated using the water surface elevation as plotted on the graphs, with the results summarized in Table 1.

## 3. Expression of Interwave Parameters

The parameters used to describe interwaves differ from those of sinusoidal, monochromatic waves. For example, where

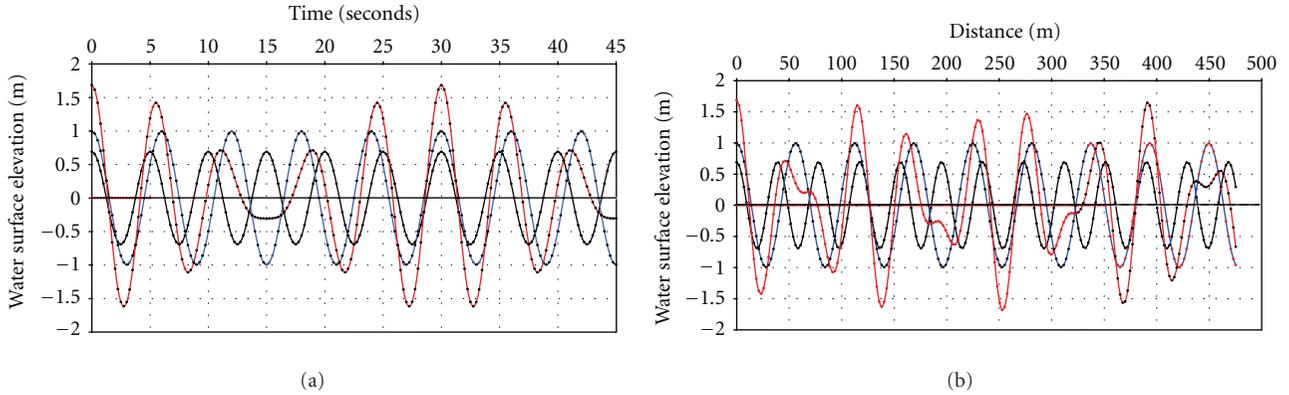


FIGURE 2: (a) Wave “groups” on a  $t/\eta$  diagram (a) for 6, 5 s interwave. This is the sea surface elevation recorded at any specific station over a time period of 45 seconds. (b) On an  $X/\eta$  diagram, the “groups” are much less prominent, that is, an observer looking at the sea surface over a distance of 500 m at any specific moment in time would hardly be able to discern groups. Blue line: dominant component wave; black lines: subordinate component waves; red line: resultant interwave.

the wave period ( $T$ ) for the latter is defined as the time interval (in seconds) between the arrival of two successive crests or troughs (which gives the same result), in the case of interwaves the mean time interval between neighboring crests will not necessarily be the same as that between neighboring troughs.

One way to determine the interwave period is to calculate the average time interval between the arrival of all successive crests ( $T_{ic}$ ) or troughs ( $T_{it}$ ) on the  $t/\eta$  record. The most commonly used method, however, describes the interwave period as the time interval between successive crossings of the mean or still water level (SWL) by the water surface in a downward or upward direction, respectively, which are known as the zero downcrossing ( $T_{izd}$ ) and zero upcrossing ( $T_{izu}$ ) periods [5].

Here, a rapid method is proposed to obtain the interwave period  $T_{ix}$ , which consists of dividing the time lapse  $t_t$  between the first and last prominent troughs on the  $t/\eta$  record by the number of prominent troughs ( $N_t$ ) between them (excluding the first but including the last trough). Troughs were used here because by design only half of the first and last crests are displayed on the  $t/\eta$  record, which might lead to ambiguity in the case of double-crested waves.

A potential problem in using this method concerns the definition of “prominent” troughs. Under certain conditions of wave interference, a whole series of crests and troughs of different sizes are generated, and while in the case of two-component interwaves there should be no problem in identifying them, it may not be so easy where three or more waves with at least one period close to that of the dominant wave interfere. In this case, the following method was found to give consistent results. For each clearly prominent trough on the record, the distance between its zero downcrossing and upcrossing point is measured and multiplied with its maximum depth below the SWL. This value is here referred to as the trough width-depth value ( $A$ ). The mean ( $\bar{A}$ ) and standard deviation ( $As$ ) of all the prominent trough width-depth values are then calculated, where the standard deviation provided in Excel is given by  $s = \sqrt{(n \sum x^2 - (\sum x)^2)/n(n-1)}$ .

For any disputed trough to be counted as a prominent trough, its  $A$ -value must be at least 50% of  $(\bar{A} - sA)$ . Figure 1 shows the case of a 12, 11, 5 s wave, where a series of undulations around the SWL is produced between 40 and 90 seconds on a  $t/\eta$  diagram. The trough between the orange arrows has an  $A$ -value of 15.109, which is more than  $0.5(\bar{A} - As) = 9.6786$ . None of the two troughs between the pink arrows reaches the 50% cutoff, but together they have a value of 10.5104. They are thus counted as one trough.

The peak interwave period  $T_{ip}$  is defined as the time interval between maximum crest interference at the same locality on the  $t/\eta$  record.

Interwave length is commonly described as the average distance between successive zero down- or upcrossing points ( $L_{izd}, L_{izu}$ ). Here, a characteristic interwave length  $L_{ix}$  was calculated by dividing the distance ( $X_t$ ) between the first and last prominent wave trough on the  $X/\eta$  record by the number of prominent troughs ( $N_t$ ) between them, including one of the end troughs. “Prominent” is defined as for the wave period. The peak interwave length ( $L_{ip}$ ) is the distance between crests of maximum interference at any specific moment on the  $X/\eta$  record.

Interwave heights are also characterized by different methods, for example, the mean height ( $H_{im}$ ) that averages every successive crest-trough elevation difference in the record and the root-mean-square height ( $H_{irms}$ ), in which the square root is obtained after squaring, summing, and averaging these individual differences. The commonly used significant wave height ( $H_{is}$ ) is the mean height of the highest one-third of all waves on the record, whereas the maximum wave height  $H_{imax}$  refers to the largest difference between any recorded adjacent trough and crest.

In modeling monochromatic waves, it is found that the standard deviation of the sea surface  $\eta s$  can be related to the wave height as obtained from (3), by multiplying  $\eta s$  by 2.8168. For example, a value of 1.3805 m is obtained for a 5 s wave by calculating the standard deviation of the water surface elevation for 240 s, compared to 1.3805 m as determined from (3). This relationship was therefore used

TABLE 1: Wave parameters as determined from  $t/\eta$  and  $X/\eta$  diagrams for different combinations of component wave periods.

(a) Two-component interwaves						
Wave parameters	6, 5 s	6, 4 s	6, 3 s	6, 2 s	6, 1 s	12, 11 s
$T_{ic}$	6.00	6.00	6.00	6.00	6.00	12.00
$T_{it}$	6.00	6.11	6.00	6.00	6.00	12.22
$T_{izd}$	6.03	6.08	6.00	6.00	6.00	12.11
$T_{izu}$	6.03	6.08	6.00	6.00	6.00	12.11
$T_{ix}$	6.06	6.11	6.00	6.00	6.00	12.11
$T_{ip}$	30.00	12.00	6.00	6.00	6.00	132.00
$L_{izd}$	56.2124	57.2489	56.2080	56.2080	56.2080	230.6821
$L_{izu}$	55.5054	55.9478	56.2080	56.2080	56.2080	230.6821
$L_{ix}$	56.2080	55.6876	56.2080	56.2080	56.2080	230.6821
$L_{ip}$	1405.18	224.832	56.2080	56.2080	56.2080	27204.3
$H_{im}$	2.2394	2.1672	1.9880	2.2088	1.9880	9.2315
$H_{irms}$	2.4736	2.2117	1.9880	2.2088	1.9880	10.0715
$H_{is}$	3.2124	2.6077	1.9880	2.2088	1.9880	14.0973
$H_{imax}$	3.3007	2.6077	1.9880	2.2088	1.9880	14.4979
$H_{ix}$	2.4296	2.1821	2.0531	2.0020	1.9892	9.8970
$C_{ip}$	9.3680	9.3680	9.3680	9.3679	9.3679	17.9550
$C_{isc}$	7.8067	6.2452	4.6839	3.1226	1.5613	17.9550

(b) Three-component interwaves							
Wave parameters	6, 5, 4 s	6, 5, 3 s	6, 5, 2 s	6, 4, 3 s	6, 4, 2 s	6, 3, 2 s	12, 11, 10 s
$T_{ic}$	5.00	5.00	4.29	6.00	6.00	6.00	10.91
$T_{it}$	5.00	5.00	4.17	6.17	6.08	6.00	10.95
$T_{izd}$	5.00	6.06	6.08	6.06	6.08	6.00	10.95
$T_{izu}$	4.98	6.16	6.03	6.06	6.19	6.00	10.90
$T_{ix}$	6.11	6.08	6.06	6.17	6.08	6.00	12.17
$T_{ip}$	60.00	30.00	30.00	12.00	12.00	6.00	660.00
$L_{izd}$	39.4794	45.9884	56.2080	56.9886	57.2478	56.2080	185.4824
$L_{izu}$	38.7331	45.9884	56.4682	55.9478	56.2080	56.2080	184.5457
$L_{ix}$	57.2489	56.2080	56.4682	55.9478	55.4273	56.2080	231.8531
$L_{ip}$	5620.72	1405.18	1405.18	224.8320	224.832	56.208	680106.6
$H_{im}$	2.2017	2.1213	1.7129	2.3535	2.3117	2.2088	9.7358
$H_{irms}$	2.4517	2.3611	2.1374	2.3946	2.3535	2.2088	11.4729
$H_{is}$	3.3410	3.2423	3.1588	2.9752	2.7534	2.2088	17.8234
$H_{imax}$	3.9894	3.3595	3.4893	2.9752	2.7534	2.2088	19.8847
$H_{ix}$	2.5947	2.4854	2.4421	2.2432	2.1956	2.0671	11.6291
$C_{ip}$	9.3679	9.3679	9.3679	9.3679	9.3679	9.3679	17.0628
$C_{isc}$	6.2452	4.6839	3.1226	4.6839	3.1226	3.1226	17.0628

to calculate  $H_{ix}$ , here termed as the characteristic interwave height, in Table 1.

As used here, the primary interwave celerity  $C_{ip}$  refers to the propagation velocity of prominent or primary wave crests ( $C_{ipc}$ ) or troughs ( $C_{ipt}$ ) within the wave train. Some of these larger wave forms may be similar to the concept of wave groups as currently used. Individual secondary crests and troughs superimposed on the primary interwave forms propagate at speeds different from  $C_{ipc}$  or  $C_{ipt}$ , which is referred to as the secondary interwave crest or trough celerity ( $C_{isc}$  or  $C_{ist}$ ), respectively. All these celerities can be

measured on the  $X/\eta$  diagram by monitoring, for example, the propagation of any specific secondary crest over the distance of 10 primary wavelengths while changing  $t$  in (1). On the  $t/\eta$  diagram this cannot be done because the graph only depicts the water surface elevation at a specific station over time.

#### 4. Two-Component Interwaves

Experimenting with different two-wave combinations, the following general observations were made.

**4.1. Interwave Period.** If the dominant wave period  $T_1$  is double or more than double the period of the subordinate wave  $T_2$ , all interwave periods including the characteristic interwave period ( $T_{ix}$ ) and peak interwave period ( $T_{ip}$ ) are equal to  $T_1$ .

If the dominant wave period is less than double that of the subordinate wave, all interwave periods except the peak interwave period are very close to that of the dominant wave, differing by less than 2%.

The peak interwave period for all two-component interwaves is given by

$$T_{ip} = \frac{T_1 T_2}{LCF}, \quad (4)$$

where LCF is the largest common factor by which  $T_1$  and  $T_2$  can be divided. For example, for a 6, 2 s wave  $T_{ip} = (6)(2)/2 = 6$  s, for a 6, 3 s wave  $T_{ip} = (6)(3)/3 = 6$  s, and for a 6, 5 s wave  $T_{ip} = (6)(5)/1 = 30$  s.

**4.2. Interwave Length.** If the dominant wave period is double or more than double the period of the subordinate wave, all interwave lengths are exactly equal to the wavelength  $L_1$  of the dominant wave. When the dominant wave period is less than double that of the subordinate wave, all interwave periods except the peak interwave period are very close to that of the dominant wave, differing by less than 3%. This means that  $L_{ix}$  can be obtained directly from the  $t/\eta$  record together with (2), substituting  $T_{ix}$  for  $T$ .

The peak interwave length for all two-component waves is also given by (2), in this case using  $T_{ip}$  instead of  $T_{ix}$ . This is the case even though  $T_{ip}$  is determined from the  $t/\eta$  diagram and  $L_{ip}$  from the  $X/\eta$  diagram. For example, for a 6, 4 s interwave,  $T_{ip} = 12$  s and  $L_{ip} = 224.832$  m as read from the  $X/\eta$  diagram, the small discrepancy (0.0034 m) being due to slight inaccuracies caused by line smoothing. This relationship is useful because waves are usually monitored at a single station and the results displayed on a  $t/\eta$  diagram where the peak interwave length will not be visible but can still be determined from (4) and (2).

**4.3. Interwave Height.** If the dominant wave period is double or more than double the period of the subordinate wave, the wave height as determined by  $H_{im}$ ,  $H_{irms}$ ,  $H_{is}$ , and  $H_{imax}$  has the same value in all four cases because the same wave form is repeated. In some combinations, this height coincides with the height of the dominant wave  $H_1$ , but in other cases it can be more than 10% larger than the latter. For example, the four different methods calculate a height of 1.988 m for both 6, 3 and 6, 1 s interwaves, agreeing closely with that of the dominant 6 s wave (1.9879 m), whereas a 6, 2 s interwave is calculated to be 2.2088 m high.

In these cases, the characteristic interwave height  $H_{ix}$  is 2.0531, 2.002, and 1.9892 m for 6; 3, 6; 2, and 6; 1 s interwaves, respectively. This is considered to better represent the characteristic wave height than the other measures, for the following reasons: a dominant wave with superimposed smaller waves must represent higher energy conditions than the dominant wave alone, and given the fact that the interwave length is the same as the dominant wavelength,

this energy should be manifested in a higher characteristic interwave height. Furthermore, the interwave height should increase with an increase in the wave period (and height) of the subordinate wave.  $H_{ix}$  is the only one of the 5 different wave height definitions that makes this distinction. In addition, it better represents the wave height as obtained from a combination of the  $t/\eta$  and  $X/\eta$  graphs. In the case of a 6, 3 s interwave, for example, the  $t/\eta$  graph depicts a peaked crest and flat trough, with the former 1.2425 m above and the trough 0.7455 m below the SWL, respectively. However, on the  $X/\eta$  graph the crest is 1.2424 m above the SWL, and there is a double trough at a depth of 0.9851 m below the SWL, with a secondary crest 0.7455 m below the SWL in the middle thereof. Taking the average trough depth as  $(2 \times 0.9851 + 0.7455)/3 = 0.9052$  m, the wave height would be 2.1476 m. Averaging this with the 1.988 m shown on the  $t/\eta$  graph gives 2.0678 m for the two graphs, which agrees better with the value of 2.0531 m obtained by  $H_{ix}$  than the value of 2.2088 given by the other 4 height definitions. Lastly, in cases where the dominant wave period is less than double that of the subordinate wave, the value of  $H_{ix}$  falls between those given by  $H_{im}$  and  $H_{irms}$ . In these cases,  $H_{is}$  and  $H_{imax}$  give larger heights than  $H_{im}$ ,  $H_{irms}$  and  $H_{ix}$ , which is logical as they represent the highest third and maximum wave heights, respectively.

**4.4. Interwave Celerity.** Interwave celerity is a somewhat controversial subject because of the concept of wave groups or “beats” as currently used. Two interfering wave trains will produce zones of reinforced crests and troughs interspersed with zones of reduced crests and troughs. The envelope curves enclosing these zones describe the wave groups, which propagate at a speed considered to be one half that of the phase velocity in deep water (e.g., [4]). However, some of these “groups” may be an artifact of the  $t/\eta$  diagrams commonly used to analyze wave trains. Consider the interference pattern of a 6, 5 s interwave on such a diagram (Figure 2(a)). In this case there are two clear “zones” with reinforced crests and troughs separated by a “zone” of reduced crests and troughs. However, on the  $X/\eta$  diagram (Figure 2(b)), which represents the real surface as it would be observed at any moment in time over a specific distance, the “groups” are far less obvious. As  $T_2$  becomes smaller relative to  $T_1$ , the secondary interwave crests and troughs diminish in size until they can hardly be distinguished, so that the primary interwave form is almost exactly that of the dominant wave and does not form groups of any description.

It was shown above that the length and period of the primary interwaves are virtually the same as the period and length of the dominant wave for all combinations of  $T_1$  and  $T_2$ . There seems to be no logical reason why such primary forms should be considered as “groups” when  $T_1$  and  $T_2$  have similar values (e.g., a 6, 5 s interwave), but not in the case of 6; 1 s interwaves, when important attributes such as period and length are shared in both cases. In this paper, therefore, all prominent wave forms produced by different combinations of  $T_1$  and  $T_2$  are simply considered to be primary interwaves, whereas individual smaller crests and troughs superimposed on them are described as secondary interwaves.

Analysis of the celerity of primary and secondary interwaves results in the following conclusions.

For all combinations of  $T_1$  and  $T_2$ , the primary interwave crest (or trough) celerity  $C_{ipc}$  (or  $C_{ipt}$ ) is equal to the celerity of the dominant wave  $C_1$ , except where  $T_2$  comes to within about 15% of  $T_1$ , when

$$C_{ipc} = \frac{g(T_1 + T_2)}{4\pi}. \quad (5)$$

For example, for a 12; 11 s interwave the observed  $C_{ipc}$  is  $17.995 \text{ m s}^{-1}$ , which agrees with  $9.81(12 + 11)/4\pi$ . This coincides with the wave celerity of an 11.5 s wave, that is,  $(T_1 + T_2)/2$ .

The celerity of any secondary interwave crest is given by

$$C_{isc} = C_1 \frac{T_2}{T_1} = C_{w2}, \quad (6)$$

except where  $T_2$  comes to within about 15% of  $T_1$ , when it is also given by (5). In these cases, no secondary interwave crests can be distinguished anymore because the two component waves merge completely into a single primary interwave form.

Where the dominant wave period is less than double the secondary wave period,  $C_{isc}$  is actually not constant, however, but varies along the crest trajectory. For example, the crest of a 6, 4 s wave originating at  $T = 0$  and  $X = 0$  (where there is maximum interference) propagates at an average celerity of  $7.026 \text{ m s}^{-1}$  during the first 6 seconds,  $4.684 \text{ m s}^{-1}$  during the next 6 seconds, and  $7.026 \text{ m s}^{-1}$  from 12 to 18 s, which is the time required to reach its next peak elevation of 1.4357 m above the SWL at a distance of 112.416 m from the origin. This time interval differs from the peak interwave period  $T_{ip}$  because the latter is reached every 12 seconds at any specific station, but represents the interference of different crests, not the cyclic decay and growth of the same crest. The mean celerity of the secondary interwave crest is  $6.2453 \text{ m s}^{-1}$  in this case, which is the same as that of the subordinate wave celerity  $C_{w2}$  given by  $gT_2^3/2\pi = 9.81 \times 4/2\pi$ .

Some of the interference patterns may be confused with wave groups, although the present analysis shows that they do not travel at half the phase velocity as true groups purportedly do, but at the same velocity as the dominant wave. Furthermore, the secondary crests actually propagate *slower* than the primary forms. This can be demonstrated by changing the time  $T$  incrementally on the  $X/\eta$  diagram, which shows that these secondary crests in fact cycle backward, thus “originating” at the front of the primary forms and “disappearing” at the back (although they simply grow and then diminish in height without actually disappearing completely).

## 5. Multicomponent Interwaves

In this series of virtual experiments, three Airy waves of different periods were superimposed to examine their interference characteristics.

**5.1. Interwave Period.** When the periods of the subordinate component waves are two-thirds or less than two-thirds that of the dominant wave, the interwave period given by all the methods considered here, including  $T_{ix}$ , are the same as or very close to that of the dominant wave. However, when the subordinate wave period  $T_2$  is more than two-thirds of the dominant wave period, the periods defined as  $T_{ic}$ ,  $T_{it}$ , and in some cases also  $T_{izu}$  and  $T_{izd}$ , may be less than the dominant wave period. This results from the fact that interference in these cases produces low-amplitude undulations varying about the SWL in some parts of the spectrum. These are counted as crests and troughs by definition of the parameters above, whereas the definition of  $T_{ix}$  in this case does not consider these undulations to be prominent troughs. For example, in the case of a 12, 11, 5 s wave, there are two troughs between 61.5 and 69 s from the origin, as defined by the zero down- and upcrossing points (Figure 1). The interwave periods  $T_{izu}$  and  $T_{izd}$  would therefore be less than  $T_{ix}$ , which in all cases falls within 2% of the period of the dominant wave,  $T_1$ .

The peak interwave period  $T_{ip}$  for multicomponent waves is given by

$$T_{ip} = \frac{T_1 T_2 T_3}{LCF_1 LCF_{21}}, \quad (7)$$

where  $LCF_1$  and  $LCF_2$  are the two largest common factors by which any pair between  $T_1$ ,  $T_2$ , and  $T_3$  can be divided. For example, for a 6, 4, 3 s interwave  $T_{ix} = 6 \times 4 \times 3/3 \times 2 = 12$ , and for a 6, 5, 4 s interwave,  $T_{ix} = 6 \times 5 \times 4/2 \times 1 = 60$  s.

**5.2. Interwave Length.** The wavelengths determined by the down- or upcrossing method ( $T_{izd}$ ,  $T_{izu}$ ) are generally the same as the wavelength of the dominant wave, but are lower in some cases where the interference surface crosses the SWL in a series of small waves, for example, in the case of 6, 5, 4 s waves.  $L_{ix}$  is in all cases close to the dominant wavelength.

The peak crest interference wavelength  $L_{ip}$  is given by (5). For example, for a 6, 5, 4 s interference wave,  $L_{ip} = 5620.716 \text{ m}$ , which is exactly divisible by 56.2072 m, 39.0327 m, and 24.981 m, the wavelengths of 6, 5, and 4 s waves, respectively.

**5.3. Interwave Height.** The wave heights as defined by  $H_{im}$  and  $H_{irms}$  appear to give somewhat haphazard results for three-component as compared to two-component waves. For example, a 6, 5 s wave has  $H_{im}$  and  $H_{irms}$  values of 2.2394 and 2.4736 m, respectively, but for a 6, 5, 4 s wave these values *reduce* to 2.2017 and 2.4517 m.  $H_{ix}$  in these cases yields characteristic heights of 2.4296 and 2.5947 m, respectively, which better represents the higher energy of 6; 5; 4 s waves as compared to 6, 5 s waves. For a 6, 4, 3 s interwave  $H_{im}$  and  $H_{irms}$  are also *higher* (2.3535 m and 2.3946 m) than for a 6, 5, 4 s wave, whereas  $H_{ix}$  correctly reflects a decrease in the characteristic wave height to 2.2432 m.

**5.4. Interwave Celerity.** The primary interwave celerity  $C_{ip}$  is in all cases, with the exception of the conditions described

below, equal to the dominant wave celerity  $C_1$ . The celerity of any secondary interwave crest is given by

$$C_1 \frac{T_3}{T_1} = C_{w3}. \quad (8)$$

All secondary interwave crests therefore propagate at the celerity of the subordinate wave with the lowest wave period.

Where both subordinate waves have periods that fall within about 15% of the dominant wave period, no secondary interwaves are observed, and the interwave crest propagates at a velocity given by

$$C_{ipc} = \frac{g(T_1 + T_2 + T_3)}{6\pi}. \quad (9)$$

## 6. Conclusions

The following general observations can be made on the basis of the analysis presented here.

Both the interwave period  $T_{ix}$  and length  $L_{ix}$  are equal to or very close to the period  $T_1$  and length  $L_1$  of the dominant component wave, irrespective of the periods of the subordinate waves. This constant relationship thus allows the dominant wave to be recognized easily in any record of fully developed, interfering colinear waves. Although  $T_{ix}$  and  $L_{ix}$  generally concur with  $T_{izd}$ ,  $T_{izu}$ ,  $L_{izd}$ , and  $L_{izu}$  of the zero up- or downcrossing methods, the latter tend to underestimate the real interwave period and length in cases where all the component waves have similar periods because of small fluctuations of the water surface around the SWL.

The interwave height  $H_{ix}$  proposed here better characterizes the “average” wave height than  $H_{im}$  or  $H_{irms}$ , because it steadily increases with increasing energy conditions, whereas the latter two heights do not show any consistent pattern. It is also easily calculated on a spreadsheet if the water surface elevation is recorded at 1 second intervals, making use of the built-in standard deviation function ( $s$ ) of Excel.

In all cases where the dominant wave period is double or more than double the periods of the subordinate waves,  $T_{ix}$ ,  $L_{ix}$ , and  $H_{ix}$  are within 4% of the dominant wave parameters. In such cases, the significant and maximum wave heights ( $H_{is}$  and  $H_{imax}$ ) are the same as  $H_{im}$  and  $H_{irms}$ , so that such interfering, irregular waves have virtually the same characteristics as monochromatic waves. This might explain why the nomograms in [9], which plot energy-based significant wave heights and periods against wind speed, duration, and fetch, show fairly constant relationships corresponding to Airy wave characteristics [10]. However, even if the wave parameters under these circumstances are very close to those of the dominant wave, small differences can produce substantial changes in wave-induced loadings and responses of deepwater offshore platforms and can also significantly affect the safety of ships encountering such waves in the deepwater environment.

Where the subordinate wave periods  $T_2$  and  $T_3$  differ by less than 15% from that of the dominant wave period, respectively, the interwave completely integrates the component waves so that no secondary interwave crests are observed. Although the interwave period and length in this case are

very close to the dominant wave period and length, the interwave height ( $H_{ix}$ ) increases significantly, whereas the interwave celerity  $C_{ip}$  ( $= C_{isc}$ ) assumes an intermediate value.

The average interwave steepness given by  $H_{ix}/L_{ix}$  varies from 0.0354 to 0.0502 for the range of conditions tested, whereas  $H_{is}/L_{ix}$  varies from 0.0354 to 0.0769 and  $H_{imax}/L_{ix}$  from 0.0354 to 0.0858. However, individual interwaves may be significantly steeper, because in the majority of cases, the highest crests occur adjacent to the deepest troughs.

Secondary interwave crests propagate at the celerity of the subordinate wave with the shortest period, whereas the primary interwaves have the same celerity as the dominant wave.

In terms of shipping and offshore design, the interrelationships outlined above can be used to forecast the highest and steepest colinear interwaves that may be expected under any particular combination of conditions ( $H_{imax}$  and  $H_{imax}/L_{ix}$ ), whereas “average” conditions are better represented by  $H_{ix}$  and  $H_{ix}/L_{ix}$ . Of particular importance here is the recurrence frequency of peak conditions at any one location, as can be determined from the  $t/\eta$  diagram (using  $T_{ip}$ ). In the case of colinear waves, such peak conditions would recur frequently at any specific location, but much less so if the waves are coming from different directions. The accompanying Excel spreadsheet (INTERWAVES) can be used to calculate the different parameters of interwaves, as well as to visualize their profiles at specific stations over time and over stretches of ocean at any moment in time.

For practical applications, it would be prudent and necessary to compare the estimates of any engineering quantity (e.g., wave forces and responses on structures or ships) derived from the methods proposed here to those based on statistical analysis of wave parameters (e.g.,  $H_{im}$ ,  $H_{irms}$ , and  $H_{max}$ ) derived from internationally accepted standards, such as the IAHR and CEM methods. Risk and design studies should use the largest of these estimates.

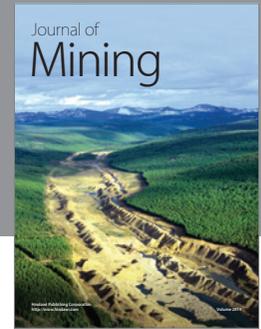
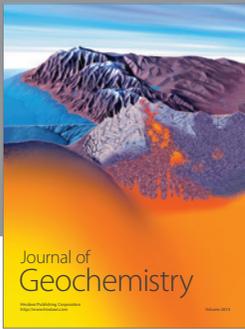
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