International Scholarly Research Network
ISRN Applied Mathematics
Volume 2012, Article ID 365927, 18 pages
doi:10.5402/2012/365927

Research Article

Repetitive Processes Based Iterative Learning Control Designed by LMIs

Jamel Dridi, Selma Ben attia, Salah Salhi, and Mekki Ksouri

Laboratory of Analysis and Control of Systems (LACS), National Engineering School of Tunis, BP 37, le Belvedere, 1002 Tunis, Tunisia

Correspondence should be addressed to Selma Ben attia, benattiaselma@gmail.com

Received 20 October 2012; Accepted 12 November 2012

Academic Editors: Y. Dimakopoulos, Z. Hou, and C. Lu

This paper addressed the stability analysis along the pass and the synthesis problem of linear 2D/repetitive systems. The algorithms for control law design are developed using a strong form of stability for discrete and differential linear repetitive processes known as stability along the pass. In particular, recent work on the use of linear matrix inequalities- (LMIs-) based methods in the design of control schemes for discrete and differential linear repetitive processes will be highlighted by the application of the resulting theory of linear model. The resulting design computations are in terms of linear matrix inequalities (LMIs). Simulation results demonstrate the good performance of the theoretical scheme.

1. Introduction

Repetitive processes are a distinct class of two-dimensional (2D) linear systems (i.e., information propagation in two independent directions) of widely spread over industrial fields. The essential unique characteristic of such process is a series of sweeps, termed passes, through a set of dynamics defined over a fixed finite duration known as the pass length [1]. On each pass, an output, termed the pass profile, is produced which acts as a forcing function on, and hence contributes to, the next pass profile, and also the initial conditions are reset before the start of each new pass. This, in turn, leads to the unique control problem for these processes in that the output sequence of pass profiles generated can contain oscillations that increase in amplitude in the pass-to-pass direction.

To introduce a formal definition, let $\alpha \prec +\infty$ pass length (assumed constant). Then, in a repetitive process, the pass profile $y_k(t)$, $0 \leq t \leq \alpha$, $k \geq 0$, generated on pass k acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile

$y_{k+1}(t)$, $0 \leq t \leq \alpha - 1$, $k \geq 0$. The fact that the pass length is finite (and hence information in this direction only occurs over a finite duration) is the key difference with other classes of 2D systems, such as those with differential dynamics described by the well-known and extensively studied Roesser and Fornasini Marchesini state space models [2].

Iterative learning control (ILC) systems have gained much attention during the last decade, which deserve investigation for theoretical development as well as for practical applications [1–3]. ILC is a technique especially developed for repetitive process, which requires repeating the same operation or task, over a finite duration and constant $\alpha \in [0, T]$. The objective of ILC is to make the output $y_k(t)$, produced on the kth pass, act as a forcing function on the next pass and hence contribute to the dynamics of the new pass profile $y_{k+1}(t)$, $0 \leq t \leq \alpha - 1$, $k \geq 0$. The original work in this area (ILC) is created by [1]. In [4], the authors determine the conditions under which the error converges from trial-to-trial; also it is possible to converge pass to pass to a limit error which is unacceptable along the trial dynamic.

Physical examples of repetitive processes use a robot that has to undertake a picking and placing manipulation. Once the task is achieved, the robot is reset to the initial position, and then the task is repeated. Also in recent years, applications have arisen where adopting a repetitive process setting for analysis which has distinct advantages over alternatives. Examples of these so-called algorithmic applications include classes of iterative learning control (ILC) schemes [1, 4, 5].

Recognizing the unique control problem, the stability theory [6, 7] for linear repetitive processes is of the bounded-input bounded-output (BIBO) form, that is, bounded inputs are required to produce bounded sequences of pass profiles (where boundedness is defined in terms of the norm on the underlying Banach space). Moreover, it consists of two concepts, one of which is defined over the finite pass length, and the other is independent of this parameter. In particular, asymptotic stability guarantees this BIBO property over the finite and fixed pass length, whereas stability along the pass is stronger since it requires this property uniformly, and, hence, it is not surprising that asymptotic stability is a necessary.

If asymptotic stability holds for a discrete or differential linear repetitive process, then any sequence of the generated pass profiles converges in the pass-to-pass direction to a limit profile which is described by a 1D discrete or differential linear systems state space model, respectively. The finite pass length, however, means that the resulting 1D linear system could have an unstable state matrix stable, since over a finite duration even an unstable 1D linear system can only produce a bounded output. There are also applications such as that in [8] where asymptotic stability is all that can be achieved.

In cases where asymptotic stability is not acceptable, stability along the pass is required, and for the processes considered here, the resulting conditions can be tested by 1D linear system tests. Such tests, however, do not lead on to effective control law design algorithms. For example, in the differential case, it is required to test that all eigenvalues of an $m * m$ transfer-function matrix $G(s)$, where m is the dimension of the pass profile vector, lie in the open unit circle in the complex plane $s = jw$, $w \geq 0$. This could clearly lead to a significant computational load and also, despite the Nyquist basis, does not provide a basis for control law design. LMI techniques have been introduced to design ILC algorithms [9, 10]. The authors proposed algorithms to design the matrix gain Γ in the update formula of iterative input $\Delta u_{k+1} = \Gamma e_k$ which satisfies the monotonic convergence condition. The matrix gains are obtained by solving LMI problems. The most effective control law design method currently available for both differential and discrete processes starts from a Lyapunov function interpretation and leads to LMI-based stability tests and control law design algorithms, but it is based on sufficient but not necessary stability conditions.

In this paper, we develop new sufficient conditions for stability along the pass of discrete and differential repetitive processes which have been developed. The given formulation can also be computed using LMIs. The results are based on dissipative theory and make extensive use of the Kalman-Yakubovich-Popov (KYP) lemma that allows us to establish the equivalence between the frequency domain inequality (FDI) for a transfer-function and an LMI defined in terms of its state space realization [11–13]. To employ the KYP lemma [11], we need the stability conditions expressed in the form of an FDI as a first step, and this means that we must restrict attention to the single-input single-output (SISO) case. In the final part of this paper, the controller performance is demonstrated via result of simulation in which the proposed controller effectiveness is compared to previous work.

This note is organized as follows. In Section 2, the iterative learning control for discrete and differential SISO system is applied, and the theoretical study of stability along the pass of a discrete and differential linear repetitive process is also introduced. In Section 3, by employing the KYP lemma, a new sufficient LMI condition is demonstrated for discrete and differential SISO system, to obtain stabilizing classes of linear repetitive process. Then, a numerical evaluation is presented to illustrate the effectiveness of the proposed approach in Section 4 for discrete and differential SISO system. Finally, the paper is concluded in Section 5.

Throughout this paper, $\sigma(A)$ and $\rho(A)$ denote the spectrum and the spectral radius of a given matrix A. $X \succ 0$ (resp., $X \prec 0$) denotes a real symmetric positive (resp., negative) definite matrix. A^T denotes the transpose of A. Furthermore, the symbol $\mathbb{C}$ indicates the set of a complex numbers and $\mathbb{C}_-$ the open left half of the complex plane. To simplify the scriptures, we will use the symbol $\mathrm{sym}\{A\} = A^T + A$. $*$ is used for the blocks induced by symmetry. Also, the identity and null matrix of the required dimensions are denotes by I and 0, respectively.

2. Application to Iterative Learning Control

2.1. Discrete Processes

The plants considered in this section are assumed to be adequately represented by discrete linear time-invariant systems described by the state space triple $\{A, B, C\}$. In an ILC setting for linear time-invariant dynamics, the state space model is written as

$$\begin{aligned} x_k(p+1) &= Ax_k(p) + Bu_k(p), \quad 0 \le p \le \alpha - 1, \\ y_k(p) &= Cx_k(p), \end{aligned} \tag{2.1}$$

where, $x_k(p) \in \mathbb{R}^n$ is the state vector, $y_k(p) \in \mathbb{R}^m$ is the output vector, $u_k(p) \in \mathbb{R}^r$ is the vector of control inputs, and $\alpha \prec \infty$ is the trial length. If the signal to be tracked is denoted by $y_d(p)$, then $e_k(p) = y_d(p) - y_k(p)$ is the error on trial k. The most basic requirement now is the control law design to force trial-to-trial error convergence (i.e., in the k direction).

The class of ILC schemes considered here is of the following form which, in effect, is a (static and dynamic) combination of previous input vectors, the current trial error, and the errors on a finite number of previous trials. In particular, on trial $(k+1)$, the control input is calculated using

$$\Delta u_{k+1}(p) = u_{k+1}(p) - u_k(p) = K_1 \eta_{k+1}(p+1) + K_2 e_k(p+1), \tag{2.2}$$

where $\Delta u_{k+1}(p)$ denotes a variation of the control input, K_1 and K_2 are matrices with compatible dimensions.

Introducing $\eta_{k+1}(p+1) = x_{k+1}(p) - x_k(p)$, $\eta_k(0) = 0$, then clearly (2.1) and (2.2) can be written as

$$\begin{aligned}
\eta_{k+1}(p+1) &= x_{k+1}(p) - x_k(p) \\
&= (A + BK_1)\eta_{k+1}(p) + BK_2 e_k(p), \\
e_{k+1}(p) &= y_d(p) - y_{k+1}(p) = y_d(p) - Cx_{k+1}(p) \\
&= -C(A + BK_1)\eta_{k+1}(p) + (I - CBK_2)e_k(p).
\end{aligned} \tag{2.3}$$

By introducing the variables:

$$\begin{aligned}
\widetilde{A} &= A + BK_1, \\
\widetilde{B}_0 &= BK_2, \\
\widetilde{C} &= -C(A + BK_1), \\
\widetilde{D}_0 &= I - CBK_2,
\end{aligned} \tag{2.4}$$

(2.3) can be written as

$$\begin{bmatrix} \eta_{k+1}(p+1) \\ e_{k+1}(p) \end{bmatrix} = \begin{bmatrix} \widetilde{A} & \widetilde{B}_0 \\ \widetilde{C} & \widetilde{D}_0 \end{bmatrix} \begin{bmatrix} \eta_{k+1}(p) \\ e_k(p) \end{bmatrix}. \tag{2.5}$$

Error convergence is connected to the monitoring path of the $y_d(p)$.

Several sets of necessary and sufficient conditions for stability along the pass of both discrete linear repetitive processes of the form considered here are known [7–14], and here we will make use of those given in terms of the corresponding 2D characteristic polynomial, where for the discrete case this is defined as [7–14]

$$C_{\mathrm{disLRP}} = \det\left(\begin{bmatrix} I - z_1\widetilde{A} & -z_1\widetilde{B}_0 \\ -z_2\widetilde{C} & I - z_2\widetilde{D}_0 \end{bmatrix} \right) \neq 0, \tag{2.6}$$

where $z_1, z_2 \in \mathbb{C}$ are the inverses of z-transform variables. See [7] for the details concerning these transform variables and, in particular, how to avoid technicalities associated with the finite pass length, which define a 2D transfer function matrix for these processes.

By [7], z_1 denotes the shift operator along the pass applied, for example, to $x_k(p)$ as follows:

$$x_k(p) = z_1 x_k(p+1), \tag{2.7}$$

and z_2 the pass-to-pass shift operator applied, for example, to $y_k(p)$ as follows:

$$x_k(p) = z_2 x_{k+1}(p). \tag{2.8}$$

Theorem 2.1 (see [7, 12, 15]). *A discrete linear repetitive process of the form* (2.5) *(controllable and observable) is stable along the pass if and only if*

(i) $\rho(\widetilde{D}_0) \prec 1$,

(ii) $\rho(\widetilde{A}) \prec 1$,

(iii) *all eigenvalues of $G_{dis}(z^{-1})$ have modulus strictly less than one.*

All the three conditions of Theorem 2.1 have well-defined physical interpretations and, unlike equivalents [16], can be tested by direct application of 1D linear time invariant systems. It is easy to show that stability along the pass guarantees that the corresponding limit profile of (2.5) is stable as a 1D linear system.

In terms of checking the conditions of these two results, the first two conditions in each case are easily solved.

$\rho(\widetilde{D}_0) \prec 1$ is the necessary and sufficient condition for asymptotic stability, that is, BIBO stability over the finite pass length. This condition, proposed in [17], insured trial-to-trial error convergence only. This last condition is precisely obtained by applying 2D discrete linear systems stability theory to (2.3) as first proposed in [17] to ensure trial-to-trial error convergence only.

It is easy to construct examples where $\rho(\widetilde{D}_0) \prec 1$, but the performance along the trial is very poor. (The source of this problem is that this condition demands that the trial output is bounded over a finite duration, and an unstable linear system can only produce a bounded output.) In repetitive process stability theory, asymptotic stability guarantees that the sequence of pass profiles generated by an example with this property converges strongly as $k \to \infty$ to a so-called limit profile whose dynamics for the processes considered here can be obtained by letting $k \to \infty$ the state space model.

Applying the second conditions of Theorem 2.1, stability of the matrix $\widetilde{A}$ (i.e., a uniformly bounded first pass profile) is, in general, only a necessary condition for stability along the pass.

The only difficulty, which can be arising, is the computational cost associated with condition (iii). For SISO examples, this condition requires that the Nyquist plot generated by $G_{\mathrm{dis}}(z^{-1})$ lies inside the unit circle in the complex plane for all $|z^{-1}| = 1$.

However, it is very difficult to provide computationally effective tests for stability in this way. It has been proved recently that any robust control problem can be turned into an LMI dilated one, in terms of converting the Lyapunov conditions to be generalized in equations by mean of lemmas [3, 5, 7, 14].

One of the ways to derive tractable tests is by applying Lyapunov theory associated with LMI techniques that became a standard tool for the stability analysis of 1D system when manipulating the state space models. These Lyapunov functions must contain contributions from the current pass state and previous pass profile vectors, for example, composed of which is the sum of quadratic terms in the current pass state and previous pass profile, respectively [7, 14].

This approach is developed by using candidate Lyapunov function for discrete models, having the following form:

$$V(k,p) = x_{k+1}^T(p)P_1x_{k+1}(p) + y_k^T(p)P_2y_k(p), \tag{2.9}$$

where $P_1 \succ 0$ and $P_2 \succ 0$.

With the associated increment,

$$\begin{aligned}\Delta V(k,p) = x_{k+1}^T(p+1)P_1x_{k+1}(p+1) - x_{k+1}^T(p)P_1x_{k+1}(p) \\ + y_{k+1}^T(p)P_2y_{k+1}(p) - y_k^T(p)P_2y_k(p).\end{aligned} \tag{2.10}$$

Then, the stability along the pass holds if $\Delta V(k,p) \prec 0$ for all (k) and (p) which is equivalent to the requirement that

$$\Phi_i^T P_{i+1}\Phi_i - P_i < 0, \tag{2.11}$$

where $P_i = \mathrm{diag}(P_1, P_2)$ and $\Phi > 0$.

2.2. Differential Processes

In this section, we considered a differential linear time-invariant systems with the following state space representation $\{A, B, C\}$ (with $T \prec \infty$):

$$\begin{aligned}\dot{x}_k(t) = Ax_k(t) + Bu_k(t), \quad 0 \le t \le \alpha,\ k \ge 0, \\ y_k(t) = Cx_k(t).\end{aligned} \tag{2.12}$$

Suppose that (t) denotes continuous time and (k) denotes learning iteration, where on trial (k), $x_k(t) \in \mathbb{R}^n$ is the state vector, $y_k(t) \in \mathbb{R}^m$ is the output vector, $u_k(t) \in \mathbb{R}^r$ is the vector of control inputs, and $\alpha \prec \infty$ is the trial length. The boundary condition is $x_{k+1}(0) = 0$ for $k = 0, 1, 2, \ldots$, and $u(t,0) = u_0(t)$ for $0 \le t \le T$.

The desired signal is denoted by $y_d(t)$ (differentiable). $e_k(t) = y_d(t) - y_k(t)$ is the error on trial (k), and the most basic requirement is to force the error to converge as $k \rightarrow \infty$. In particular, the objective of constructing a sequence of input functions such that the performance is gradually improving with each successive trial can be refined to a convergence condition on the input and the error $\lim_{k\to\infty}\|e_k\| = 0$.

Consider also a control law of the form:

$$\Delta u_{k+1}(t) = u_{k+1}(t) - u_k(t) = k_1\dot{\eta}_{k+1}(t) + k_2\dot{e}_k(t). \tag{2.13}$$

Introducing

$$\eta_{k+1}(t) = \int_0^t [x_{k+1}(\tau) - x_k(\tau)]d\tau, \qquad \eta_k(0) = 0, \tag{2.14}$$

then clearly (2.12) and (2.13) can be written as

$$\begin{aligned}\dot{\eta}_{k+1}(t) &= \int_0^t [\dot{x}_{k+1}(\tau) - \dot{x}_k(\tau)]d\tau \\ &= (A + BK_1)\eta_{k+1}(t) + BK_2 e_k(t), \\ e_{k+1}(t) - e_k(t) &= -y_{k+1}(t) + y_k(t) = -C(x_{k+1}(t) - x_k(t)), \\ e_{k+1}(t) &= -C(A + BK_1)\eta_{k+1}(t) + (I - CBK_2)e_k(t).\end{aligned} \tag{2.15}$$

Variables are introduced as follows:

$$\begin{aligned}\widetilde{A} &= A + BK_1, \\ \widetilde{B}_0 &= BK_2, \\ \widetilde{C} &= -C(A + BK_1), \\ \widetilde{D}_0 &= I - CBK_2.\end{aligned} \tag{2.16}$$

Then, clearly (2.15) can be written as

$$\begin{bmatrix} \dot{\eta}_{k+1}(t) \\ e_{k+1}(t) \end{bmatrix} = \begin{bmatrix} \widetilde{A} & \widetilde{B}_0 \\ \widetilde{C} & \widetilde{D}_0 \end{bmatrix} \begin{bmatrix} \eta_{k+1}(t) \\ e_k(t) \end{bmatrix}. \tag{2.17}$$

For the differential processes, the 2D characteristic polynomial is defined by

$$C_{\text{diffLRP}}(s, z_2) = \det\left(\begin{bmatrix} sI - \widetilde{A} & -\widetilde{B}_0 \\ -z_2\widetilde{C} & I - z_2\widetilde{D}_0 \end{bmatrix}\right) \neq 0, \quad \forall\{s, z_2\} \in \{(s, z_2) : \operatorname{Re}(s) \geq 0,\ |z_2| \leq 1\}, \tag{2.18}$$

where $s \in \mathbb{C}$, is the Laplace transform indeterminate and $z_2 \in \mathbb{C}$ is the inverse of z-transform variable, respectively, as

$$x_k(t) = s x_k(t+1), \qquad x_k(t) = z_2 x_{k+1}(t), \tag{2.19}$$

which is of the form (2.17), and hence the repetitive process stability theory can be applied to the ILC control scheme (2.13). In particular, stability along the pass is equivalent to uniform BIBO stability (defined in terms of the norm on the underlying function space), that is, independent of the trial length, and hence it may be possible to achieve acceptable pass-to-pass error convergence along the pass dynamics.

Theorem 2.2 (see [7–12]). *A differential linear repetitive process of the form* (2.17) *(controllable and observable) is stable along the pass if and only if*

(i) $\rho(\widetilde{D}_0) \prec 1$,

(ii) $\rho(\widetilde{A}) \in \mathbb{C}_-$,

(iii) *all eigenvalues of* $G_{diff}(s^{-1})$ *have modulus strictly less than one, for all* $\omega \geq 0$.

All the three conditions of Theorem 2.2 have well-defined physical interpretations and, unlike equivalents [6], can be tested by direct application of 1D linear time-invariant systems.

It is easy to show that stability along the pass guarantees that the corresponding limit profile of (2.17) is stable as a 1D linear system, that is, all eigenvalues of the state matrix $\widetilde{A}$ have strictly negative real parts.

In terms of checking the conditions of these two results, the first two conditions in each case are easily solved.

Consider condition (i), this is the necessary and the sufficient condition for asymptotic stability, that is, BIBO stability over the finite pass length. This condition, proposed in [7, 17], insured trial-to-trial error convergence only.

Applying the second conditions of Theorem 2.2, stability of the matrix A (i.e., a uniformly bounded first pass profile) is, in general, only a necessary condition for stability along the pass. The only difficulty, which can be arising, is the computational cost associated with condition (iii). For SISO examples, this condition requires that the Nyquist plot generated by $G_{\text{diff}}(s)$ lies inside the unit circle in the complex plane for all $s = i\omega$.

However, it is very difficult to provide computationally effective tests for stability in this way. It has been proved recently that any robust control problem can be turned into an LMI dilated one, in terms of converting the Lyapunov conditions to be generalized in equations by mean of lemmas [7, 12, 13].

One of the ways to derive tractable tests is by applying Lyapunov theory associated with LMI techniques that became a standard tool for the stability analysis of 1D system when manipulating the state space models. These Lyapunov functions must contain contributions from the current pass state and previous pass profile vectors, for example, composed of which is the sum of quadratic terms in the current pass state and previous pass profile, respectively [12].

An alternative approach that does lead to control law design algorithms are presented in [7–14]. This approach is developed by using candidate Lyapunov function for differential models of the form

$$\Delta V(k,t) = \dot{V}(k,t) + \Delta v(k,t) \prec 0, \tag{2.20}$$

where $P_1 \succ 0$ and $P_2 \succ 0$.

With the associated increment,

$$\Delta V(k,t) = \dot{x}_{k+1}^T(t) P_1 x_{k+1}(t) + x_{k+1}^T(t) P_1 \dot{x}_{k+1}(t) + y_{k+1}^T(t) P_2 y_{k+1}(t) - y_k^T(t) P_2 y_k(t). \tag{2.21}$$

Then, the stability along the pass holds if $\Delta V(k,t) \prec 0$ for all (k) and (t) which is equivalent to the requirement that

$$\Phi_i^T P_{i+1} \Phi_i - P_i < 0, \tag{2.22}$$

where $P_i = \text{diag}(P_1, P_2)$.

3. LMI-Based Iterative Learning Control

Iterative learning control (ILC) is a simple and effective method for the control of systems doing a defined task repetitively and periodically in a limited and constant time interval. In this section, the main contribution of this paper is provided. First, two necessary and sufficient scaling LMI conditions for particulars class of systems are given. Then, a sufficient condition is presented for the discrete and differential process.

The Kalman-Yakubovich-Popov (KYP) lemma [11, 13] is used to develop necessary and sufficient conditions for stability along the pass of the SISO of the discrete/differential linear repetitive processes (2.5) and (2.17), respectively.

3.1. Discrete Processes

Considering the SISO version of (2.5) and by introducing (2.4), the following theorem is established.

Theorem 3.1 (see [15]). *The SISO version of* (2.5) *is stable along the pass if and only if there exist matrices $r > 0$, $S > 0$, $Q > 0$ and a symmetric matrix P such that the following LMIs are feasible:*

$$\begin{aligned}
&(1)\quad \widetilde{D}_0^T r \widetilde{D}_0 - r \prec 0,\\
&(2)\quad \widetilde{A}^T S \widetilde{A} - S \prec 0,\\
&(3)\quad \begin{bmatrix} \widetilde{A}P\widetilde{A}^T - P - Q\widetilde{A}^T - \widetilde{A}Q + 2Q & \widetilde{A}P\widetilde{C}^T - Q\widetilde{C}^T & \widetilde{B}_0 \\ \widetilde{C}P\widetilde{A}^T - \widetilde{C}Q & \widetilde{C}P\widetilde{C}^T - I & \widetilde{D}_0 \\ \widetilde{B}_0^T & \widetilde{D}_0^T & -I \end{bmatrix} \prec 0.
\end{aligned} \tag{3.1}$$

The difficulty with the condition of Theorem 3.1 is that it is nonlinear in its parameters. It can, however, be controlled into the following results, where the inequality is a strict LMI, a linear constraint which also gives a formula for computing K_1 and K_2.

Theorem 3.2. *The SISO version of* (2.5) *is stable along the pass if there exist matrices $S > 0$, $Q > 0$, G, N_g, K_2 and a symmetric matrix P such that the following LMIs are feasible:*

$$(1)\quad \begin{bmatrix} -CBK_2 & 0 \\ 0 & CBK_2 - 2 \end{bmatrix} \prec 0, \tag{3.2}$$

$$(2)\quad \begin{bmatrix} -S & AG + BN_g \\ * & S - G - G^T \end{bmatrix} \prec 0, \tag{3.3}$$

$$(3)\quad \begin{bmatrix} -P + 2Q + sym\{\alpha AG + \alpha BN_g\} & * & -Q - \alpha G^T + AG + BN_g & BK_2 \\ -\alpha CAG - \alpha CBN_g & -I & -CAG - CBN_g & I - CBK_2 \\ * & * & P - G - G^T & 0 \\ * & * & * & -I \end{bmatrix} \prec 0, \tag{3.4}$$

where $\alpha > 0$. If these LMIs are feasible, the controller gain is computed by

$$K_1 = N_g G^{-1}. \tag{3.5}$$

To simplify the proof, we consider each LMI of the previous result separately.

Proof. (1) *The First LMI*. First, note that both r and $\widetilde{D}_0$ are real numbers, and hence $r(\widetilde{D}_0^2 - 1) < 0$, with $r > 0$. By using (2.4), it is obvious that $(1 - CBK_2)^2 - 1 < 0$ or $CBK_2(CBK_2 - 2) < 0$.

Hence, we require that $0 < CBK_2 < 2$. The value of CBK_2 greatly influences the pass-to-pass error convergence, which is equivalent to (3.2) since here CBK_2 is a scalar.

(2) *The Second LMI*. We apply the Schur complement on $\widetilde{A}^T S \widetilde{A} - S < 0$, this inequality is equivalent to: $\begin{bmatrix} -S & S\widetilde{A}^T \\ * & -S \end{bmatrix} < 0$, and by using the projection lemma [18], we obtain

$$\text{(i)} \quad \begin{bmatrix} -S & 0 \\ 0 & -S \end{bmatrix} + \begin{bmatrix} S \\ 0 \end{bmatrix} \begin{bmatrix} 0 & \widetilde{A}^T \end{bmatrix} + \begin{bmatrix} 0 \\ \widetilde{A} \end{bmatrix} \begin{bmatrix} S & 0 \end{bmatrix} < 0, \tag{3.6a}$$

$$\text{(ii)} \quad \begin{bmatrix} -S & 0 & S \\ * & -S & \widetilde{A}G \\ * & * & -G - G^T \end{bmatrix} < 0. \tag{3.6b}$$

By applying the Schur complement, the inequality (3.6b) is transformed into

$$\begin{bmatrix} -S & \widetilde{A}G \\ * & S - G - G^T \end{bmatrix} < 0. \tag{3.6c}$$

Substituting (2.4) in this LMI (3.6c), (3.3) is done.

(3) *The Third LMI*. Multiplying $\begin{bmatrix} 1 & 0 & \widetilde{A} \\ 0 & 1 & \widetilde{C} \end{bmatrix}$ by the right side of (3.4) and its transpose by the left one.

Introducing (2.16), and by applying the projection lemma [18], the inequality (3.1) is obtained.

Moreover, (3.4) follows on setting $N_g = K_1 G$. □

3.2. *Differential Processes*

Considering the SISO version of (2.4) and by introducing (2.16), the following theorem is given.

Theorem 3.3 (see [15]). *The SISO version of* (2.17) *is stable along the pass if and only if there exist matrices* $r > 0$, $X > 0$, $Q > 0$ *and a symmetric matrix* P *such that the following LMIs are feasible:*

$$
\begin{aligned}
&(1)\quad \tilde{D}_0^T r \tilde{D}_0 - r \prec 0,\\
&(2)\quad \tilde{A}^T X + X\tilde{A} \prec 0,\\
&(3)\quad \begin{bmatrix} \tilde{A}Q\tilde{A}^T + P\tilde{A}^T + \tilde{A}P & \tilde{A}QC^T + P\tilde{C}^T & \tilde{B}_0 \\ \tilde{C}Q\tilde{A}^T + \tilde{C}P & \tilde{C}Q\tilde{C}^T - I & \tilde{D}_0 \\ \tilde{B}_0^T & \tilde{D}_0^T & -I \end{bmatrix} \prec 0.
\end{aligned}
\tag{3.7}
$$

The difficulty with the condition of Theorem 3.3 is that it is nonlinear in its parameters. It can, however, be controlled into the following results, where the inequality is a strict LMI, a linear constraint which also gives a formula for computing the gain K_1 and K_2.

Theorem 3.4. *The SISO version of* (2.17) *is stable along the pass if there exist matrices* $X > 0$, $Q > 0$, G, N_g, K_2 *and a symmetric matrix* P *such that the following LMIs are feasible:*

$$
\begin{aligned}
&(1)\quad \begin{bmatrix} -CBK_2 & 0 \\ 0 & CBK_2 - 2 \end{bmatrix} \prec 0,\\
&(2)\quad \begin{bmatrix} sym\{AG + BN_g\} & X + AG + BN_g - G^T \\ * & -G - G^T \end{bmatrix} \prec 0,\\
&(3)\quad \begin{bmatrix} sym\{\alpha(AG + BN_g)\} & * & P + AG + BN_g - \alpha G^T & BK \\ -\alpha CAG - \alpha CBN_g & -I & -CAG - CBN_g & I - CBK_2 \\ * & * & Q - G - G^T & 0 \\ * & * & 0 & -I \end{bmatrix} \prec 0,
\end{aligned}
\tag{3.8}
$$

where $\alpha > 0$. *If these LMIs are feasible, the controller gain is computed by* $K_1 = N_g G^{-1}$.

Proof. It follows in a direct way from the LMI given in Section 3.1 applied on the extended controlled system (2.17). The resulting LMIs are treated by Lemma projection and the Schur complement already well known and given in [18] for instance. □

4. Simulation Results

4.1. Numerical Evaluation for a Discrete Process

In this section, we compare the control performances of two ILC algorithms (Theorem 3 in [19] and Theorem 3.2 of our proposed approach) described previously through a numerical evaluation summarized in Table 1.

The system is characterized by order (n) and number of inputs (m). For fixed values of (n, m), we generate randomly 100 ILC systems of the form (2.1).

Method 1. It uses the conditions given in Theorem 3 in [19], which are sufficient conditions $(\gamma = 0.1)$.

Table 1: Numerical evaluation.

	Method	Success
$n = 2$	Method 1	72
$m = 1$	Method 2	86
$n = 3$	Method 1	43
$m = 1$	Method 2	68
$n = 4$	Method 1	24
$m = 1$	Method 2	40
$n = 5$	Method 1	05
$m = 1$	Method 2	15
$n = 6$	Method 1	01
$m = 1$	Method 2	06

Method 2. It uses the conditions given in Theorem 3.2 proposed in Section 3, which are sufficient conditions ($\alpha = 0.1$).

By using the Matlab LMI Control Toolbox to check the feasibility of the LMI conditions, a counter is increased if the corresponding method succeeds in providing stabilizing control.

4.2. *Differential Processes*

To our knowledge, the ILC control for the continuous case has not been studied. The given example highlights the contribution of the developed condition for the synthesis problem of differential process.

4.2.1. Illustrative Example 1

Considering a differential linear time-invariant systems described by (2.12) when

$$A = \begin{bmatrix} -1.054 & -1.051 \\ -0.011 & -1.105 \end{bmatrix}, \qquad B = \begin{bmatrix} 1.04 \\ 2.55 \end{bmatrix}, \qquad C = \begin{bmatrix} 0.50 & 1.22 \end{bmatrix}. \tag{4.1}$$

Applying the control law (2.13), the system is stable in the closed loop, and the conditions in Theorem 3.4 provide the following gains:

$$K_1 = \begin{bmatrix} 0.4848 & 0.5160 \end{bmatrix}, \qquad K_2 = 0.3305. \tag{4.2}$$

Here,

$$\rho\left(\tilde{A}\right) = \begin{Bmatrix} -0.3390 \\ -0.0000 \end{Bmatrix}, \qquad \rho\left(\tilde{D}_0\right) = -0.2000. \tag{4.3}$$

For $\omega > 0$, the three condition proposed in Theorem 3.4 in Section 3 are verified; this process is stable along the pass and as confirmed by the Nyquist plot of Figure 1.

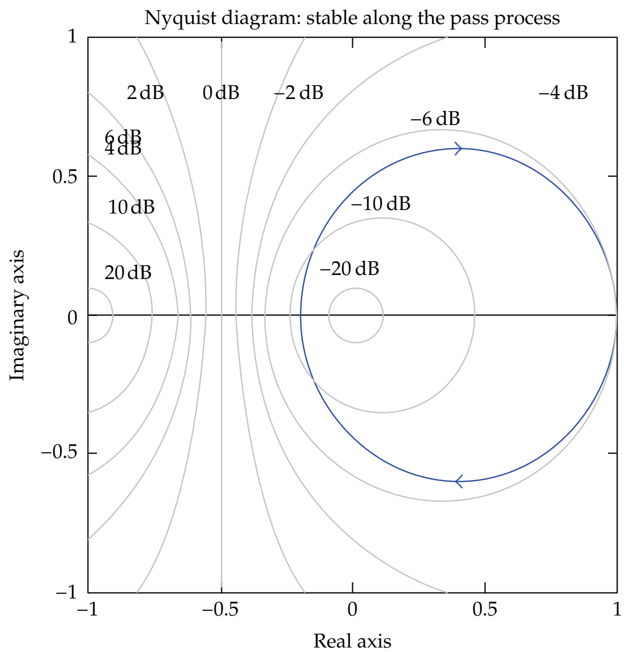


Figure 1: Nyquist plots for the stable along the pass process.

4.2.2. Illustrative Example 2

In this section, an example is given to demonstrate the effectiveness of the proposed method. As shown in Figure 2, we consider a DC motor which its armature is driven by a constant current source, but its field winding current is variable. So, the motor rotational angle control is done by varying the voltage of the source connected to the field winding. The motor rotates a mechanical load. In this situation, the state space equations of the motor are as follows [20]:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bv_f(t), \\ y(t) &= Cx(t), \quad t \geq 0, \end{aligned} \tag{4.4}$$

where $x(t) = [i_f(t) \;\; \omega(t) \;\; \theta(t)]^T$, $y(t) = \theta(t)$,

$$A = \begin{bmatrix} -\dfrac{R_f}{L_f} & 0 & 0 \\ \dfrac{K_m}{J} & \dfrac{-f}{J} & 0 \\ 0 & 1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} \dfrac{1}{L_f} \\ 0 \\ 0 \end{bmatrix}, \qquad C = [0 \;\; 0 \;\; 1], \tag{4.5}$$

where R_f and L_f are the field winding resistance and inductance, respectively, K_m is the motor torque ratio, and J and f are the mechanical load inertia momentum and friction ratio, respectively. Also $v_f(t)$ and $i_f(t)$ are the field winding source voltage and current, respectively, and $\omega(t)$ and $\theta(t)$ are the motor shaft rotational speed and angle, respectively.

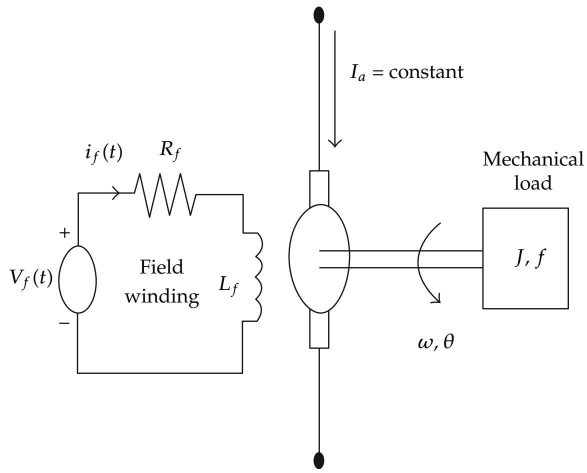


Figure 2: DC motor with constant armature current.

We purpose to determine the input voltage of motor ($v_f(t)$), so that the motor output $y(t)$ periodically follows the desired given signal $y_d(t)$ in time interval $[0, t_f]$, such that by increasing the iterations number, error between $y(t)$ and $y_d(t)$ vanishes.

To determine the input voltage of motor, we use the proposed method in this paper. For this reason, the state equations of the motor should be written as discrete-time form. We discretize the motor state equations by choosing the sampling period $T = 0.01$ sec and the following amounts for parameters:

$$R_f = 20\,\Omega, \qquad L_f = 1\,\mathrm{H}, \qquad K_m = 100\,\mathrm{Nm/A}, \qquad f = 0.5\,\mathrm{Nms/rad}, \qquad J = 2\,\mathrm{Nms^2/rad},$$
$$t_f = 12\,\mathrm{sec}. \tag{4.6}$$

By considering variable k as the iteration number, the obtained discrete state equations are as follows:

$$\begin{gathered} x_k(p+1) = A_D x_k(p) + B_D v_{fk}(p), \\ y_k(p) = C_D x_k(p), \\ p = 0, 1, \ldots, 1200, \qquad k = 0, 1, \ldots, \end{gathered} \tag{4.7}$$

where the coefficient matrices are as the following:

$$A_D = \begin{bmatrix} 0.8187 & 0 & 0 \\ 0.4526 & 0.9975 & 0 \\ 0.0023 & 0.0100 & 1 \end{bmatrix}, \qquad B_D = \begin{bmatrix} 0 \\ 0.0197 \\ 0.0211 \end{bmatrix}, \qquad C_D = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}. \tag{4.8}$$

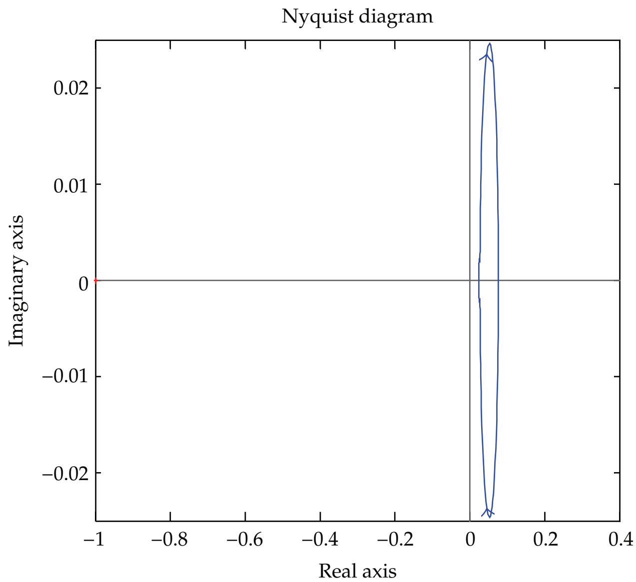


Figure 3: Graphical representation of condition (iii).

Applying the control law (2.2), the system is stable in the closed loop, and also the LMI of Theorem 3.2 is feasible:

$$\widetilde{A} = \begin{bmatrix} 0.8170 & 0 & 0 \\ 0.4920 & 0.9846 & -0.9060 \\ 0.0445 & -0.0038 & 0.0296 \end{bmatrix}, \qquad \widetilde{B}_0 = \begin{bmatrix} 0 \\ 0.8847 \\ 0.9476 \end{bmatrix},$$
$$\widetilde{C} = \begin{bmatrix} -0.0445 & 0.0038 & -0.0296 \end{bmatrix}, \qquad \widetilde{D}_0 = 0.0524, \tag{4.9}$$

and the control gains are computed:

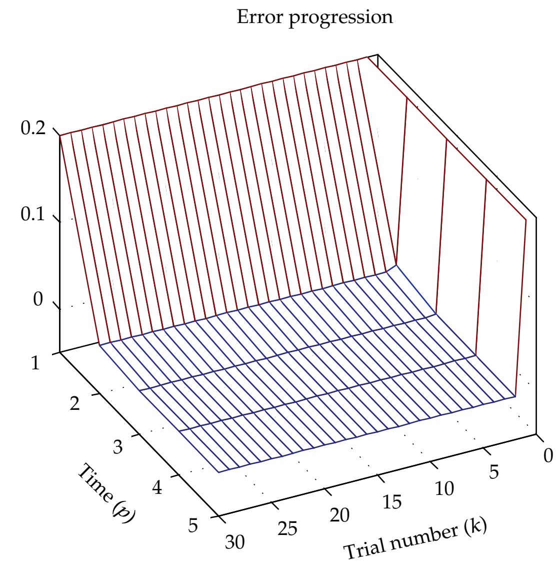
$$K_1 = \begin{bmatrix} 2.0005 & -0.6529 & -45.9904 \end{bmatrix}, \qquad K_2 = 44.9098. \tag{4.10}$$

Here,

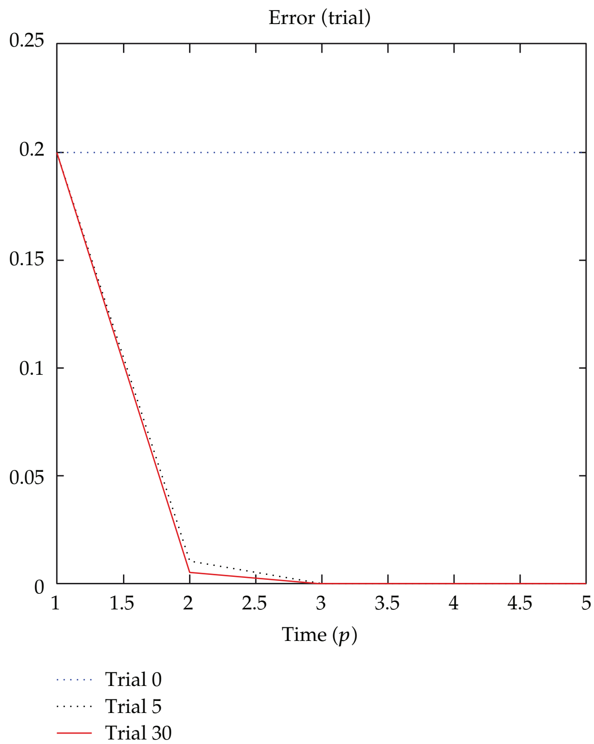
$$\rho\left(\widetilde{A}\right) = \begin{Bmatrix} 0.0260 \\ 0.9882 \\ 0.8170 \end{Bmatrix}, \qquad \rho\left(\widetilde{D}_0\right) = 0.0524. \tag{4.11}$$

The three conditions of Theorem 3.2 proposed in Section 3 are verified, and this process is stable along the pass, and as confirmed by the Nyquist plot of Figure 3, and give the stability condition 3.

The next figure (Figures 4(a) and 4(b)) presents the converged error signals for the ILC architecture with the feedback controller along the trial dynamics. These show that the objective of trial-to-trial error convergence and along the trial performance has been reached.



(a)



(b)

Figure 4: (a) The motor error at the iterations. (b) The output-error on the iterations 0, 5 and 30.

The simulation results have been shown in Figures 4(a) and 4(b), as shown, by increasing the iteration number, the motor rotational angle quickly is stable along the pass, following selection of K_1 and K_2.

5. Conclusion

The contribution of this paper stands in the combination of the concept of stability along the pass with "slack" scalars α, with ad hoc changes of variables to provide improved LMI conditions by applying the ILC design.

These conditions reduce significantly the conservations and show the advantage of using the scalar variables in the case of the ILC. Numerical evaluations are given to demonstrate the applicability and the conservatism reduction of the proposed conditions, and a comparison with recent conditions in the literature has been described.

These formulations enable to consider the case of uncertain repetitive process later. It is possible to consider these new conditions to deal with performances in the context of H_2 and H_∞ settings. These extensions are under study.

References

[1] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operations of robots by learning," *Journal of Robotic Systems*, vol. 1, no. 2, pp. 123–140, 1984.

[2] E. Fornasini and G. Marchesini, "State-space realization theory of two-dimensional filters," *IEEE Transactions on Automatic Control*, vol. 21, no. 4, pp. 484–492, 1976.

[3] M. Norrlf, *Iterative learning control, analysis, design and experiments [Ph.D. thesis]*, Linköping University, Linkopings, Sweden, 2000.

[4] D. H. Owens, N. Amann, E. Rogers, and M. French, "Analysis of linear iterative learning control schemes—a 2D systems/repetitive processes approach," *Multidimensional Systems and Signal Processing*, vol. 11, no. 1-2, pp. 125–177, 2000.

[5] K. L. Moore, *Iterative Learning Control for Deterministic Systems*, Advances in Industrial Control Series, Springer, London, UK, 1993.

[6] K. Gakowski, R. W. Longman, and E. Rogers, "Special issue: multidimensional systems (nD) and iterative learning control," *International Journal of Applied Mathematics and Computer Science*, vol. 13, no. 1, 2003.

[7] E. Rogers, K. Galkowski, and D. H. Owens, *Control Systems Theory and Applications for Linear Repetitive Processes*, vol. 349 of *Lecture Notes in Control and Information Sciences*, Springer, Berlin, Germany, 2007.

[8] P. D. Roberts, "Two-dimensional analysis of an iterative nonlinear optimal control algorithm," *IEEE Transactions on Circuits and Systems*, vol. 49, no. 6, pp. 872–882, 2002, Special issue on multidimensional signals and systems.

[9] J. Xu, M. Sun, and L. Yu, "LMI-based robust iterative learning controller design for discrete linear uncertain systems," *Journal of Control Theory and Applications*, vol. 3, no. 3, pp. 259–265, 2005.

[10] H. S. Ahn, K. L. Moore, and Y. Chen, "LMI approach to iterative learning control design," in *Proceedings of the IEEE Mountain Workshop on Adaptive and Learning Systems (SMCals '06)*, pp. 72–77, July 2006.

[11] A. Rantzer, "On the Kalman-Yakubovich-Popov lemma," *Systems & Control Letters*, vol. 28, no. 1, pp. 7–10, 1996.

[12] B. Brogliato, R. Lozano, B. Maschke, and O. Egeland, *Dissipative Systems Analysis and Control. Theory and Applications*, Communications and Control Engineering Series, Springer, London, UK, 2nd edition, 2007.

[13] T. Iwasaki and S. Hara, "Generalized KYP lemma: unified frequency domain inequalities with design applications," *IEEE Transactions on Automatic Control*, vol. 50, no. 1, pp. 41–59, 2005.

[14] E. Rogers and D. H. Owens, *Stability Analysis for Linear Repetitive Processes*, vol. 175 of *Lecture Notes in Control and Information Sciences*, Springer, Berlin, UK, 1992.

[15] W. Paszke, P. Rapisarda, E. Rogers, and M. Steinbuch, "Using dissipativity theory to formulate necessary and sufficient conditions for stability of linear repetitive processes," Research Report, School of Electronics and Computer Science, University of Southampton, Hampshire, UK, 2008.

[16] T. Iwasaki, G. Meinsma, and M. Fu, "Generalized *S*-procedure and finite frequency KYP lemma," *Mathematical Problems in Engineering*, vol. 6, no. 2-3, pp. 305–320, 2000.

[17] J. E. Kurek and M. B. Zaremba, "Iterative learning control synthesis based on 2-D system theory," *IEEE Transactions on Automatic Control*, vol. 38, no. 1, pp. 121–125, 1993.

[18] P. Gahinet and P. Apkarian, "A linear matrix inequality approach to h∞ control," *International Journal of Robust and Nonlinear Control*, vol. 4, no. 4, pp. 421–448, 1994.

[19] L. Hladowski, Z. Cai, K. Galkowski et al., "Repetitive process based iterative learning control designed by LMIs and experimentally verified on a gantry robot," in *American Control Conference (ACC '09)*, pp. 949–954, Institute of Control and Computation Engineering of the University of Zielona, June 2009.

[20] R. C. Dorf and R. H. Bishop, *Modern Control Systems*, Prentice Hall, 10th edition, 2004.

Advances in Operations Research
Advances in Decision Sciences
Journal of Applied Mathematics
Algebra
Journal of Probability and Statistics
The Scientific World Journal
International Journal of Differential Equations
Hindawi
Submit your manuscripts at
http://www.hindawi.com
International Journal of Combinatorics
Advances in Mathematical Physics
Journal of Complex Analysis
Journal of Mathematics
Mathematical Problems in Engineering
Abstract and Applied Analysis
Discrete Dynamics in Nature and Society
International Journal of Mathematics and Mathematical Sciences
Journal of Discrete Mathematics
Journal of Function Spaces
International Journal of Stochastic Analysis
Journal of Optimization