Research Article

Exponential Stability for a Class of Switched Nonlinear Systems with Mixed Time-Varying Delays via an Average Dwell-Time Method

N. Yotha,^{1,2} T. Botmart,^{2,3} and T. Mouktonglang^{1,2}

¹ *Department of Mathematics, Chiang Mai University, Chiang Mai 50200, Thailand*

² *Centre of Excellence in Mathematics, CHE, Si Ayutthaya Road, Bangkok 10400, Thailand*

³ *Department of Mathematics, Srinakharinwirot University, Bangkok 10110, Thailand*

Correspondence should be addressed to T. Botmart, thongchaib@swu.ac.th

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The problem of exponential stability for a class of switched nonlinear systems with discrete and distributed time-varying delays is studied. The constraint on the derivative of the time-varying delay is not required which allows the time delay to be a fast time-varying function. We study the stability properties of switched nonlinear systems consisting of both stable and unstable subsystems. Average dwell-time approached and improved piecewise Lyapunov functional combined with Leibniz-Newton are formulated. New delay-dependent sufficient conditions for the exponential stabilization of the switched systems are first established in terms of LMIs. A numerical example is also given to illustrate the effectiveness of the proposed method.

1. Introduction

The switched systems are an important class of hybrid systems. They are described by a family of continuous or discrete-time subsystems and a rule that orchestrates the switching between the subsystems. Recently, switched systems have attracted much attention due to the widespread application in control, chemical engineering processing [1], communication networks, traffic control [2, 3], and control of manufacturing systems [4–6]. A switched nonlinear system with time delay is called switched nonlinear delay system, where delay may be contained in the system state, control input, or switching signals. In [7–9], some stability properties of switched linear delay systems composed of both stable and unstable subsystems have been studied by using an average dwell-time approach and piecewise

Lyapunov functions. It is shown that when the average dwell time is sufficiently large and the total activation time of the unstable subsystems is relatively small compared with that of the Hurwitz stable subsystems, global exponential stability is guaranteed. The concept of dwell time was extended to average dwell time by Hespanha and Morse [10] with switching among stable subsystems. Furthermore, [11] generalized the results to the case where stable and unstable subsystems coexist.

The stability analysis of nonlinear time-delay systems has received increasing attention. Time-delay systems are frequently encountered in various areas such as chemical engineering systems, biological modeling, and economics. The stability analysis for nonlinear time-delay systems has been investigated extensively. Various approaches to such problems have been proposed, see [12–14] and the references therein. It is well known that the existences of time delay in a system may cause instability and oscillations system. Thus, the stability analysis of nonlinear time-delay systems has received considerable attention for the last few decades, see [15, 16].

Recently, the exponential stability problems for time-delay systems have been studied by many researchers, see for examples [17–21]. The problems have been dealt with for various control areas such as exponential stabilization for linear time-delay systems [22], exponential stabilization for uncertain time-delay systems [21, 23]. Hien and Phat [23] presented exponential stability and stabilization conditions for a class of uncertain linear system with time-varying delay, based on an improved Lyapunov-Krasovskii functional combined with Leibniz-Newton formula. The robust stability conditions are derived in terms of LMIs, but the time-varying delays are required to be differentiable and the lower bound is restricted to zero. However, in most cases, these conditions are difficult to satisfy. Therefore, in this paper, we will employ some new techniques so that the above conditions can be removed.

Stability analysis of nonlinear systems with distributed delays is of both practical and theoretical importance. For some systems, delay phenomena may not be simply considered as delays in the velocity terms and/or discrete delays in the states. Therefore, it is desirable to extend the system model to include distributed delays. Practical applications, modeled by systems with distributed delays, can be found in [24–26]. In [27], Gao et al. studied the problem of linear switching systems with discrete and continuous time delay. By constructing Lyapunov functional under a condition on the time delay, we show that it stabilizes the system for sufficiently small delays.

In this paper, the problem of exponential stability for a class of switched nonlinear systems with discrete and distributed time-varying delays is studied. The discrete time delay is a continuous function belonging to a given interval, which means that the lower and upper bounds for the time-varying delay are available. However, the delay function is not necessary to be differentiable. We study the stability properties of switched nonlinear systems consisting of both stable and unstable subsystems. Average dwell-time approached and improved piecewise Lyapunov functional combined with Leibniz-Newton's formula, and new delay-dependent sufficient conditions for the exponential stabilization of the switched systems are first established in terms of LMIs. A numerical example is also given to illustrate the effectiveness of the proposed method.

The rest of this paper is organized as follows. In Section 2, we give notations, definition, propositions, and lemma which will be used in the proof of the main results. Delay-dependent sufficient conditions for the exponential stability of switched nonlinear time-varying delay systems are presented in Section 3. A Numerical example illustrated the obtained results is given in Section 4. The paper ends with conclusions in Section 5 and cited references.

2. Preliminaries

The following notation will be used in this paper. $\mathbb{R}^+$ denotes the set of all real nonnegative numbers. $\mathbb{R}^n$ denotes the n -dimensional space and the vector norm $\|\cdot\|$. $M^{n \times r}$ denotes the space of all $(n \times r)$ matrices.

We also let A^T denote the transpose of matrix A : A is symmetric if $A = A^T$; I denotes the identity matrix. $\lambda(A)$ denotes the set of all eigenvalues of A . $\lambda_{\max}(A) = \max\{\operatorname{Re} \lambda : \lambda \in \lambda(A)\}$; $x_t := \{x(t+s) : s \in [-h, 0]\}$, $\|x_t\| = \sup_{s \in [-h, 0]} \|x(t+s)\|$; $C([0, t], \mathbb{R}^n)$ denotes the set of all $\mathbb{R}^n$ -valued continuous functions on $[0, t]$. $L_2([0, t], \mathbb{R}^m)$ denotes the set of all the $\mathbb{R}^m$ -valued square integrable functions on $[0, t]$: matrix A is called semipositive definite ($A \geq 0$) if $\langle Ax, x \rangle \geq 0$, for all $x \in \mathbb{R}^n$; A is positive definite ($A > 0$) if $\langle Ax, x \rangle > 0$ for all $x \neq 0$; $A > B$ means $A - B > 0$: the symmetric term in a matrix is denoted by $*$.

Consider a nonlinear system with interval time-varying and distributed delays of the form

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t) + D_{\sigma(t)}x(t - h_{\sigma(t)}(t)) + C_{\sigma(t)} \int_{t-k_{\sigma(t)}(t)}^t x(s)ds \\ &\quad + E_{\sigma(t)}f_{\sigma(t)}\left(t, x(t), x(t - h_{\sigma(t)}(t)), \int_{t-k_{\sigma(t)}(t)}^t x(s)ds\right), \\ x(t) &= \phi(t), \quad t \in [-\tau, 0], \quad \tau = \max\{h_2, k\}, \end{aligned} \quad (2.1)$$

where $x(t) \in \mathbb{R}^n$ is the state. $A_{\sigma(t)}, C_{\sigma(t)}, D_{\sigma(t)}$, and $E_{\sigma(t)}$ are given matrices of appropriate dimensions. $\sigma(t) : [0, +\infty) \rightarrow M = \{1, 2, \dots, m\}$ is piecewise constant and left continuous and is called a switching signal and $\phi(t) \in C([-h_2, 0], \mathbb{R}^n)$ is the initial function with the norm

$$\|\phi\| = \sup_{s \in [-\tau, 0]} \sqrt{\|\phi(s)\|^2 + \|\dot{\phi}(s)\|^2}. \quad (2.2)$$

The time delays $h(t)$ and $k(t)$ denoted the discrete time-varying delay and distributed time-varying delay, respectively. They are assumed to satisfy the following conditions:

$$0 \leq h_{1\sigma(t)} \leq h_{\sigma(t)}(t) \leq h_{2\sigma(t)}, \quad 0 \leq k_{\sigma(t)}(t) \leq k_{\sigma(t)}. \quad (2.3)$$

The nonlinear perturbation satisfies the following growth condition:

$$\begin{aligned} \exists a_i, b_i, c_i > 0, \quad \|f_i(t, x, y, z)\| &\leq a_i\|x\| + b_i\|y\| + c_i\|z\|, \\ \forall (t, x, y, z) &\in \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n, \quad i \in M. \end{aligned} \quad (2.4)$$

We introduce the following technical well-known propositions, which will be used in the proof of our results.

Definition 2.1 (see [7]). For any switching signal $\sigma(t)$ and any $T_2 > T_1 \geq 0$, let $N_{\sigma}(T_1, T_2)$ denote the number of switching of $\sigma(t)$ over the interval (T_1, T_2) . For given $T_a > 0, N_0 \geq 0$,

if the inequality $N_\sigma(T_1, T_2) \leq N_0 + ((T_2 - T_1)/T_a)$ holds, then T_a is call average dwell time and N_0 is called chattering bound. As commonly used in the literature, for convenience, we choose $N_0 = 0$ in this paper.

Proposition 2.2 (Cauchy inequality). *For any symmetric positive definite matrix $N \in M^{n \times n}$ and $x, y \in \mathbb{R}^n$ one has*

$$\pm 2x^T y \leq x^T N x + y^T N^{-1} y. \quad (2.5)$$

Proposition 2.3 (see [28]). *For any symmetric positive definite matrix $M > 0$, scalar $\gamma > 0$ and vector function $\omega : [0, \gamma] \rightarrow \mathbb{R}^n$ such that the integrations concerned are well defined, the following inequality holds:*

$$\left(\int_0^\gamma \omega(s) ds \right)^T M \left(\int_0^\gamma \omega(s) ds \right) \leq \gamma \left(\int_0^\gamma \omega^T(s) M \omega(s) ds \right). \quad (2.6)$$

Proposition 2.4 (see [28] (Schur complement lemma)). *Given constant symmetric matrices X, Y, Z with appropriate dimensions satisfying $X = X^T, Y = Y^T > 0$. Then $X + Z^T Y^{-1} Z < 0$ if and only if*

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0 \quad \text{or} \quad \begin{pmatrix} -Y & Z \\ Z^T & X \end{pmatrix} < 0. \quad (2.7)$$

3. Main Results

3.1. Stability of Switched Nonlinear Time-Varying Delay Systems

First, we present a delay-dependent exponential stabilizability analysis conditions for the given nonlinear time-varying delay systems (2.1). We consider the case when the stable and unstable subsystems coexist. We assume that the subsystem i ($1 \leq i \leq k$) is stable, where the positive integer k satisfies $1 \leq k \leq m$ and the subsystem j ($k+1 \leq j \leq m$) is unstable. Let us set $\eta_i = (h_{2i} - h_{1i})$, $i = 1, 2, \dots, m$.

For the system (2.1), given $\alpha > 0$, the following lemma provides a change estimation of Lyapunov-Krasovskii functional candidate:

$$V_i(x(t)) = V_{i1} + V_{i2} + V_{i3} + V_{i4} + V_{i5} + V_{i6} + V_{i7}, \quad (3.1)$$

where

$$V_{i1} = x^T(t) Y_i x(t),$$

$$V_{i2} = \int_{t-h_{1i}}^t e^{\alpha(s-t)} x^T(s) Y_i Q_i Y_i x(s) ds,$$

$$V_{i3} = \int_{t-h_{2i}}^t e^{\alpha(s-t)} x^T(s) Y_i Q_i Y_i x(s) ds,$$

$$\begin{aligned}
V_{i4} &= h_{1i} \int_{-h_{1i}}^0 \int_{t+s}^t e^{\alpha(\tau-t)} \dot{x}^T(\tau) Y_i R_i Y_i \dot{x}(\tau) d\tau ds, \\
V_{i5} &= h_{2i} \int_{-h_{2i}}^0 \int_{t+s}^t e^{\alpha(\tau-t)} \dot{x}^T(\tau) Y_i R_i Y_i \dot{x}(\tau) d\tau ds, \\
V_{i6} &= \eta_i \int_{-h_{2i}}^{-h_{1i}} \int_{t+s}^t e^{\alpha(\tau-t)} \dot{x}^T(\tau) Y_i U_i Y_i \dot{x}(\tau) d\tau ds, \\
V_{i7} &= \int_{k_i}^0 \int_{t+s}^t e^{\alpha(\tau-t)} x^T(\tau) Y_i S_i Y_i x(\tau) d\tau ds.
\end{aligned} \tag{3.2}$$

Lemma 3.1. For a given constant $\alpha > 0$, suppose (2.3) holds. If there exist symmetric positive definite matrices P_i, Q_i, R_i, U_i, S_i such that the following LMI holds:

$$\begin{aligned}
\Sigma_{1i} &= W_i - [0 \ 0 \ 0 \ -I \ I \ 0 \ 0 \ 0]^T \\
&\quad \times e^{-\alpha h_{2i}} U_i [0 \ 0 \ 0 \ -I \ I \ 0 \ 0 \ 0] < 0,
\end{aligned} \tag{3.3}$$

$$\begin{aligned}
\Sigma_{2i} &= W_i - [0 \ 0 \ I \ 0 \ -I \ 0 \ 0 \ 0]^T \\
&\quad \times e^{-\alpha h_{2i}} U_i [0 \ 0 \ I \ 0 \ -I \ 0 \ 0 \ 0] < 0,
\end{aligned} \tag{3.4}$$

$$\Sigma_{3i} = \begin{bmatrix} \Psi_i & k_i C_i P_i & 2a_i P_i E_i^T & k_i c_i^2 E_i P_i \\ * & \frac{-k_i e^{-\alpha k_i} S_i}{4} & 0 & 0 \\ * & * & -2a_i I & 0 \\ * & * & * & \frac{-k_i c_i^2 e^{-\alpha k_i} S_i}{4} \end{bmatrix} < 0, \tag{3.5}$$

$$W_i = \begin{bmatrix} W_{11i} & W_{12i} & W_{13i} & W_{14i} & W_{15i} & 0 & 0 & 0 \\ * & W_{22i} & 0 & 0 & W_{25i} & k_i C_i P_i & k_i c_i^2 E_i P_i & 0 \\ * & * & W_{33i} & 0 & W_{35i} & 0 & 0 & 0 \\ * & * & * & W_{44i} & W_{45i} & 0 & 0 & 0 \\ * & * & * & * & W_{55i} & 0 & 0 & 2b_i P_i E_i^T \\ * & * & * & * & * & W_{66i} & 0 & 0 \\ * & * & * & * & * & * & W_{77i} & 0 \\ * & * & * & * & * & * & * & -2b_i I \end{bmatrix}, \tag{3.6}$$

$$i = 1, 2, 3, \dots, k,$$

where

$$\Psi_i = -0.1e^{-\alpha h_{1i}} R_i - 0.1e^{-\alpha h_{2i}} R_i,$$

$$W_{11i} = [A_i + \alpha I] P_i + P_i [A_i + \alpha I]^T - 0.9(e^{-\alpha h_{1i}} + e^{-\alpha h_{2i}}) R_i + 2Q_i + k_i S_i + (a_i + b_i) I,$$

$$W_{12i} = P_i A_i^T,$$

$$\begin{aligned}
W_{13i} &= e^{-\alpha h_{1i}} R_i, \\
W_{14i} &= e^{-\alpha h_{2i}} R_i, \\
W_{15i} &= D_i P_i, \\
W_{22i} &= \left(h_{1i}^2 + h_{2i}^2 \right) R_i + \eta_i^2 U_i - 2P_i + (a_i + b_i) I, \\
W_{25i} &= D_i P_i, \\
W_{33i} &= -e^{-\alpha h_{1i}} (Q_i + R_i) - e^{-\alpha h_{2i}} U_i, \\
W_{35i} &= e^{-\alpha h_{2i}} U_i, \\
W_{44i} &= -e^{-\alpha h_{2i}} (Q_i + R_i + U_i), \\
W_{45i} &= e^{-\alpha h_{2i}} U_i, \\
W_{55i} &= -2e^{-\alpha h_{2i}} U_i, \\
W_{66i} &= -\frac{k_i e^{-\alpha k_i} S_i}{4}, \\
W_{77i} &= -\frac{k_i c_i^2 e^{-\alpha k_i} S_i}{4}.
\end{aligned} \tag{3.7}$$

Then, along the trajectory of system (2.1), one has

$$V_i(x(t)) \leq e^{-\alpha(t-t_0)} V_i(x(t_0)), \quad t \geq t_0 \geq 0. \tag{3.8}$$

Proof. Let $Y_i = P_i^{-1}$, $y(t) = Y_i x(t)$. We consider the following Lyapunov-Krasovskii functional (3.1). By taking the derivative of Lyapunov-Krasovskii functional candidate (3.1) along the trajectory of the system (2.1) we have

$$\begin{aligned}
\dot{V}_{i1} &= y^T(t) \left[P_i A_i^T + A_i P_i \right] y(t) + 2y^T(t) D_i P_i y(t - h_i(t)) + 2y^T(t) C_i P_i \int_{t-k_i(t)}^t y(s) ds \\
&\quad + 2y^T(t) E_i f_i \left(t, x(t), x(t - h_i(t)), \int_{t-k_i(t)}^t x(s) ds \right), \\
\dot{V}_{i2} &= y^T(t) Q_i y(t) - e^{-\alpha h_{1i}} y^T(t - h_{1i}) Q_i y(t - h_{1i}) - \alpha V_{i2}, \\
\dot{V}_{i3} &= y^T(t) Q_i y(t) - e^{-\alpha h_{2i}} y^T(t - h_{2i}) Q_i y(t - h_{2i}) - \alpha V_{i3}, \\
\dot{V}_{i4} &\leq h_{1i}^2 \dot{y}^T(t) R_i \dot{y}(t) - h_{1i} e^{-\alpha h_{1i}} \int_{t-h_{1i}}^t \dot{y}^T(s) R_i \dot{y}(s) ds - \alpha V_{i4},
\end{aligned}$$

$$\begin{aligned}
\dot{V}_{i5} &\leq h_{2i}^2 \dot{y}^T(t) R_i \dot{y}(t) - h_{2i} e^{-\alpha h_{2i}} \int_{t-h_{2i}}^t \dot{y}^T(s) R_i \dot{y}(s) ds - \alpha V_{i5}, \\
\dot{V}_{i6} &\leq \eta_i^2 \dot{y}^T(t) U_i \dot{y}(t) - \eta_i e^{-\alpha h_{2i}} \int_{t-h_{2i}}^{t-h_{1i}} \dot{y}^T(s) U_i \dot{y}(s) ds - \alpha V_{i6}, \\
\dot{V}_{i7} &\leq k_i y^T(t) S_i y(t) - e^{-\alpha k_i} \int_{t-k_i(t)}^t y^T(s) S_i y(s) ds - \alpha V_{i7}.
\end{aligned} \tag{3.9}$$

Applying Proposition 2.3 and the Leibniz-Newton formula, we have

$$\begin{aligned}
-h_1 \int_{t-h_{1i}}^t \dot{y}^T(s) R_i \dot{y}(s) ds &\leq - \left[\int_{t-h_{1i}}^t \dot{y}(s) ds \right]^T R_i \left[\int_{t-h_{1i}}^t \dot{y}(s) ds \right] \\
&\leq - [y(t) - y(t-h_{1i})]^T R_i [y(t) - y(t-h_{1i})] \\
&= -y^T(t) R_i y(t) + 2y^T(t) R_i y(t-h_{1i}) - y^T(t-h_{1i}) R_i y(t-h_{1i}), \\
-h_2 \int_{t-h_{2i}}^t \dot{y}^T(s) R_i \dot{y}(s) ds &\leq - \left[\int_{t-h_{2i}}^t \dot{y}(s) ds \right]^T R_i \left[\int_{t-h_{2i}}^t \dot{y}(s) ds \right] \\
&\leq - [y(t) - y(t-h_{2i})]^T R_i [y(t) - y(t-h_{2i})] \\
&= -y^T(t) R_i y(t) + 2y^T(t) R_i y(t-h_{2i}) - y^T(t-h_{2i}) R_i y(t-h_{2i}).
\end{aligned} \tag{3.10}$$

Note that

$$\begin{aligned}
-(h_{2i} - h_{1i}) \int_{t-h_{2i}}^{t-h_{1i}} \dot{y}^T(s) U_i \dot{y}(s) ds &= -(h_{2i} - h_{1i}) \int_{t-h_{2i}}^{t-h_i(t)} \dot{y}^T(s) U_i \dot{y}(s) ds \\
&\quad - (h_{2i} - h_{1i}) \int_{t-h_i(t)}^{t-h_{1i}} \dot{y}^T(s) U_i \dot{y}(s) ds \\
&= -(h_{2i} - h_i(t)) \int_{t-h_{2i}}^{t-h_i(t)} \dot{y}^T(s) U_i \dot{y}(s) ds \\
&\quad - (h_i(t) - h_{1i}) \int_{t-h_{2i}}^{t-h_i(t)} \dot{y}^T(s) U_i \dot{y}(s) ds \\
&\quad - (h_i(t) - h_{1i}) \int_{t-h_i(t)}^{t-h_{1i}} \dot{y}^T(s) U_i \dot{y}(s) ds \\
&\quad - (h_{2i} - h_i(t)) \int_{t-h_i(t)}^{t-h_{1i}} \dot{y}^T(s) U_i \dot{y}(s) ds.
\end{aligned} \tag{3.11}$$

Using Proposition 2.3 gives

$$\begin{aligned}
-(h_{2i} - h_i(t)) \int_{t-h_{2i}}^{t-h_i(t)} \dot{y}^T(s) U_i \dot{y}(s) ds &\leq - \left[\int_{t-h_{2i}}^{t-h_i(t)} \dot{y}(s) ds \right]^T U_i \left[\int_{t-h_{2i}}^{t-h_i(t)} \dot{y}(s) ds \right] \\
&\leq - [y(t - h_i(t)) - y(t - h_{2i})]^T U_i [y(t - h_i(t)) - y(t - h_{2i})], \\
-(h_i(t) - h_{1i}) \int_{t-h_i(t)}^{t-h_{1i}} \dot{y}^T(s) U_i \dot{y}(s) ds &\leq - \left[\int_{t-h_i(t)}^{t-h_{1i}} \dot{y}(s) ds \right]^T U_i \left[\int_{t-h_i(t)}^{t-h_{1i}} \dot{y}(s) ds \right] \\
&\leq - [y(t - h_{1i}) - y(t - h_i(t))]^T U_i [y(t - h_{1i}) - y(t - h_i(t))].
\end{aligned} \tag{3.12}$$

Let $\beta_i = ((h_{2i} - h_i(t)) / (h_{2i} - h_{1i})) \leq 1$. Then

$$\begin{aligned}
-(h_{2i} - h_i(t)) \int_{t-h_{2i}}^{t-h_{1i}} \dot{y}^T(s) U_i \dot{y}(s) ds &= -\beta_i \int_{t-h_i(t)}^{t-h_{1i}} (h_{2i} - h_{1i}) \dot{y}^T(s) U_i \dot{y}(s) ds \\
&\leq -\beta_i \int_{t-h_i(t)}^{t-h_{1i}} (h_i(t) - h_{1i}) \dot{y}^T(s) U_i \dot{y}(s) ds \\
&\leq -\beta_i [y(t - h_{1i}) - y(t - h_i(t))]^T U_i [y(t - h_{1i}) - y(t - h_i(t))] \\
-(h_i(t) - h_{1i}) \int_{t-h_{2i}}^{t-h_i(t)} \dot{y}^T(s) U_i \dot{y}(s) ds &= -(1 - \beta_i) \int_{t-h_{2i}}^{t-h_i(t)} (h_{2i} - h_{1i}) \dot{y}^T(s) U_i \dot{y}(s) ds \\
&\leq -(1 - \beta_i) \int_{t-h_{2i}}^{t-h_i(t)} (h_{2i} - h(t)) \dot{y}^T(s) U_i \dot{y}(s) ds \\
&\leq -(1 - \beta_i) [y(t - h_i(t)) - y(t - h_{2i})]^T \\
&\quad \times U_i [y(t - h_i(t)) - y(t - h_{2i})].
\end{aligned} \tag{3.13}$$

Therefore, from (3.12) and (3.13), we obtain

$$\begin{aligned}
&-(h_{2i} - h_{1i}) \int_{t-h_{2i}}^{t-h_{1i}} \dot{y}^T(s) U_i \dot{y}(s) ds \\
&\leq - [y(t - h_i(t)) - y(t - h_{2i})]^T U_i [y(t - h_i(t)) - y(t - h_{2i})] \\
&\quad - [y(t - h_{1i}) - y(t - h_i(t))]^T U_i [y(t - h_{1i}) - y(t - h_i(t))] \\
&\quad - \beta_i [y(t - h_{1i}) - y(t - h_i(t))]^T U_i [y(t - h_{1i}) - y(t - h_i(t))] \\
&\quad - (1 - \beta_i) [y(t - h_i(t)) - y(t - h_{2i})]^T U_i [y(t - h_i(t)) - y(t - h_{2i})].
\end{aligned} \tag{3.14}$$

By using the following identity relation

$$\begin{aligned}
 & P_i \dot{y}(t) - A_i P_i y(t) - D_i P_i y(t - h_i(t)) - C_i P_i \int_{t-k_i(t)}^t y(s) ds \\
 & - E_i f_i \left(t, x(t), x(t - h_i(t)), \int_{t-k_i(t)}^t x(s) ds \right) = 0,
 \end{aligned} \tag{3.15}$$

we have

$$\begin{aligned}
 & -2\dot{y}^T(t) P_i \dot{y}(t) + 2\dot{y}^T(t) A_i P_i y(t) + 2\dot{y}^T(t) D_i P_i y(t - h_i(t)) + 2\dot{y}^T(t) C_i P_i \int_{t-k_i(t)}^t y(s) ds \\
 & + 2\dot{y}^T(t) E_i f_i \left(t, x(t), x(t - h_i(t)), \int_{t-k_i(t)}^t x(s) ds \right) = 0.
 \end{aligned} \tag{3.16}$$

Applying Propositions 2.2 and 2.3 gives

$$\begin{aligned}
 2\dot{y}^T(t) C_i P_i \int_{t-k_i(t)}^t y(s) ds & \leq 4k_i e^{\alpha k_i} \dot{y}^T(t) C_i P_i S_i^{-1} P_i C_i^T y(t) \\
 & + \frac{1}{4k_i} e^{-\alpha k_i} \left(\int_{t-k_i(t)}^t y(s) ds \right)^T S_i \left(\int_{t-k_i(t)}^t y(s) ds \right) \\
 & \leq 4k_i e^{\alpha k_i} \dot{y}^T(t) C_i P_i S_i^{-1} P_i C_i^T y(t) \\
 & + \frac{1}{4} e^{-\alpha k_i} \int_{t-k_i(t)}^t \dot{y}^T(s) S_i y(s) ds, \\
 2\dot{y}^T(t) C_i P_i \int_{t-k_i(t)}^t y(s) ds & \leq 4k_i e^{\alpha k_i} \dot{y}^T(t) C_i P_i S_i^{-1} P_i C_i^T \dot{y}(t) \\
 & + \frac{1}{4k_i} e^{-\alpha k_i} \left(\int_{t-k_i(t)}^t y(s) ds \right)^T S_i \left(\int_{t-k_i(t)}^t y(s) ds \right) \\
 & \leq 4k_i e^{\alpha k_i} \dot{y}^T(t) C_i P_i S_i^{-1} P_i C_i^T \dot{y}(t) \\
 & + \frac{1}{4} e^{-\alpha k_i} \int_{t-k_i(t)}^t \dot{y}^T(s) S_i y(s) ds.
 \end{aligned} \tag{3.17}$$

By using Proposition 2.2 and condition (2.4), we have

$$\begin{aligned}
& 2\mathbf{y}^T(t)E_i f_i\left(t, \mathbf{x}(t), \mathbf{x}(t-h(t)), \int_{t-k(t)}^t \mathbf{x}(s)ds\right) \\
& \leq 2\|\mathbf{y}(t)\|E_i\left\|f_i\left(t, \mathbf{x}(t), \mathbf{x}(t-h(t)), \int_{t-k(t)}^t \mathbf{x}(s)ds\right)\right\| \\
& \leq 2\|\mathbf{y}(t)\|E_i\left(a_i\|\mathbf{x}\| + b_i\|\mathbf{x}(t-h_i(t))\| + c_i\left\|\int_{t-k_i(t)}^t \mathbf{x}(s)ds\right\|\right) \\
& = 2a_i\|\mathbf{y}(t)\|\|E_i P_i \mathbf{y}(t)\| + 2b_i\|\mathbf{y}(t)\|\|E_i P_i \mathbf{y}(t-h_i(t))\| + 2c_i\|\mathbf{y}(t)\|\left\|E_i P_i \int_{t-k_i(t)}^t \mathbf{y}(s)ds\right\| \\
& \leq (a_i + b_i)\mathbf{y}^T(t)\mathbf{y}(t) + a_i\mathbf{y}^T(t)P_i E_i^T E_i P_i \mathbf{y}(t) + b_i\mathbf{y}^T(t-h_i(t))P_i E_i^T E_i P_i \mathbf{y}(t-h_i(t)) \\
& \quad + 4k_i c_i^2 e^{\alpha k_i} \mathbf{y}^T(t)E_i P_i S_i^{-1} P_i E_i^T \mathbf{y}(t) + \frac{1}{4}e^{-\alpha k_i} \int_{t-k_i(t)}^t \mathbf{y}^T(s)S_i \mathbf{y}(s)ds, \\
& 2\dot{\mathbf{y}}^T(t)E_i f_i\left(t, \mathbf{x}(t), \mathbf{x}(t-h(t)), \int_{t-k(t)}^t \mathbf{x}(s)ds\right) \\
& \leq (a_i + b_i)\dot{\mathbf{y}}^T(t)\dot{\mathbf{y}}(t) + a_i\dot{\mathbf{y}}^T(t)P_i E_i^T E_i P_i \dot{\mathbf{y}}(t) + b_i\dot{\mathbf{y}}^T(t-h_i(t))P_i E_i^T E_i P_i \dot{\mathbf{y}}(t-h_i(t)) \\
& \quad + 4k_i c_i^2 e^{\alpha k_i} \dot{\mathbf{y}}^T(t)E_i P_i S_i^{-1} P_i E_i^T \dot{\mathbf{y}}(t) + \frac{1}{4}e^{-\alpha k_i} \int_{t-k_i(t)}^t \dot{\mathbf{y}}^T(s)S_i \dot{\mathbf{y}}(s)ds.
\end{aligned} \tag{3.18}$$

Hence, according to (3.10)–(3.18), we have

$$\begin{aligned}
\dot{V}_i(\mathbf{x}(t)) + \alpha V_i(\mathbf{x}(t)) & \leq \zeta^T(t)W_i \zeta(t) \\
& \quad - \beta_i [\mathbf{y}(t-h_{1i}) - \mathbf{y}(t-h_i(t))]^T e^{-\alpha h_{2i}} \mathcal{U}_i [\mathbf{y}(t-h_{1i}) - \mathbf{y}(t-h_i(t))] \\
& \quad - (1-\beta_i) [\mathbf{y}(t-h_i(t)) - \mathbf{y}(t-h_{2i})]^T e^{-\alpha h_{2i}} \mathcal{U}_i [\mathbf{y}(t-h_i(t)) - \mathbf{y}(t-h_{2i})] \\
& = \zeta^T(t) [(1-\beta_i)\Sigma_{1i} + \beta_i \Sigma_{2i}] \zeta(t) + \mathbf{y}^T(t)\Sigma_i \mathbf{y}(t),
\end{aligned} \tag{3.19}$$

where Σ_{1i} and Σ_{2i} are defined in (3.3), (3.4), respectively, and

$$\begin{aligned}
\Sigma_i & = \Psi_i + 4k_i e^{\alpha k_i} C_i P_i S_i^{-1} P_i C_i^T + 2a_i P_i E_i^T E_i P_i + 4k_i c_i^2 e^{\alpha k_i} E_i P_i S_i^{-1} P_i E_i^T, \\
\zeta(t) & = [\mathbf{y}(t), \dot{\mathbf{y}}(t), \mathbf{y}(t-h_{1i}), \mathbf{y}(t-h_{2i}), \mathbf{y}(t-h(t))].
\end{aligned} \tag{3.20}$$

Since $0 \leq \beta_i \leq 1$, $(1 - \beta_i)\Sigma_{1i} + \beta_i\Sigma_{2i}$ is a convex combination of Σ_{1i} and Σ_{2i} , therefore, $(1 - \beta_i)\Sigma_{1i} + \beta_i\Sigma_{2i} < 0$ is equivalent to $\Sigma_{1i} < 0$ and $\Sigma_{2i} < 0$. Applying Shur complement lemma, the inequalities Σ_i is equivalent to $\Sigma_{3i} < 0$. Thus, it follows from (3.3)–(3.5) and (3.19), we obtain

$$\dot{V}(x(t)) + \alpha V(x(t)) \leq 0, \quad t \geq t_0 \geq 0. \quad (3.21)$$

Thus, by the above differential inequality, we have

$$V_i(x(t)) \leq e^{-\alpha(t-t_0)} V_i(x(t_0)), \quad t \geq t_0 \geq 0. \quad (3.22)$$

□

For the system (2.1) and given $\beta > 0$, the following lemma provides a change estimate of Lyapunov-Krasovskii functional candidate:

$$V_j(x(t)) = V_{j1} + V_{j2} + V_{j3} + V_{j4} + V_{j5} + V_{j6} + V_{j7}, \quad (3.23)$$

where

$$\begin{aligned} V_{j1} &= x^T(t) Y_j x(t), \\ V_{j2} &= \int_{t-h_{1j}}^t e^{\beta(t-s)} x^T(s) Y_j Q_j Y_j x(s) ds, \\ V_{j3} &= \int_{t-h_{2j}}^t e^{\beta(t-s)} x^T(s) Y_j Q_j Y_j x(s) ds, \\ V_{j4} &= h_{1i} \int_{-h_{1j}}^0 \int_{t+s}^t e^{\beta(t-\tau)} \dot{x}^T(\tau) Y_j R_j Y_j \dot{x}(\tau) d\tau ds, \\ V_{j5} &= h_{2j} \int_{-h_{2j}}^0 \int_{t+s}^t e^{\beta(t-\tau)} \dot{x}^T(\tau) Y_j R_j Y_j \dot{x}(\tau) d\tau ds, \\ V_{j6} &= \eta_j \int_{-h_{2j}}^{-h_{1j}} \int_{t+s}^t e^{\beta(t-\tau)} \dot{x}^T(\tau) Y_j U_j Y_j \dot{x}(\tau) d\tau ds, \\ V_{j7} &= \int_{k_j}^0 \int_{t+s}^t e^{\beta(t-\tau)} x^T(\tau) Y_j S_j Y_j x(\tau) d\tau ds. \end{aligned} \quad (3.24)$$

Lemma 3.2. For a given constant $\beta > 0$, suppose (2.3) holds. If there exist symmetric positive definite matrices P_j, Q_j, R_j, U_j, S_j such that the following LMI holds:

$$\begin{aligned} \Sigma_{1j} &= W_j - [0 \ 0 \ 0 \ -I \ I \ 0 \ 0 \ 0]^T \\ &\quad \times e^{\beta h_{2j}} U_j [0 \ 0 \ 0 \ -I \ I \ 0 \ 0 \ 0] < 0, \end{aligned} \quad (3.25)$$

$$\begin{aligned} \Sigma_{2j} &= W_j - [0 \ 0 \ I \ 0 \ -I \ 0 \ 0 \ 0]^T \\ &\quad \times e^{\beta h_{2j}} U_j [0 \ 0 \ I \ 0 \ -I \ 0 \ 0 \ 0] < 0, \end{aligned} \quad (3.26)$$

$$\Sigma_{3j} = \begin{bmatrix} \Psi_j & k_j C_j P_j & 2a_j P_j E_j^T & k_j c_j^2 E_j P_j \\ * & \frac{-k_j e^{\beta k_j} S_j}{4} & 0 & 0 \\ * & * & -2a_j I & 0 \\ * & * & * & \frac{-k_j c_j^2 e^{\beta k_j} S_j}{4} \end{bmatrix} < 0, \quad (3.27)$$

$$W_j = \begin{bmatrix} W_{11j} & W_{12j} & W_{13j} & W_{14j} & W_{15j} & 0 & 0 & 0 \\ * & W_{22j} & 0 & 0 & W_{25j} & k_j C_j P_j & k_j c_j^2 E_j P_j & 0 \\ * & * & W_{33j} & 0 & W_{35j} & 0 & 0 & 0 \\ * & * & * & W_{44j} & W_{45j} & 0 & 0 & 0 \\ * & * & * & * & W_{55j} & 0 & 0 & 2b_j P_j E_j^T \\ * & * & * & * & * & W_{66j} & 0 & 0 \\ * & * & * & * & * & * & W_{77j} & 0 \\ * & * & * & * & * & * & * & -2b_j I \end{bmatrix}, \quad (3.28)$$

$$j = k + 1, \dots, m,$$

where

$$\Psi_j = -0.1e^{\beta h_{1j}} R_j - 0.1e^{\beta h_{2j}} R_j,$$

$$W_{11j} = [A_j - \beta I] P_j + P_j [A_j - \beta I]^T - 0.9(e^{\beta h_{1j}} + e^{\beta h_{2j}}) R_j + 2Q_j + k_j S_j + (a_j + b_j) I,$$

$$W_{12j} = P_j A_j^T,$$

$$W_{13j} = e^{\beta h_{1j}} R_j,$$

$$W_{14j} = e^{\beta h_{2j}} R_j,$$

$$W_{15j} = D_j P_j,$$

$$W_{22j} = (h_{1j}^2 + h_{2j}^2) R_j + \eta_j^2 U_j - 2P_j + (a_j + b_j) I,$$

$$W_{25j} = D_j P_j,$$

$$W_{33j} = -e^{\beta h_{1j}} (Q_j + R_j) - e^{\beta h_{2j}} U_j,$$

$$W_{35j} = e^{\beta h_{2j}} U_j,$$

$$\begin{aligned}
W_{44j} &= -e^{\beta h_{2j}}(Q_j + R_j + U_j), \\
W_{45j} &= e^{\beta h_{2j}}U_j, \\
W_{55j} &= -2e^{\beta h_{2j}}U_j, \\
W_{66j} &= -\frac{k_j e^{\beta k_j} S_j}{4}, \\
W_{77j} &= -\frac{k_j c_j^2 e^{\beta k_j} S_j}{4}.
\end{aligned} \tag{3.29}$$

Then, along the trajectory of system (2.1), one has

$$V_j(x(t)) \leq e^{\beta(t-t_0)} V_j(x(t_0)), \quad t \geq t_0 \geq 0. \tag{3.30}$$

Proof. By taking the derivative of Lyapunov-Krasovskii functional candidate (3.23) along the trajectory of the system (2.1), we are able to do similar estimation as we did for the Lemma 3.1. We have the following:

$$\dot{V}_j(t) - \beta V_j(t) \leq 0, \quad t \geq t_0 \geq 0. \tag{3.31}$$

Thus, by the above differential inequality, we have

$$V_j(x(t)) \leq e^{\beta(t-t_0)} V_j(x(t_0)), \quad t \geq t_0 \geq 0. \tag{3.32}$$

□

From Lemmas 3.1 and 3.2, it is easy to show the following properties of the Lyapunov functional candidate (3.1) and (3.23).

(i) There exist scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, such that

$$\varepsilon_1 \|x(t)\|^2 \leq V_{\sigma(t)}(x(t)) \leq \varepsilon_2 \|x(t_0)\|_{cl}^2, \quad \sigma(t) \in M. \tag{3.33}$$

(ii) There exists a constant scalar $\mu \geq 1$ such that

$$V_i(x(t)) \leq \mu V_j(x(t)), \quad i, j \in M. \tag{3.34}$$

(iii) The Lyapunov functional candidate (3.1) and (3.23) whose derivative along the trajectory of the corresponding subnetwork satisfies

$$V(x(t)) \leq \begin{cases} e^{-\alpha(t-t_k)} V_i(x(t_k)), & \text{if } i \in 1, 2, \dots, k, \text{ for } t \in [t_k, t_{k+1}), \\ e^{\beta(t-t_k)} V_j(x(t_k)), & \text{if } j \in k+1, k+2, \dots, m, \text{ for } t \in [t_k, t_{k+1}). \end{cases} \tag{3.35}$$

Now, for any piecewise constant switching signal $\sigma(t)$ and any $0 \leq t_0 < t$, we let $T^-(t_0, t)$ (resp., $T^+(t_0, t)$) denote the total activation time of the stable subsystem (resp., the ones of unstable subsystem) during (t_0, t) . Then, we choose a scalar $\alpha^* \in (0, \alpha)$ arbitrarily to propose the following switching law:

(S1): determine the switching signal $\sigma(t)$ so that

$$\inf_{t \geq t_0} \frac{T^-(t_0, t)}{T^+(t_0, t)} \geq \frac{\beta + \alpha^*}{\alpha - \alpha^*} \quad (3.36)$$

holds on time interval (t_0, t) . Meanwhile, we choose $\alpha^* < \alpha$ as the average dwell-time scheme: for any $t > t_0$,

$$N_\sigma(t_0, t) \leq N_0 + \frac{t - t_0}{T_a}, \quad T_a \geq T_a^* = \frac{\ln \mu}{\alpha^*}. \quad (3.37)$$

Theorem 3.3. For a given constant $\alpha > 0$, $\beta > 0$ and time-varying delay satisfying (2.3), suppose that the subsystem $(1 \leq i \leq k)$ of switched system (2.1) satisfies the conditions of Lemma 3.1, and the others satisfy the conditions of Lemma 3.2. Then the system (2.1) is exponentially stable for switching signal satisfying (3.36), (3.37) and the state decay estimate is given by

$$\|x(t)\| \leq \sqrt{\frac{c_0 b_i}{a_i}} e^{-\lambda(t-t_0)} \|x(t_0)\|_{\text{cl}}, \quad (3.38)$$

where

$$\begin{aligned} c_0 &= e^{N_0 \ln \mu}, \quad \lambda = \frac{1}{2} \left(\alpha^* - \frac{\ln \mu}{T_a} \right), \quad \mathcal{L} = \min_{1 \leq i \leq m} \lambda_{\min}(P_i^{-1}), \\ \mathcal{M} &= \max_{1 \leq i \leq m} \lambda_{\max}(P_i^{-1}) + \max_{1 \leq i \leq m} 2h_{2i} \lambda_{\max}[P_i^{-1} Q_i P_i^{-1}] + \max_{1 \leq i \leq m} 2h_{2i}^3 \lambda_{\max}[P_i^{-1} R_i P_i^{-1}] \\ &\quad + \max_{1 \leq i \leq m} \eta_i^3 \lambda_{\max}[P_i^{-1} U_i P_i^{-1}] + \max_{1 \leq i \leq m} k_i^2 \lambda_{\max}[P_i^{-1} S_i P_i^{-1}], \end{aligned} \quad (3.39)$$

and there exists $\mu \geq 1$ such that

$$P_i \leq \mu P_j, \quad Q_i \leq \mu Q_j, \quad R_i \leq \mu R_j, \quad U_i \leq \mu U_j, \quad S_i \leq \mu S_j, \quad \forall i, j \in M. \quad (3.40)$$

Proof. Suppose that $t \in [t_k, t_{k+1})$. For piecewise Lyapunov functional candidate (3.1) and (3.23) along trajectory of network system (2.1), we have

$$V(x(t)) \leq \begin{cases} e^{-\alpha(t-t_k)} V_i(x(t_k)), & \text{if } i \in 1, 2, \dots, k, \\ e^{\beta(t-t_k)} V_j(x(t_k)), & \text{if } j \in k+1, k+2, \dots, m. \end{cases} \quad (3.41)$$

Since $V_{\sigma(t_k)}(x(t_k)) \leq \mu V_{\sigma(t_k^-)}(x(t_k^-))$ is true from (3.34) at the switching point t_k , where $t_k^- = \lim_{t \rightarrow t_k} t$ and the relation $k = N_{\sigma}(t_0, t) \leq N_0 + ((t - t_0)/T_a)$, we obtain

$$\begin{aligned}
 V(x(t)) &\leq e^{\beta T^+(t_k, t) - \alpha T^-(t_k, t)} V_{\sigma(t_k)}(x(t_k)) \\
 &\leq e^{\beta T^+(t_k, t) - \alpha T^-(t_k, t)} \mu V_{\sigma(t_k^-)}(x(t_k^-)) \\
 &\leq e^{\beta T^+(t_k, t) - \alpha T^-(t_k, t)} \mu e^{\beta T^+(t_{k-1}, t_k) - \alpha T^-(t_{k-1}, t_k)} V_{\sigma(t_{k-1})}(x(t_{k-1})) \\
 &\leq \mu e^{\beta T^+(t_{k-1}, t) - \alpha T^-(t_{k-1}, t)} V_{\sigma(t_{k-1})}(x(t_{k-1})) \\
 &\vdots \\
 &\leq \mu^k e^{\beta T^+(t_0, t) - \alpha T^-(t_0, t)} V_{\sigma(t_0)}(x(t_0)) \\
 &\leq e^{\beta T^+(t_0, t) - \alpha T^-(t_0, t) + (N_0 + ((t - t_0)/T_a)) \ln \mu} V_{\sigma(t_0)}(x(t_0)).
 \end{aligned} \tag{3.42}$$

Under the switching law (S1) for any t_0, t , we have

$$\begin{aligned}
 \beta T^+(t_0, t) - \alpha T^-(t_0, t) &\leq -\alpha^*(T^+(t_0, t) + T^-(t_0, t)) \\
 &= -\alpha^*(t - t_0).
 \end{aligned} \tag{3.43}$$

Thus,

$$\begin{aligned}
 V(x(t)) &\leq e^{N_0 \ln \mu} e^{-(\alpha^* - (\ln \mu / T_a))(t - t_0)} V_{\sigma(t_0)}(x(t_0)) \\
 &\leq c_0 e^{-2\lambda(t - t_0)} V_{\sigma(t_0)}(x(t_0)),
 \end{aligned} \tag{3.44}$$

where $c_0 = e^{N_0 \ln \mu}$, $\lambda = (1/2)(\alpha^* - (\ln \mu / T_a))$. According to (3.33), we have

$$\mathcal{L}\|x(t)\|^2 \leq V(x(t)), \quad V_{\sigma(t_0)}(x(t_0)) \leq \mathcal{M}\|x(t_0)\|_{\text{cl}}^2. \tag{3.45}$$

Combining (3.44) and (3.45) leads to

$$\|x(t)\|^2 \leq \frac{1}{\mathcal{L}} V(x(t)) \leq \frac{\mathcal{M}}{\mathcal{L}} c_0 e^{-2\lambda(t - t_0)} \|x(t_0)\|_{\text{cl}}^2. \tag{3.46}$$

Therefore,

$$\|x(t)\| \leq \sqrt{\frac{c_0 \mathcal{M}}{\mathcal{L}}} e^{-\lambda(t - t_0)} \|x(t_0)\|_{\text{cl}}. \tag{3.47}$$

which means that solution to (2.1) is exponentially stable. The proof is thus completed. $\square$

4. Numerical Example

In this section, we now provide an example to show the effectiveness of the result in Theorem 3.3.

Example 4.1. Consider the nonlinear systems with interval time-varying and distributed delays (2.1) with the following parameters:

$$\begin{aligned} \dot{x}(t) = & A_{\sigma(t)}x(t) + D_{\sigma(t)}x(t - h_{\sigma(t)}(t)) + C_{\sigma(t)} \int_{t-k_{\sigma(t)}(t)}^t x(s)ds \\ & + E_{\sigma(t)}f_{\sigma(t)}\left(t, x(t), x(t - h_{\sigma(t)}(t)), \int_{t-k_{\sigma(t)}(t)}^t x(s)ds\right), \end{aligned} \quad (4.1)$$

where

$$\begin{aligned} A_1 &= \begin{bmatrix} -2 & 0.1 \\ 0 & -1 \end{bmatrix}, & D_1 &= \begin{bmatrix} 0.1 & 0 \\ 0.3 & -0.5 \end{bmatrix}, & C_1 &= \begin{bmatrix} 0.1 & 2 \\ 0 & 0.2 \end{bmatrix}, & E_1 &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}, & D_2 &= \begin{bmatrix} 0.2 & 0 \\ 0.1 & -1.3 \end{bmatrix}, & C_2 &= \begin{bmatrix} 0.25 & 0.2 \\ 0 & 0.1 \end{bmatrix}, & E_2 &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ f_1\left(t, x_1(t), x_1(t - h(t)), \int_{t-k_1(t)}^t x_1(s)ds\right) &\leq 0.1\|x_1(t)\| + 0.1\|x_1(t - h(t))\| \\ &\quad + 0.2\left\|\int_{t-k_1(t)}^t x_1(s)ds\right\|, \\ f_2\left(t, x_2(t), x_2(t - h(t)), \int_{t-k_2(t)}^t x_2(s)ds\right) &\leq 0.1\|x_2(t)\| + 0.05\|x_2(t - h(t))\| \\ &\quad + 0.1\left\|\int_{t-k_2(t)}^t x_2(s)ds\right\|. \end{aligned} \quad (4.2)$$

Given $h_{11} = 0.1$, $h_{12} = 0.2$, $h_{21} = 0.15$, $h_{22} = 0.25$, $k_1 = 0.22$, $k_2 = 0.2$, $\alpha = 0.25$, and $\beta = 0.2$, it is found that LMIs (3.3), (3.4), (3.5), (3.25), (3.26), and (3.27) of Lemmas 3.1 and 3.2 have feasible solutions. Thus, we can get $\mu = 1.35$. We now choose $\alpha^* = 0.155 < \alpha$. Then, the switching law (S1) will require $\inf_{t \geq t_0} T^-(t_0, t)/T^+(t_0, t) \geq ((\beta + \alpha^*)/(\alpha - \alpha^*)) = 4.2632$ and the average dwell time is computed as $T_a \geq T_a^* = \ln \mu / \alpha^* = 3.6472$.

We let $h_1(t) = 0.1 + 0.05|\cos t|$, $k_1(t) = 0.1|\sin t|$, $h_2(t) = 0.1 + 0.05|\sin t|$, $k_2(t) = 0.1|\cos t|$ and $\phi(t) = [-\cos t, \cos t]$, for all $t \in [-0.25, 0]$. Figure 1 shows the trajectories of solutions $x_1(t)$ and $x_2(t)$ of the interval time-varying and distributed delays system.

5. Conclusions

In this paper, we have investigated the exponential stability for a class of switched nonlinear systems with discrete and distributed time-varying delays. The interval time-varying delay

Figure 1: The trajectories of $x_1(t)$ and $x_2(t)$ of the interval time-varying delay system (4.1).

function is not necessary to be differentiable which allows time-delay function to be a fast time-varying function. By using the average dwell-time approached and improved piecewise Lyapunov-Krasovskii functionals which are constructed based on and combined with Leibniz-Newton's formula, for the cases that subsystems are stable and unstable, some new delay-dependent stability criteria are derived by a set of linear matrix inequalities. Numerical examples are given to illustrate the effectiveness of our theoretical results.

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