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Research Article

A Convenient Utility Function with Giffen Behaviour

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The paper proposes a simple utility function that can generate Giffen behaviour. The function suggests an alternative direction where Giffen behaviour can be found and also implies a convenient framework for empirical testing. Moreover, because of its simple form, the utility function is well-suited for teaching purposes.

1. Introduction

It was not until the third [1, 2] edition of his *Principles* that Alfred Marshall stated that the law of demand may not always hold. Marshall inserted a new paragraph with the famous "Giffen paradox," in which he argues that, under subsistence conditions, a rise in the price of a cheap foodstuff (bread) can force poor families to consume more, rather than less of it. ("(A)s Mr Giffen has pointed out, a rise in the price of bread makes so large a drain on the resources of the poorer labouring families and raises so much the marginal utility of money to them, that they are forced to curtail their consumption of meat and the more expensive farinaceous foods: and, bread being still the cheapest food which they can get and will take, they consume more, and not less of it. But such cases are rare; when they are met with they must be treated separately" ([1], page 208; [2], pages 109, 110).) The precise meaning of this paragraph has kept the minds of many economists occupied for more than a century, as has the more general issue of the possibility of an upward sloping segment of the demand curve (see, [3–8]). The discourse has been fuelled by the difficulty experienced in finding convincing empirical evidence of Giffen behaviour. The standard textbook example of the Irish potato, popularized by Paul Samuelson's *Economics* ([9], page 432), has been discredited (see, [10, 11]). Only recently, Jensen and Miller [12] claimed to have found the first rigorous evidence of Giffen behaviour—rice consumption by very poor Chinese households. Another difficulty has been that, although it was long recognized that the axioms of consumer theory allow for an upward sloping demand curve, concrete utility functions with this property were hard to formulate. Thus, textbooks usually illustrate the Giffen phenomenon by a picture with an indifference map and some arrows indicating a substitution effect that is offset by a positive income effect.

This paper deals with the latter problem by proposing a convenient utility function that implies Giffen behaviour in the case of utility maximization under a fixed-income constraint. As far as I know, there are only a few publications with explicit utility functions with Giffen behaviour; a brief overview suffices (functional forms are reported in the appendix). (It should be noted that particularly in the older literature mentioned here, the Giffen problem is just a side issue. A more extensive and rigorous overview is given by Heijman and van Mouche [13].) The numerical example by Wold [14], reprinted in the textbook of Wold and Juréen ([15], page 102), is probably the first. It is used as an exercise by Katzner ([16], page 62) and later discussed by Weber [17]. Another utility function is given by Vandermeulen [18]. However, because of some additional constraints, the relevant domain is not a cartesian product, which may be seen as a drawback. This also holds for the utility function of Silberberg and Walker [19], who revert to a completely numerical example (including prices). Also Spiegel [20] provides a utility function of which the Giffen property is discovered by way of a numerical exercise. Sörenson [21] proposes a modified Leontief utility function, which eliminates the substitution effect at the kink of the indifference curve and offers some economic justification for such a functional form (see also, [22]). As a classroom

example, however, the function is less suitable, because the nondifferentiability at the kinks prevents the use of the relative price rule (Gossen's Second Law) for the derivation of the demand curves. Finally, Doi et al. [23] are probably the first to present a utility function with convex indifference curves that have no kinks in the entire positive quadrant. Some limitations are that the demand functions are not in explicit form and that the evaluation of the price effect is not straightforward.

Compared with this literature, the utility function below stands out in three respects. First, the function has a simple form that lends itself easily for interpretation. This not only provides a clue as to what may cause Giffen behaviour, but also gives suggestions for where to look for empirical examples. Second, and related to this, the system of demand functions generated by this function is suitable for linear regression analysis. Thus, a suggestion is offered on how to look for empirical evidence of Giffen behaviour. Third, the utility function is well-suited for teaching purposes. The derivation of demand curves and the evaluation of income and price effects can be done in the usual way, and also the convexity of the indifference curves is easily verified.

2. A Simple Utility Function with the Giffen Property

The utility function specified below is based on the example of Wold [14] and a remark by Slutsky [24]. Although Slutsky is widely credited for his study of the generalized utility function (already in 1915), it is a result he derived for the additive utility function that is of interest here. Slutsky provided the by now familiar argument that the assumption of diminishing marginal utility is not necessary for downward sloping convex indifference curves. In particular, he found that in the case of additive utility an appropriate indifference map may also be obtained if one—but only one—good has *increasing* marginal utility. ("(...) If only one of the u_{ii} is positive, the budget is stable if $\Omega > 0$, unstable if $\Omega < 0$. (...) The budget can never be stable if more than one of the u_{ii} is positive" ([24], page 46). Here u_{ii} refers to the second-order partial derivative of the utility function with respect to the amount of good i; "budget" refers to a consumer's optimum goods bundle; "(un)stable" refers to whether the second-order condition is satisfied; and Ω is the relevant bordered Hessian determinant. See also Liebhafsky [25] and Yaari [26].) Moreover, if one good has increasing marginal utility and all the other goods have decreasing marginal utility, then the former is a normal good and all the other ones are inferior goods (given additive utility and convex indifference curves). This result is important, since for an upward sloping segment of the demand curve the good must be inferior.

Fix two positive parameters γ_x and γ_y and consider the utility function:

$$u(x,y) := \alpha_1 \ln(x-\gamma_x) - \alpha_2 \ln\left(\gamma_y - y\right), \qquad (0<\alpha_1<\alpha_2) \tag{1}$$

with the domain $x > \gamma_x$ and $0 \le y < \gamma_y$. Variables x and y refer to the quantities consumed of goods X and Y. As in a Stone-Geary utility function, γ_x may be interpreted as a minimum subsistence quantity of good X. Consumption of the other good Y is subject to a maximum quantity γ_y; sometimes too much of a good may damage one's health. Clearly the marginal utility of each good is positive. Also, in the case of good Y, marginal utility increases as more of it is consumed. The marginal rate of substitution of X for Y is

$$-\frac{dy}{dx} = \frac{\alpha_1\left(\gamma_y - y\right)}{\alpha_2(x-\gamma_x)} > 0, \tag{2}$$

so the indifference curves slope downward. By further differentiation, the rate of change of their slope is

$$\frac{d^2y}{dx^2} = \frac{\alpha_1\left(\gamma_y - y\right)}{\alpha_2^2(x-\gamma_x)^2}(\alpha_2-\alpha_1) > 0, \tag{3}$$

so the indifference curves are strictly convex. In a diagram, the indifference curves converge on the point $(x,y) = (\gamma_x,\gamma_y)$ and, starting from this point, widen as x increases. Note that, just as in the original paper of Wold [14], a convergence point can be avoided by an appropriate extension of the utility function (see Discussion and appendix below). The widening (or flattening) of the indifference curves implies that the income elasticity of good X is negative. (This may be seen as follows. Given some arbitrary suitable utility function $u(x,y)$, the marginal rate of substitution of X for Y (MRS) equals u_x/u_y (subscripts refer to partial derivatives). Differentiating with respect to y gives $\partial\text{MRS}/\partial y = (u_y u_{xy} - u_x u_{yy})/u_y^2$. Note that if this term is negative, then the indifference curves widen as x increases. Now, with income m, and after some manipulations, differentiation of the first-order conditions of constrained utility maximization gives $\partial x/\partial m = \lambda(u_y u_{xy} - u_x u_{yy})/\Omega$, where λ is the positive marginal utility of money and Ω is the Hessian determinant of u bordered by $(u_x, u_y, 0)$, which is positive in the case of convex indifference curves. Hence, $\text{sign}(\partial x/\partial m) = \text{sign}(\partial\text{MRS}/\partial y)$. In our specific case, $u_{xy} = 0$ and $u_{yy} > 0$; therefore, $\text{sign}(\partial x/\partial m) = \text{sign}(-u_{yy}) < 0$.) So, in agreement with Slutsky's argument, X is inferior.

Let $\beta_i := \alpha_i/(\alpha_2-\alpha_1)$ $(i = 1,2)$, and note that $\beta_i > 0$. With income m and prices p_x and p_y, we obtain the demand functions:

$$\begin{aligned} x &= \gamma_x - \frac{\beta_1}{p_x}\left(m - p_x\gamma_x - p_y\gamma_y\right), \\ y &= \gamma_y + \frac{\beta_2}{p_y}\left(m - p_x\gamma_x - p_y\gamma_y\right), \end{aligned} \tag{4}$$

provided that income and prices are such that

$$m - p_y\gamma_y < p_x\gamma_x < m - p_y\gamma_y + \frac{p_y\gamma_y}{\beta_2}. \tag{5}$$

The inequalities ensure that the maximizers lie in the (interior) domain. (For $x > \gamma_x$, it is required that $m - p_y\gamma_y < p_x\gamma_x$; and for $0 < y < \gamma_y$, it is required that

$m - p_y\gamma_y < p_x\gamma_x < m - p_y\gamma_y + p_y\gamma_y/\beta_2$. Further, note that $m - p_y\gamma_y + p_y\gamma_y/\beta_2 < m$ and that the left-hand term approaches m as α_2/α_1 increases (and so β_2 approaches 1).)

So suppose that these conditions hold and consider the demand function of good X. First note that X indeed is an inferior good (Y is a normal—even luxury—good). The derivative with respect to p_x implies that

$$\operatorname{sign}\left(\frac{\partial x}{\partial p_x}\right) = \operatorname{sign}\left(m - p_y\gamma_y\right). \tag{6}$$

If $A := m - p_y\gamma_y \leq 0$, then demand for X is defined for all $p_x \in (0, (A + p_y\gamma_y/\beta_2)/\gamma_x)$ and slopes downward (or is constant). If $m - p_y\gamma_y > 0$, then demand is defined for all $p_x \in (A/\gamma_x, (A + p_y\gamma_y/\beta_2)/\gamma_x)$ and slopes upward. By rewriting the demand for X as $x - \gamma_x(1 + \beta_1) = -\beta_1(m - p_y\gamma_y)/p_x$, we see that $m - p_y\gamma_y > 0$ is equivalent to $x - \gamma_x(1 + \beta_1) < 0$, so that

$$\gamma_x < x < \gamma_x\left(\frac{\alpha_2}{\alpha_2 - \alpha_1}\right). \tag{7}$$

Hence, upward-sloping demand is obtained if and only if the optimal quantity of good X is relatively low.

Finally, note that Giffen behaviour arises if and only if the maximum constraint on the consumption of good Y is binding in the sense that $\gamma_y < m/p_y$. That is, if and only if not the entire income can be spent on good Y. This requires high incomes rather than low incomes. Further, as expected from this Stone-Geary type of utility function, multiplying the demand functions with their respective prices generates a system of expenditure functions which are linear in income and prices, and thus, suitable for regression analysis.

3. Discussion

Symbolic notation allows us to discover what many numerical examples may not: the kind of preferences that give rise to Giffen behaviour. The proposed utility function suggests that Giffen behaviour can be found in the presence of activities that ultimately damage one's health, but the desire for which increases as an activity proceeds. Likely examples are drinking, smoking, and drug intake and maybe also certain sports activities with addictive elements. An increase of income raises the demand for such hazardous activities while lowering the demand for other harmless activities. If the income level is high, and so also is the pleasure from additional hazardous behaviour, then a price fall of some harmless activity may cause an income effect that stimulates hazardous enterprises so much that consumption of the harmless activity falls.

It is clear that this explanation considerably differs from the "Giffen paradox." Marshall tended to believe that in reality Giffen behaviour is only likely under subsistence conditions. Also Hicks and Allen ([27], pages 68,69) appeal to empirical observation to downplay the significance of Giffen behaviour, arguing that as the standard of living rises, such behaviour becomes increasingly improbable. It explains why the subsistence example has long been the standard explanation and the principal guide in the search for empirical evidence. It should be emphasized, however, that economic theory does not tell us that the specific properties of the indifference map that give rise to Giffen behaviour only prevail at relatively low levels of utility (see [4, 5]). Our result that Giffen behaviour occurs at high incomes rather than low incomes indeed exemplifies that there are no a priori reasons to believe that Giffen behaviour can happen only at low standards of living (see also [18, 23]).

To be fair to Marshall, it should be mentioned that in his time the possibility of increasing marginal utility—where "the appetite comes with eating"—was regarded as somewhat disturbing. Often one tried to bypass the problem by referring to a change in the unit of analysis or a change in preferences ([28], page 395). Thus, Marshall conceded that "the more good music a man hears, the stronger is his taste for it likely to become," but added that, since this all takes time, "the man is not the same at the beginning as at the end of it" ([1], page 97). Our assumption of increasing marginal utility can be modified, however, by extending the utility function to the case, where more consumption of good Y after some point, instead of reaching a maximum constraint, yields diminishing marginal utility. This exercise would keep the possibility of Giffen behaviour intact (see the appendix). Since many goods seem to exhibit increasing marginal utility at low consumption levels, it is suggested that the scope for actual Giffen behaviour is much larger than what is generally believed (Also Blaug's ([29], page 314) remark that many goods, if defined narrowly enough, are inferior for some ranges of income, suggests that actual Giffen behaviour can be far more pervasive than existing data disclose.).

Appendix

The relevant part (i.e., with the Giffen property) of the utility function of Wold [14] is $u(x, y) := (x - 1)/(y - 2)^2$ for $x > 1$ and $0 \leq y \leq 1.6$. Wold also reports an extension of the utility function to the full-nonnegative quadrant.

With parameters $k > 0$, $m < 0$, and $n > -1$, the utility function of Vandermeulen [18] is $u(x, y) := (n + 1)^{-1}y^{n+1}x^{-1} - (1 - m)^{-1}kx^{m-1}$ for all $x > 0$ and $y \geq 0$. In addition, the inequalities $(n + 1)k/(1 - m) \leq y^{n+1}x^{-m} \leq (n + 1)k$ must hold. The first inequality guarantees convex indifference curves, the second one nonnegative marginal utility with respect to x.

With a a positive or negative parameter, the utility function of Silberberg and Walker [19] is $u(x, y) := ax + \log x + y^2/2$ for $x > 0$ and $y \geq 0$. To obtain convex indifference curves, the domain is restricted by $(y/x)^2 \geq (a + 1/x)^2$.

With parameters $\alpha, \beta, \gamma, \delta > 0$, the relevant part of the utility function of Spiegel [20] is $u(x, y) := \alpha x - \beta x^2/2 + \gamma y + \delta y^2/2$ for $0 \leq x \leq \alpha/\beta$ and $y \geq 0$.

The utility function of Sörenson [21] is $u(x, y) := \min(u_1(x, y), u_2(x, y))$ where, with parameters $A > 1$ and $B > 0$, $u_1(x, y) := x + B$ and $u_2(x, y) := A(x + y)$. He also provides an alternative specification for u_1 and u_2 with strictly convex indifference curves.

The relevant part of the utility function of Doi et al. [23] is $u(x, y) := \alpha \ln x + \beta \ln y - \gamma xy$ for all $x > 0$ and $y > 0$ for which $xy < \alpha/\gamma$, where α, β, and γ satisfy $\beta > \alpha > \beta/2 > 0$ and $\gamma > 0$. The demand functions are not in explicit form and the determination of price effects is not straightforward.

The utility function in the text can be extended as follows (see remarks in Sections 2 and 3). Fix a parameter ϵ such that $0 < \epsilon < \gamma_y$, and define for the domain $x > \gamma_x$ and $y \geq 0$

$$u(x, y) := \alpha_1 \ln(x - \gamma_x) - \alpha_2 \begin{cases} \ln\left(\gamma_y - y\right) & \text{if } 0 \leq y < \gamma_y - \epsilon \\ -\ln\left(y - \gamma_y + \epsilon + \dfrac{1}{\epsilon}\right) & \text{if } y \geq \gamma_y - \epsilon \end{cases} \tag{A.1}$$

$(0 < \alpha_1 < \alpha_2)$. This function is continuous and has downward-sloping and, if $\epsilon \leq 1$, strictly convex indifference curves (with no kinks if $\epsilon = 1$). For the case $0 \leq y < \gamma_y - \epsilon$, the demand functions are the same as in the text, provided that

$$C := m - p_y\gamma_y + \epsilon\frac{p_y}{\beta_2} < p_x\gamma_x < D := m - p_y\gamma_y + \frac{p_y\gamma_y}{\beta_2}. \tag{A.2}$$

Note that $C < D$ and recall $\text{sign}(\partial x/\partial p_x) = \text{sign}(m - p_y\gamma_y)$. Now, if $C \leq 0$, and so $m - p_y\gamma_y < 0$, then demand for X is defined for all $p_x \in (0, D/\gamma_x)$ and slopes downward. If $C > 0$, then demand is defined for all $p_x \in (C/\gamma_x, D/\gamma_x)$; it then slopes downward (or is constant) if $C \leq \epsilon(p_y/\beta_2)$ and *upward* if $C > \epsilon(p_y/\beta_2)$. (It is a simple exercise to extend this utility function to the full-nonnegative quadrant.)

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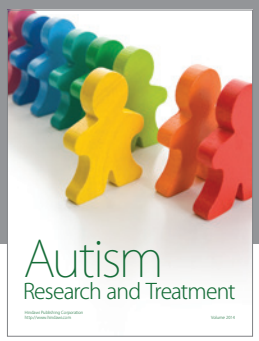

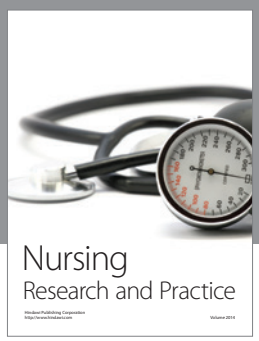

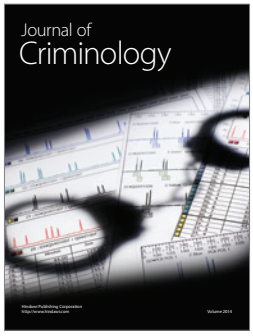

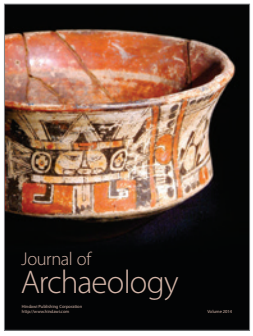

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