International Scholarly Research Network
ISRN Optics
Volume 2012, Article ID 659642, 6 pages
doi:10.5402/2012/659642

Research Article

Novel Robust Design Method of Multilayer Optical Coatings

Suyong Wu, Xingwu Long, and Kaiyong Yang

Department of Optoelectronic Engineering, College of Optoelectronic Science and Engineering, National University of Defense Technology, Changsha 410073, China

Correspondence should be addressed to Suyong Wu, sywu2001@163.com

Received 16 November 2011; Accepted 3 January 2012

Academic Editors: D. Monzon-Hernandez and A. A. Sukhorukov

We present a novel fast robust design method of multilayer optical coatings. The sensitivity of optical films to production errors is controlled in the whole optimization design procedure. We derive an analytical calculation model for fast robust design of multilayer optical coatings. We demonstrate its effectiveness by successful application of the robust design method to a neutral beam splitter. It is showed that the novel robust design method owns an inherent fast computation characteristic and the designed film is insensitive to the monitoring thickness errors in deposition process. This method is especially of practical significance to improve the mass production yields and repetitive production of high-quality optical coatings.

The control concept of film sensitivity with respect to layer parameters errors might be firstly put forward in the design of multilayer optical coatings by Dobrowolski in their further publication of the film program FILTER developed at National Research Council of Canada [1, 2]. It was done by adding the sum of derivatives of the basic design merit function with respect to any chosen set of construction parameters into the basic design merit function in the optimization procedure, but no actual applications were given. Zorc also pointed the feasibility of choosing practically optimum design film in relation to production errors in his research on a V-type AR coating at 1064 nm [3]. Actually the true application of robust optical coating design might be realized by Zheng and Tang with the effect of monitoring errors on the spectral characteristics considered [4]. The merit function in their design includes two parts, one is the optical property and the other is the calculated yield of finished products of the multilayer coatings, which is taken as an end optimization goal. The disadvantage of Zheng's method is that the computation accuracy and time of the calculated yield depend on the number of computer simulation, which is the worst when the layer bumper is large. Another notable progress of robust optical coating design was accomplished by Greiner with the expected optical performance optimized in the design procedure under some known statistical distribution of the layer parameters variations [5]. However, there is also a confliction between the computation accuracy and time of the expected optical performance in Greiner's method. In the following design contests of 1998 and 2001 topical meeting on Optical Interference Coatings, practical demands of robustness with manufacturing errors were emphasized in the design contests of a dual-band antireflection coating and a gain-flattening filter (GFF) for erbium-doped fiber amplifier [6, 7]. Verly, the winner of GFF contest, reported his design philosophy using semiautomatic techniques to optimize a standard RMS-type merit function without errors stability of the solutions. The errors stability was checked by plotting the sensitivity curve at intervals during the design process and lastly confirmed by random error calculations [8]. In the year of 2003, Tikhonravov et al. present a new robust design concept based on optimization of a design merit function including the spectral characteristics stability of optical coatings with respect to layer thicknesses errors, but no actual applications or algorithm details were published [9]. Recently Trubestkov et al. proposed a robust synthesis method based on simultaneous optimization of the spectral characteristcs of many local designs in the neighborhood of a pivotal design (caused by layer thicknesses errors) in order to solve the most complicated design problems such as dispersive mirrors [10, 11].

One common disadvantage of the main robust, thin-film design methods is the considerable long time consumption for the stability calculation of multilayer coatings with respect to layer thicknesses and refractive indices errors. In its early stage thin-film stability calculation was solved approximatively by statistical simulation of random layer parameters errors with large simulation number. Compromise must be made between computation accuracy and time consumption by using a moderate simulation number when incorporating the stability with respect to layer parameters errors into the optimization procedure. In this paper we will derive a fast robust design method of multilayer optical coatings based on an analytical calculation model for film sensitivity to layer parameters errors. We demonstrate its fast computation characteristic and effectiveness by numerical design results of a neutral beam splitter.

One of the most popular basic design merit functions in the numerical optimization of multilayer optical coatings is defined as follows:

$$F(\mathbf{n},\mathbf{d}) = \frac{1}{L}\sum_{l=1}^{L}\left(\frac{T_l(\mathbf{n},\mathbf{d},\lambda_l) - \hat{T}_l(\lambda_l)}{\Delta T_l}\right)^2, \tag{1}$$

where $T_l(\mathbf{n},\mathbf{d},\lambda_l)$, $\hat{T}_l(\lambda_l)$, and ΔT_l are the theoretical, target, tolerance transmittance of the design film at wavelength point λ_l from a given wavelength grid with total number of L points, respectively, $\mathbf{n},\mathbf{d}$ are the design film's refractive indices and geometric thickness vector. For the sake of convenience and clarity, we call the thin film design based on optimization of expression (1) "Conventional Design" to distinguish from the robust design to be presented in the paper. Suppose the error vectors of refractive indices and thicknesses are $\Delta\mathbf{n} = (\Delta n_1, \Delta n_2, \ldots, \Delta n_m)$ and $\Delta\mathbf{d} = (\Delta d_1, \Delta d_2, \ldots, \Delta d_m)$, respectively, (where m is the layer number of design film), we can define the film errors sensitivity $S(\mathbf{n},\mathbf{d})$ to layer parameters errors approximately as

$$\begin{aligned} S(\mathbf{n},\mathbf{d}) &= E\left(\frac{1}{N}\sum_{i=1}^{N}\left(F_i\left(\mathbf{n}+\Delta\mathbf{n}^{(i)}, \mathbf{d}+\Delta\mathbf{d}^{(i)}\right)\right) - F(\mathbf{n},\mathbf{d})\right) \\ &= E(F(\mathbf{n}+\Delta\mathbf{n}, \mathbf{d}+\Delta\mathbf{d})) - F(\mathbf{n},\mathbf{d}), \end{aligned} \tag{2}$$

where $\Delta\mathbf{n}^{(i)}, \Delta\mathbf{d}^{(i)}$ are the ith simulated film refractive indices and thicknesses errors vector with the total simulation number of N, $E(\square)$ refering to the mathematical expectation operation. When N increases to infinite, expression (2) converges to the accurate film errors sensitivity. However, it is really time consuming for numerical calculation, which is formidable in the optimization procedure of multilayer optical coatings.

When the design film is to be deposited by quartz crystal monitoring or time monitoring method of layer thicknesses, errors in layer refractive indices and thickness are independent and not correlated between different layers [12–14], and we can simulate these errors as random ones distributed in accordance with the normal distribution law with zero mathematical expectation and some given standard deviations. Thus according to the second-order segment of the Taylor series of perturbed basic design merit function listed as follows:

$$\begin{aligned} &F(\mathbf{n}+\Delta\mathbf{n}, \mathbf{d}+\Delta\mathbf{d}) \\ &\quad = F(\mathbf{n},\mathbf{d}) + \sum_{i=1}^{m}\left(\frac{\partial F}{\partial n_i}\Delta n_i\right) + \sum_{i=1}^{m}\left(\frac{\partial F}{\partial d_i}\Delta d_i\right) \\ &\qquad + \frac{1}{2}\sum_{i,j=1}^{m}\left(\frac{\partial^2 F}{\partial n_i \partial n_j}\Delta n_i \Delta n_j + \frac{\partial^2 F}{\partial d_i \partial d_j}\Delta d_i \Delta d_j\right. \\ &\qquad \left. + 2\frac{\partial^2 F}{\partial n_i \partial d_j}\Delta n_i \Delta d_j\right), \end{aligned} \tag{3}$$

we can derive an accurate analytical expression of the film errors sensitivity $S(\mathbf{n},\mathbf{d})$ to layer parameters errors by the probability statistics principle on the mathematical expectation of independent random variables. When we calculate the mathematical expectation of the above Taylor series, most of the summation parts become to be zeros except the product of $\Delta n_i \Delta n_j$ and $\Delta d_i \Delta d_j$ in the last summation part when $i = j = 1, 2, \ldots, m$, because all the random errors are independent variables. So we can apply this conclusion into the mathematical expectation operation of expression (2) and can obtain

$$S(\mathbf{n},\mathbf{d}) = \frac{1}{2}\sum_{j=1}^{m}\left(\frac{\partial^2 F}{\partial n_j^2}\sigma_{n,j}^2 + \frac{\partial^2 F}{\partial d_j^2}\sigma_{d,j}^2\right), \tag{4}$$

where $\sigma_{n,j}, \sigma_{d,j}$ are the standard deviation of the jth layer's refractive index error and thickness error, respectively, $\partial^2 F/\partial n_j^2, \partial^2 F/\partial d_j^2$ are the main diagonal elements of basic design merit function's Hess Matrix with respect to layer refractive indices and thicknesses. In order to improve the calculation speed of film errors sensitivity by expression (4), it is advisable to use the analytical model of spectral coefficient's first- and second-order partial derivatives of multilayer optical coatings with respect to layer parameters [14–18]. It is necessary to mention that the reason why the optical property used here in the basic design merit function is transmittance rather than reflectance is that the second-order partial derivatives of transmittance are less in calculation amounts than reflectance [17, 18].

Now we can obtain a robust design merit function of multilayer optical coatings by incorporating the film errors sensitivity into the basic design merit function as follows:

$$FF(\mathbf{n},\mathbf{d}) = F(\mathbf{n},\mathbf{d}) + S(\mathbf{n},\mathbf{d}). \tag{5}$$

For the sake of convenience and clarity, we call the thin film design based on optimization of expressions (1), (4), and (5) as "Sensitivity Control Design".

In order to test the calculation accuracy and time consumption of thin film robust designs based on film errors sensitivity by expression (4) compared with expression (2), we use a 21-layer ramp transmittance filter to calculate the new robust design merit function FF by expressions (1), (2),

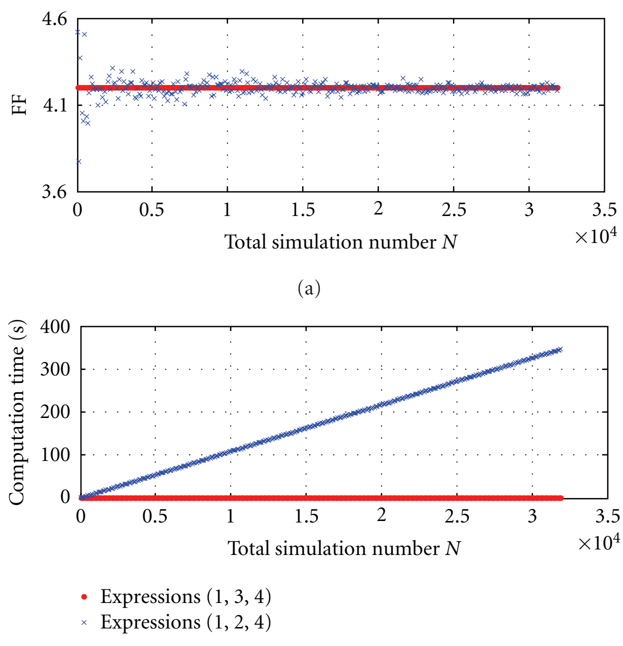

Figure 1: The robust design merit function value and the time consumption calculated by expressions (1), (2), (5) or (1), (4), (5) with different total simulation numbers.

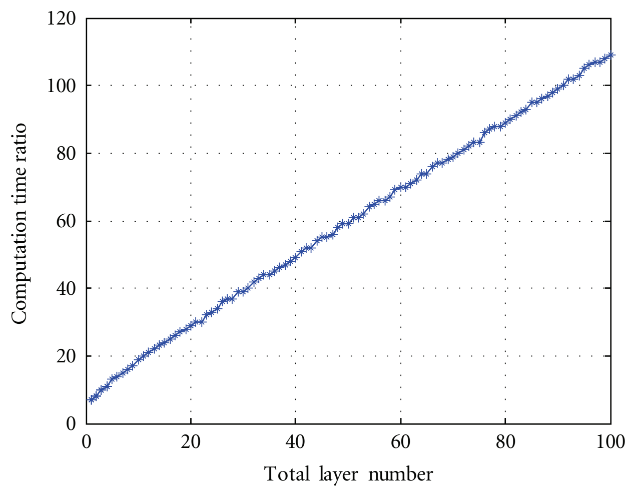

Figure 2: The computation time ratio of the robust design merit function based on expressions (1), (4), (5) to basic design merit function based on expression (1) with different total layer numbers.

(5) and (1), (4), (5), and the corresponding computation time as showed in Figure 1. As we can see from Figure 1, the robust design merit function value based on expressions (1), (2), (5) is converged gradually to the real value obtained by expressions (1), (4), (5). And the computation time by expressions (1), (2), (5) when the total simulation number N is large is many times longer than the time by expressions (1), (4), (5). So we demonstrate that the film errors sensitivity model based on expressions (1), (4), (5) is really accurate and inherently fast.

When the same optimization technique is used to solve the Conventional Design and Sensitivity Control Design of multilayer optical coatings defined as above, the total design time consumption comparison can be approximately made by comparison of their corresponding merit function computation time. In order to find a more universal conclusion, we plot the computation time ratio of the robust design merit function based on expressions (1), (4), (5) to the basic design merit function based on expression (1) with respect to different total layer number as showed in Figure 2. A linear relationship is found between the total layer number and the computation time ratio of Sensitivity Control Design to Conventional Design of multilayer optical coatings. And the computation time ratio is about $(m + 9)$ (where m is the total layer number). This time ratio is much less than the smallest time ratio (usually equal N to whose value is very large, seen from Figure 1) based on expressions (1), (2), (5) in order to obtain a good accuracy of film errors sensitivity, because the analytical computation model of film errors sensitivity plays a vital role in speeding up. This quantitative conclusion is applicable to a variety of optical coatings.

In order to illustrate the effectiveness of the above novel fast robust design method, we use the neutral beam splitter in the spectral region from 400 to 700 nm cited from [19] as a design example. The target transmittance in the whole spectral band is 50% at normal incidence with the reflection from the substrate back side considered together. The refractive indices of the substrate, incident medium, and available materials are 1.52, 1.0, 2.35, and 1.45. Most modern coating machines are capabale of reproducing the refractive index as such and eventual variations in the refractive index are negligible. So we do not take random layer refractive indices errors into account here. We also assume that random thickness errors with normal distributions and standard deviations of 0.2 nm that were made directly in all layers which is the same as [19]. The specified transmittance is allowed in the corridor of $\pm 1\%$ deviations from 50% in the whole given spectral band.

Our optimization technique is based on a hybrid genetic algorithm which might be firstly presented by Martin et al. [20]. The optimization technique utilizes genetic algorithm based on elite selection strategy to find the neighborhood region of overall minimum or quasi-minimum and then utilizes nonlinear least-squares method based on Levenberg-Marquardt algorithm to refine the genetic algorithm designed coating to quickly and stably converge to the overall minimum or quasiminimum [21]. This hybrid optimization algorithm incorporates the overall search ability of genetic algorithm and refinement accuracy and speed of nonlinear least-squares method together, which avoids the deficiency of slow refinement speed and some kind of random characteristic of standard genetic algorithm. We have verified the ability of the hybrid genetic algorithm to effectively design many kinds of multilayer optical coatings [21]. When the hybrid genetic algorithm is used to deal with the robust thin film design problems under layer parameters errors, we take the robust design merit function based on expressions (1), (4), (5) as the fitness function of the

Table 1: Properties of the neutral beam splitter ($n_0 = 1.0$, $n_L = 1.45$, $n_H = 2.35$, $n_s = 1.52$).

d(nm)		Conventional design 1	Conventional design 2	Sensitivity control design 1	Sensitivity control design 2
Substrate				Massive	
1	H	9.416	7.836	**11.3461**	16.6216
2	L	70.040	55.305	**35.4709**	28.0701
3	H	24.597	31.419	**113.1081**	139.6693
4	L	87.878	23.621	**164.3371**	7.5744
5	H	21.941	82.439	**72.6831**	135.0514
6	L	87.775	44.960	**93.5646**	85.1263
7	H	58.533	26.888	**35.5054**	54.3485
8	L	114.755	86.482		91.1863
9	H	57.279	55.507		215.1898
10	L	65.784	115.317		87.6283
11	H		52.603		
12	L		64.619		
Air				Massive	
$\sqrt{F}$		0.0200	0.0026	0.1046	0.0832
S		0.1391	0.1466	0.0603	0.0595
$\sqrt{FF}$		0.3735	0.3829	0.2668	0.2577

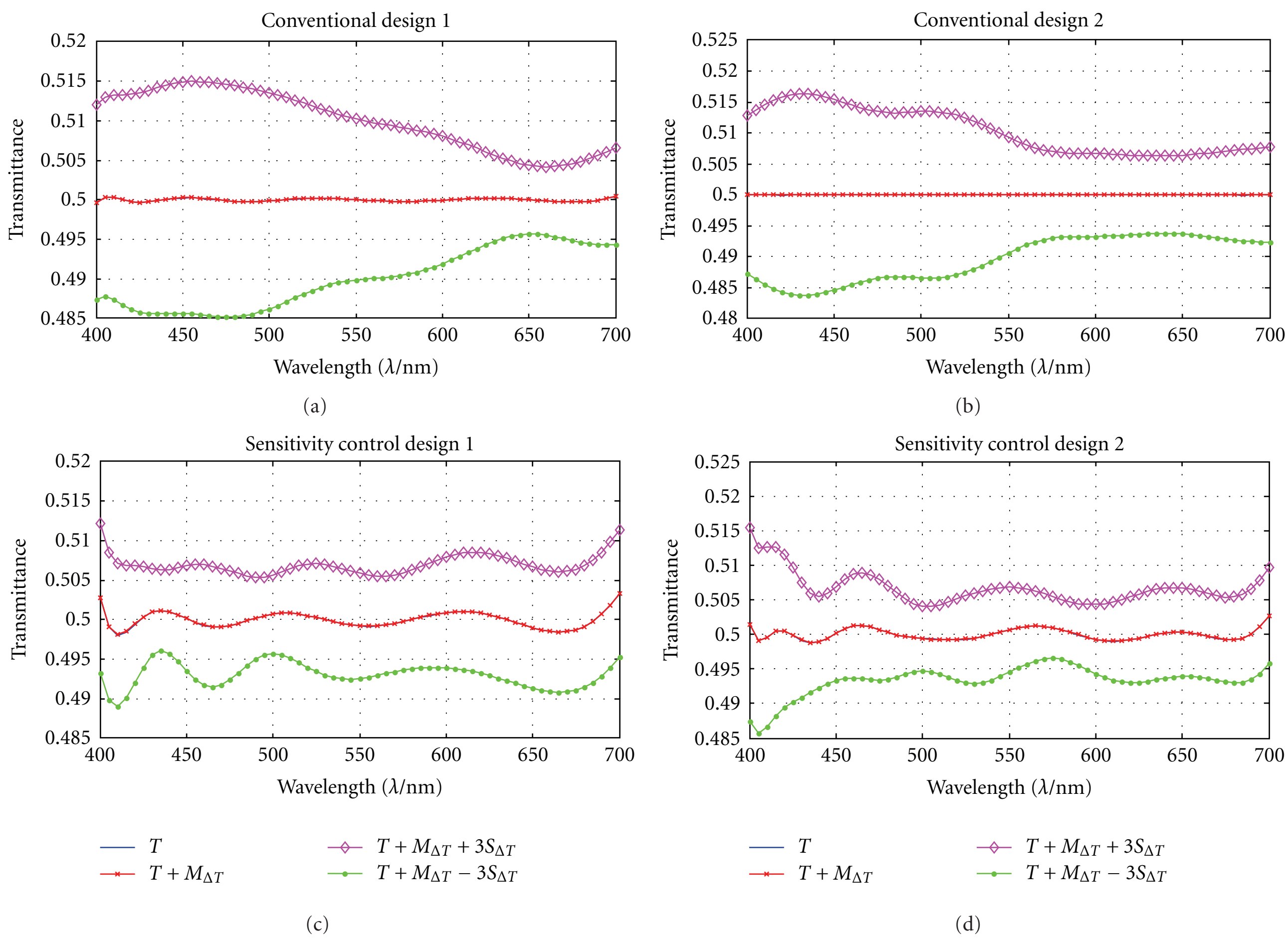

Figure 3: The theoretical transmittance T, mathematical expectation transmittance $T + M_{\Delta T}$, and its triple standard deviation corridor $T + M_{\Delta T} \pm 3S_{\Delta T}$ of the neutral beam splitters listed in Table 1 at normal incidence with the given 0.2 nm layer thicknesses errors level.

population in the whole genetic optimization procedure. After the stopping criteria are fulfilled, the robust design merit function of the genetic algorithm designed coating is further minimized by nonlinear least-squares method based on Levenberg-Marquardt algorithm to its minimum. At present we have compiled this optimization technique into our home-made film design software package [21].

In our design experiment, we use uniform wavelength grid with 61 total spectral points and the transmittance tolerance are set to 1%. Based on our home-made film design software package [21], we obtain two Conventional Design coatings based on optimization of expression (1) and two Sensitivity Control Design coatings based on optimization of expressions (1), (4), (5) with layer thicknesses listed in Table 1 and perturbed spectral properties by random thicknesses errors showed in Figure 3. The perturbed spectral transmittance under the given standard deviations of 0.2 nm thicknesses errors is supposed to be within the triple standard deviation transmittance corridor with a probability of 99.74% according to the probability statistics principle. As we can see from Figure 3, the theoretical transmittance curves of the two Conventional Design coatings are excellently flat at 50%. But, under the given standard deviations of 0.2 nm thicknesses errors, the perturbed transmittance can run out of the allowed region of $\pm 1\%$ deviations from 50% in the whole given spectral band. However, the two Sensitivity Control Design Coatings have more theoretical transmittance curve ripples but obviously controlled smaller film errors sensitivities with almost 100% production yield. Another better feature of the two Sensitivity Control Design Coatings is their smaller layer number compared with the two Conventional Design coatings and the 11-layer coating in [19]. In a word, the novel robust design method is demonstrated to be able to design coatings insensitive to layer parameters errors.

Let us talk about the application conditions of our method. The deduction of expression (4) is under the assumption that the deposited errors in layer refractive indices and thickness are independent and not correlated between different layers. This situation is typical of coatings deposited by quartz crystal monitoring or time monitoring method of layer thickness, which is commonly used in thin film factories and laboratories. So our robust design method is more significant from the practical point of view. At present our method cannot be applied to coatings deposited with correlated layer parameters errors. Secondly, it is hard to estimate the probability of our method to find a design with a low sensitivity to manufacturing errors for all types of coatings. The success of our method depends partly on the design possibility of the spectral targets of the required thin films. So it is advisable to try to search with convetional design methods first to get an estimate of the difficulty of the success of our robust design method.

In conclusion, we present a novel fast robust design method of multilayer optical coatings. The sensitivity of optical films to production errors is controlled in the whole optimization design procedure. We derive an analytical calculation model for fast robust design of multilayer optical coatings. We demonstrate its effectiveness by successful application of the robust design method to a neutral beam splitter. It is showed that the novel robust design method owns an inherent fast computation characteristic and the designed film is insensitive to the monitoring thickness errors in deposition process. This method is especially of practical significance to improve the mass production yields and repetitive production of high-quality optical coatings. Additionally this design technique can make the planning of manufacture process possible without expensive sampling and make a shortening facility cycle for new coating systems.

References

[1] J. A. Dobrowolski, "Versatile computer program for absorbing optical thin film systems," *Applied Optics*, vol. 20, no. 1, pp. 74–81, 1981.

[2] J. A. Dobrowolski, F. C. Ho, A. Belkind, and V. A. Koss, "Merit functions for more effective thin film calculations," *Applied Optics*, vol. 28, no. 14, pp. 2824–2831, 1989.

[3] H. Zorc, "Optimum multilayer design selection in relation to production errors," *Vacuum*, vol. 37, no. 1-2, pp. 101–102, 1987.

[4] Y.-F. Zheng and J.-F. Tang, "New automatic design of optical coatings," *Applied Optics*, vol. 26, no. 8, pp. 1546–1549, 1987.

[5] H. Greiner, "Robust optical coating design with evolutionary strategies," *Applied Optics*, vol. 35, no. 28, pp. 5477–5483, 1996.

[6] P. Baumeister, "Evaluation of the solutions for two design problems presented at the 1998 Optical Interference Coatings Conference," *Applied Optics*, vol. 39, no. 13, pp. 2230–2234, 2000.

[7] A. Thelen, M. Tilsch, A. V. Tikhonravov, M. K. Trubetskov, and U. Brauneck, "Topical meeting on optical interference coatings (OIC'2001): design contest results," *Applied Optics*, vol. 41, no. 16, pp. 3022–3038, 2002.

[8] P. G. Verly, "Design of a robust thin-film interference filter for erbium-doped fiber amplifier gain equalization," *Applied Optics*, vol. 41, no. 16, pp. 3092–3096, 2002.

[9] A. V. Tikhonravov, M. K. Trubetskov, T. V. Amotchkina, and A. A. Tikhonravov, "Application of advanced optimization concepts to the design of high quality optical coatings," in *Proceedings of the International Society for Optics and Photonics (SPIE '03)*, vol. 4829, pp. 1061–1062, 2003.

[10] M. K. Trubetskov and A. V. Tikhonravov, "Robust synthesis of multilayer coatings," in *Proceedings of the Optical Interference Coatings, OSA Technical Digest (Optical Society of America '10)*, 2010, TuA4.

[11] V. Pervak, M. K. Trubetskov, and A. V. Tikhonravov, "Robust synthesis of dispersive mirrors," *Optics Express*, vol. 19, pp. 2371–2380, 2011.

[12] B. T. Sullivan and J. A. Dobrowolski, "Deposition error compensation for optical multilayer coatings. I. Theoretical description," *Applied Optics*, vol. 31, no. 19, pp. 3821–3835, 1992.

[13] H. A. Macleod, *Thin Film Optical Filters*, Institute of Physics Publishing, Bristol, UK, 2001.

[14] S. h. Furman and A. V. Tikhonravov, *Basics of Optics of Multilayer Systems*, Editions Frontieres, Paris, France, 1992.

[15] J. Hrdina, "Parallelization of computation of derivatives in thin-film systems," *Journal Of Modern Optics*, vol. 37, no. 6, pp. 1087–1094, 1990.

[16] J. Hrdina, "Fast computation of second-order derivatives of reflectance, transmittance and corresponding phase shifts in thin film synthesis," *Journal of Modern Optics*, vol. 41, no. 1, pp. 129–136, 1994.

[17] W. u. Suyong, L. Xingwu, and Y. Kaiyong, "Accurate calculation and Matlab based fast realization of merit function's Hesse matrix for the design of multilayer optical coating," *Optoelectronics Letters*, vol. 5, no. 5, pp. 359–363, 2009.

[18] S. Wu, W. Suyong, L. Xingwu, and H. Yun, "Calculation model for spectral coefficient's first and second order partial derivatives of multilayer optical coatings with respect to layer parameters," *High Power Laser and Particle Beams*, vol. 22, no. 1, pp. 81–90, 2010 (Chinese).

[19] A. V. Tikhonravov, M. K. Trubestkov, and T. V. Amotchkina, "Computational manufacturing experiments for choosing optimal design and monitoring strategy," in *Proceedings of the Optical Interference Coatings, OSA Technical Digest (Optical Society of America '10)*, 2010, TuA5.

[20] S. Martin, J. Rivory, and M. Schoenauer, "Synthesis of optical multilayer systems using genetic algorithm," *Applied Optics*, vol. 34, no. 13, pp. 2247–2254, 1995.

[21] W. u. Suyong, *Application of Genetic Algorithm in the Design of Optical Coatings*, National University of Defense Technology, Changsha, China, 2007.