Research Article

The Characteristics of Cosmic Rays in a Fractal Medium

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We consider the galactic propagation of cosmic rays with energies below the knee of the cosmic ray spectrum (at 3×10^6 GeV). Lagutin and his colleagues suggested that propagation in a fractal medium would be an appropriate model for such particles and pointed out that a different diffusion equation to that usually assumed is then required. We present results of calculations using a diffusion equation appropriate to an inhomogeneous medium. These results are encouraging and can fit properties of the energy density of cosmic rays using the supernova model of Erlykin and Wolfendale. The containment time of cosmic rays, in such a modeling of propagation in a fractal medium, is discussed.

1. Introduction

It has long been considered that the supernova remnants (SNRs) are responsible for galactic cosmic rays [1, 2]. They totally release about 10^{51} erg as their explosion energy which makes them as very likely candidates in the galaxy, capable of delivering the requisite cosmic ray (CR) power (10^{41} erg/s) to the interstellar medium [3]. The most likely particle acceleration mechanism would be diffusive shock acceleration which can also satisfy the requirement of producing a power law energy spectrum such as what is observed. Although there is evidence in the favor of SNR models [4, 5], they also face with some serious problems. One problem is the energy of the knee ($\sim 10^6$ GeV) which is above the maximum energy of accelerated protons in all the models (e.g., Axford and Berezhko model [3]). For instance, none of the existing supernova models can easily provide the necessary acceleration for galactic CRs to reach the highest observed energies, higher than 10^6 GeV.

Also, the total particle energy required to be extracted from the shock energy can be at least a few percent, based on conventional galactic propagation models. It is necessary to ensure that any alternative propagation models do not require a substantial increase in

this fraction. The interstellar medium (ISM) is known as an inhomogeneous environment of matter and magnetic fields in different scales with a fractal character. Galactic structures and physical parameters such as shells, clouds, filaments, temperature, density, and degree of ionization are distributed over ranges of spatial scales [6].

However, during the recent decades, from both theoretical and observational points of view, more evidence (e.g., [7]) has been about the existence of multiscale structures in the galaxy. Hence, we like to see the effect of CR propagation in such a fractal medium. In this search, we will examine the issue of the flux, or energy density, of CRs at the Earth in the energy range below the spectral knee. By assuming that the diffusion of CRs in ISM has a fractal structure, a question arises here about whether the calculated particle density as a function of galactic radius is consistent with the observations of the radial gradient and energy density of CRs. We will also examine more directly how results of such propagation differ from more conventional propagation modelling of galactic CR containment times. It is commonly assumed that CR propagation through the galaxy has a form of normal diffusion and causes a spatial distribution of particles around the source.

In this paper, we will examine the suggestion by Lagutin et al. [8] to consider an interstellar medium with scales described by a fractal structure, resulting in an anomalous diffusion equation. Lagutin et al. introduced a parameter, α , as a key parameter for CR propagation in such interstellar medium, where its value is related to the ISM spectrum of magnetic irregularities. The normal diffusion in a homogeneous medium, which yields a Gaussian distribution of particle densities, is achieved when $\alpha = 2$. The range of $\alpha < 2$ corresponds to the so-called superdiffusive regime of anomalous diffusion. In this paper, we will show the usefulness of assuming the inhomogeneous ISM [9, 10] in the range $\alpha < 2$ [8]. We will attempt to identify a likely value of α , using CR flux data plus the assumption that those particles originate in supernovae. This supposition which is in agreement with the physical pattern of galactic magnetic fields provides us with a parameter to be included in comparing supernovae source models with observation.

2. Calculating the Radial Gradient of Galactic CRs and the Local Energy Density

We will assume that all CRs in the energy range of interest are of supernova origin. We will also assume that the spatial distribution function of supernovae within our galaxy is as $\rho_{sn} = AF(R, Z)$ [11, 12] that

$$F(R, Z) = \left(\frac{R}{R_0}\right)^a \exp\left(-b\left(\frac{R}{R_0} - 1\right) - \frac{Z}{Hz}\right). \quad (2.1)$$

The assumed values of the parameters in (2.1) are $a = 1.69$, $b = 3.33$, $R_0 = 8.5 \text{ Kpc}$ (the galactic radius of the solar system), and $Hz = 0.2 \text{ Kpc}$ (the vertical scale parameter of galactic supernovae). Z is distance perpendicular to the galactic disk.

Using the solution of Lagutin et al. [8] for the steady-state superdiffusion model of CR particles, we first calculated the radial gradient of the galactic CR intensity using the supernova distribution described by (2.1) with the assumption of cylindrical symmetry. Assuming the production rate of type II SN explosions is $10^{-2} \text{ year}^{-1}$ in the galaxy, we examined how well the model would predict the observed value of the energy density of CRs. To have the concentration of CRs with energy E at a distance r from a point source

Figure 1: The intensity (arb. Units) of CRs as a function of galactic radial distance assuming a supernova origin with CRs propagating in various fractal media (for different values of the parameter α , see text).

(supernova) which has a power law energy spectrum, we apply the steady-state diffusion equation suggested by Lagutin et al. as follows:

$$D(-\Delta)^{\alpha/2}N(r, E) = S(r, E), \quad (2.2)$$

where $S(r, E)$ describes the density distribution of sources, D is the anomalous diffusivity, $N(r, E)$ is the number of particles at distance r from SN, and $(-\Delta)^{\alpha/2}$ is the fractional Laplacian called the “Riss” operator [13, 14]. Following Lagutin et al. through the Greens function for the above equation, we can find the solution for the steady-state case with CRs propagating diffusively from sources having the assumed spatial distribution of supernovae. We assume that τ_0 , the galactic lifetime of CRs with 1 GeV energy, equals 4×10^7 years [15] and that H_G , the vertical scale of the galactic magnetic fields, is 1 Kpc.

The observed intensity of CRs, I , at an energy E is the sum over all sources of the product of the intensity from a source at a particular location and the density of sources there, that is, the sum of all $N(r, E) F(R, Z)$. The intensity, in arbitrary units, in terms of R for different α , (an intrinsic parameter of interstellar medium), is shown in Figure 1. We calculated the radial gradient of CRs, for each α value in Figure 1, and obtained an average galactic radial gradient of CRs in each case. Our results for the range $1.6 < \alpha < 2$ are consistent with experimental determinations of the fractional radial gradient, ranging from 3×10^{-2} to $6 \times 10^{-2} \text{ kpc}^{-1}$ [16].

We wished to determine how well our anomalous diffusion model would fit the CR energy density as measured at the Earth. This was based on supernovae occurring relatively locally, in an annulus between 4.5 Kpc and 12.5 Kpc from the galactic centre, each radiating for a total effective time of T years, and assuming that a supernova explosion occurs once per 100 years anywhere in the galaxy. We used the flux $F(R, Z)$ from a single supernova derived above (following Lagutin et al.) from the superdiffusion Greens function and integrated this with a Monte Carlo technique over the appropriate part of our galaxy model. The energy density, I , is now a function of Z, r, α , and T plus the fraction of supernova energy transferred to CRs. Using our model for the energy density contributed by a single supernova, we

Figure 2: The containment time ratio for conventional diffusion compared to superdiffusion in a turbulent galactic magnetic field.

determined the propagation parameters which result in the best fit to the observed CR energy density. Testing different values of parameters, such as α in a range of 0.5 to 2, T from 10^4 to 10^7 years and fraction of supernova energy transferred to particles from 0.01 to 0.1, we obtained a series of resulting values of CR energy density in the vicinity of the sun.

To compare our simulated energy density of CRs with observational results, we used a model of Erlykin and Wolfendale which gives an energy spectrum for CRs measured in the vicinity of the sun [16]. This allowed us to calculate a total CR energy density above $10^{4.4}$ GeV as $1.8 \times 10^{-4} \text{ eV cm}^{-3}$. A comparison of our derived energy densities, as a function of propagation parameters, with the energy density of the Erlykin and Wolfendale model, indicates a best fit for $\alpha = 1.8$ (with a total possible range 1.6 to 1.9 given uncertainty in the supernova emission time and the fraction of supernova energy being transferred to particles). This value, being below 2, implies a fractal structure for ISM. In this case, we found a best fit of 10% for the fraction of energy going into energetic particles. This fraction is comparable with the value of 16% for the kinetic energy of the supernova ejecta found by A.M. Hillas [2]. The effective emitting time of the supernova was then best fitted at 7×10^4 years. Our calculations thus support a model of the interstellar medium, within which galactic CRs produced by supernova propagate, being most consistent with CR observations if a fractal and nonhomogeneous structure is incorporated. We expect the resulting flux to be reduced for the superdiffusion case when compared to earlier modeling, but the required energy transfer to CRs is still not excessive, even for our conservative assumption of one supernova per hundred years.

In order to examine this result from another point of view, we have followed the propagation of CRs explicitly through a model of the galactic magnetic field which assumes a fractal structure through a power law distribution of characteristic magnetic field scale sizes. At any location, we assume that one such scale size applies to the magnetic field, which has itself a turbulent structure with a Kolmogorov spectrum up to that scale size. This is an extension of the type of propagation often considered [17] which assumes a Kolmogorov spatial spectrum up to a fixed scale size. Figure 2 shows the result of such modeling. The superdiffusion, fractal model results in shorter galactic containment times. Those times are strongly influenced by the possibility of encountering a region associated with a particularly

large turbulence scale which makes particle loss more likely. In this way, the final CR energy density is less than for conventional diffusion for a given distribution and frequency of supernova sources. In the case presented, which corresponds closely to superdiffusion α of 1.7, there is an overall reduction in containment by about a factor of five. This is the approximate value of α which we found to be required to fit the galactic CR energy density.

3. Conclusions

We have examined CR propagation of 10^{12} to 10^{19} eV in a galactic model incorporating fractal structure which is shown in Figure 2. Our calculated result with the observed values of radial gradient of galactic CRs given in the range of 3×10^{-2} to $6 \times 10^{-2} \text{ Kpc}^{-1}$ and also comparison of our simulated values of energy density of CRs from the expected supernova model ($1.8 \times 10^{-4} \text{ eV cm}^{-3}$) confirm a diffusion of cosmic particles of supernova origin in the fractal medium and reject their diffusion in homogenous medium with Gaussian distribution ($\alpha = 2$). We find that a superdiffusion model results in lower galactic containment times for nearly the whole CR energy spectrum, and as a result lower energy density below that expected from conventional propagation in turbulent magnetic field.

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