Research Article

Viscous Bianchi Type I Universe with Stiff Matter and Decaying Vacuum Energy Density

Raj Bali,¹ Pratibha Singh,¹ and J. P. Singh²

¹ Department of Mathematics, University of Rajasthan, Jaipur 302004, India

² Department of Mathematical Sciences, A. P. S. University, Rewa 486003, India

Correspondence should be addressed to Raj Bali, balir5@yahoo.co.in

Received 11 April 2012; Accepted 7 May 2012

Academic Editors: S. C. Lim and M. Rasetti

Copyright © 2012 Raj Bali et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

We examine homogeneous and anisotropic Bianchi type I cosmological model with viscous stiff matter and time-varying cosmological term Λ which scales with Hubble parameter H . The resulting model approaches isotropy. The cosmological term Λ relaxes to a genuine cosmological constant, and the model in the absence of bulk viscosity tends to a deSitter universe asymptotically. Our scenario presents an initial epoch with decelerating expansion followed by late-time acceleration consistent with observations. Bulk viscosity advances the accelerating phase in the model and prevents the matter density to vanish for large times.

1. Introduction

Stiff fluid cosmological models create more interest in the study because, for these models, the speed of light equals the speed of sound, and its governing equations have the same characteristics as those of gravitational field [1]. Barrow [2] in his investigation has also pointed out that the entropy level of the universe makes it likely that its initial state was isotropic and quiescent ($p = \omega\rho$, $\omega \in (-1, 0)$) rather than chaotic only if the equation of state for high density of matter tends to stiff $\rho = p$ (ρ being matter density and p , the isotropic pressure). Keeping in view the importance of stiff fluid models, Bali and Sharma [3], Bali et al. [4], and Mak and Harko [5, 6] have investigated cosmological models for stiff fluid distribution in different contexts.

It has been argued for a long time that the dissipative process in early stages of cosmic expansion may well account for the high degree of isotropy we observe today. Dissipative effects involving both the bulk and shear viscosity play a significant role in the early evolution of the universe. The inclusion of dissipative terms in the energy-momentum

tensor of cosmic fluid seems to be the best motivated generalization of the matter term of the gravitational field equations. Eckart [7] developed the first relativistic theory of nonequilibrium thermodynamics to the effect of bulk viscosity. Padmanabhan and Chitre [8] pointed out that the presence of bulk viscosity leads to inflationary-like situations in general relativity. Johri and Sudarshan [9] have shown that bulk viscosity acts like a negative energy field in an expanding universe. Romano and Pavón [10] have investigated the evolution of Bianchi type I universe with viscous fluid. The effect of bulk viscosity on the cosmological evolution has been investigated by a number of authors namely, Saha [11–13], Singh et al. [14], Sahni and Starobinsky [15], Peebles and Ratra [16], Bali and Pradhan [17], Bali and Kumawat [18], and Bali [19].

A wide range of observations suggest that universe possesses a non-zero cosmological constant. The cosmological constant Λ is the most favoured candidate of dark energy representing energy density of vacuum. Zel'dovich [20], Dreitlein [21], and Krauss and Turner [22] have studied its significance. Recently Barrow and Shaw [23] suggested that cosmological term corresponds to a very small value of the order 10^{-122} when applied to Friedmann universe. Linde [24] has investigated that Λ is a function of temperature and is related to the spontaneous symmetry-breaking process. A number of cosmological models in which Λ decays with time have been investigated by Sahni and Starobinsky [15], Bertolami [25], Beesham [26], Berman [27], Singh and Desikan [28], Abdussattar and Vishwakarma [29], Bronnikov et al. [30], Bali and Singh [31], and Ram and Verma [32].

In this paper we investigate viscous Bianchi type I cosmological model containing stiff matter and a cosmological term Λ scaling with Hubble parameter H . We find that the model evolves with decelerating expansion in the initial epoch followed by late-time accelerating phase. The model approaches isotropy in the limit of large times, and shear viscosity accelerates the isotropization process. The presence of bulk viscosity prevents the matter density to vanish asymptotically. In the absence of viscosity, the model tends to a deSitter universe for large time [33].

2. Basic Equations

The line element for homogeneous, anisotropic Bianchi type I space-time with flat space sections in synchronous coordinate frame is given by

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2. \quad (2.1)$$

We consider the matter component of source field to be viscous fluid described by the energy-momentum tensor

$$T_{ij}^M = (\rho + \bar{p})v_i v_j + \bar{p}g_{ij} - 2\eta\sigma_{ij}, \quad (2.2)$$

where $\bar{p}$ is the effective pressure related to the equilibrium pressure p by

$$\bar{p} = p - \zeta v^i_{;i}. \quad (2.3)$$

Here ρ is the energy density of matter, $\eta \geq 0$ and $\zeta \geq 0$ are the coefficients of shear and bulk viscosity respectively, v^i , the four-velocity vector of the fluid satisfying, $v_i v^i = -1$, θ is the expansion scalar and σ_{ij} is the shear tensor defined by

$$\begin{aligned}\theta &= v^i_{;i}, \\ \sigma_{ij} &= \frac{1}{2} \left(v_{i;k} h^k_j + v_{j;k} h^k_i \right) - \frac{1}{3} \theta h_{ij},\end{aligned}\tag{2.4}$$

where $h_{ij} = g_{ij} + v_i v_j$ is the projection tensor.

The shear viscosity characterizes a change in shape of a fixed volume of the fluid, whereas the bulk viscosity characterizes a change in volume of the fluid of a fixed shape.

We choose gravitational units such that $8\pi G = c = 1$. Since the vacuum has the symmetry of the background, its energy-momentum tensor has the form $T_{ij}^\Lambda = \Lambda g_{ij}$, where Λ is a function of time in a homogeneous space. In comoving coordinates system ($v^i = \delta^i_4$), it corresponds to a perfect fluid with energy density $\rho_\Lambda = \Lambda$ and pressure $p_\Lambda = -\Lambda$. The Einstein's field equations with viscous matter and vacuum energy are given by

$$R_{ij} - \frac{1}{2} R_k^k g_{ij} = -T_{ij}^{\text{total}},\tag{2.5}$$

where $T_{ij}^{\text{total}} = T_{ij}^M + T_{ij}^\Lambda = (\rho_t + p_t) v_i v_j + p_t g_{ij}$ with the understanding that $\rho_t = \rho + \Lambda$ and $p_t = p - \Lambda$ are total energy density and total pressure, respectively. The Bianchi identities require that T_{ij}^{total} has a vanishing divergence. The surviving components of the field equations (2.5) for the Bianchi type I metric are

$$p - \left(\zeta - \frac{2}{3} \eta \right) \theta - \Lambda = -\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC} + 2\eta \frac{\dot{A}}{A},\tag{2.6}$$

$$p - \left(\zeta - \frac{2}{3} \eta \right) \theta - \Lambda = -\frac{\ddot{C}}{C} - \frac{\ddot{A}}{A} - \frac{\dot{C}\dot{A}}{CA} + 2\eta \frac{\dot{B}}{B},\tag{2.7}$$

$$p - \left(\zeta - \frac{2}{3} \eta \right) \theta - \Lambda = -\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB} + 2\eta \frac{\dot{C}}{C},\tag{2.8}$$

$$\rho + \Lambda = \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC}.\tag{2.9}$$

Here and henceforth, an overhead dot ($\dot{}$) denotes ordinary derivative with respect to cosmic time t , and semicolon ($;$) stands for covariant derivative. We define average scale factor R for Bianchi type I space-time as

$$R^3 = ABC.\tag{2.10}$$

Generalized Hubble parameter H and generalized deceleration parameter q are defined as

$$\begin{aligned} H &= \frac{\dot{R}}{R} = \frac{1}{3}(H_1 + H_2 + H_3), \\ q &= -\frac{R\ddot{R}}{\dot{R}^2} = -\frac{\dot{H}}{H^2} - 1, \end{aligned} \quad (2.11)$$

where $H_1 = \dot{A}/A$, $H_2 = \dot{B}/B$, and $H_3 = \dot{C}/C$ are directional Hubble's factors along x , y , and z directions, respectively. For Bianchi type I metric expressions for volume expansion scalar θ and shear tensor σ_{ij} come out to be

$$\begin{aligned} \theta &= 3H, \\ \sigma_1^1 &= H_1 - H, \quad \sigma_2^2 = H_2 - H, \quad \sigma_3^3 = H_3 - H, \quad \sigma_4^4 = 0. \end{aligned} \quad (2.12)$$

Magnitude σ of the shear tensor σ_{ij} is given by

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{6}\left\{(H_1 - H_2)^2 + (H_2 - H_3)^2 + (H_3 - H_1)^2\right\}. \quad (2.13)$$

From (2.6) to (2.8), we obtain the following two equations after integration:

$$\begin{aligned} \frac{\dot{A}}{A} - \frac{\dot{B}}{B} &= \frac{k_1}{ABC}e^{-2\int\eta dt}, \\ \frac{\dot{B}}{B} - \frac{\dot{C}}{C} &= \frac{k_2}{ABC}e^{-2\int\eta dt}. \end{aligned} \quad (2.14)$$

From (2.10) and (2.14), we obtain that

$$\begin{aligned} \frac{\dot{A}}{A} &= \frac{\dot{R}}{R} + \frac{2k_1 + k_2}{3R^3}e^{-2\int\eta dt}, \\ \frac{\dot{B}}{B} &= \frac{\dot{R}}{R} + \frac{k_2 - k_1}{3R^3}e^{-2\int\eta dt}, \\ \frac{\dot{C}}{C} &= \frac{\dot{R}}{R} - \frac{k_1 + 2k_2}{3R^3}e^{-2\int\eta dt}. \end{aligned} \quad (2.15)$$

We consider the shear viscosity η scaling with the expansion scalar θ [11], that is,

$$\eta = 3\eta_0 \frac{\dot{R}}{R}, \quad (2.16)$$

η_0 being constant and bulk viscosity ζ of the form given by Meng and Ren [34]

$$\zeta = \zeta_0 + \zeta_1 \frac{\dot{R}}{R}, \quad (2.17)$$

where ζ_0 and ζ_1 are constants. Using (2.16) in (2.15), we obtain that

$$\frac{\dot{A}}{A} = \frac{\dot{R}}{R} + \frac{2k_1 + k_2}{3R^{3+6\eta_0}}, \quad \frac{\dot{B}}{B} = \frac{\dot{R}}{R} + \frac{k_2 - k_1}{3R^{3+6\eta_0}}, \quad \frac{\dot{C}}{C} = \frac{\dot{R}}{R} - \frac{k_1 + 2k_2}{3R^{3+6\eta_0}}. \quad (2.18)$$

Shear scalar σ in this case turns out to be

$$\sigma = \frac{k}{\sqrt{3}R^{3+6\eta_0}}; \quad k^2 = k_1^2 + k_2^2 + k_1k_2 \quad (2.19)$$

implying that the shear viscosity accelerates the isotropization. Field equations (2.6)–(2.9) can be expressed in terms of H , and q as

$$\bar{p} - \Lambda = (2q - 1)H^2 - \sigma^2, \quad (2.20)$$

$$\rho + \Lambda = 3H^2 - \sigma^2. \quad (2.21)$$

From (2.20) and (2.21), we get the following:

$$\begin{aligned} \frac{\dot{R}}{R} &= -\frac{1}{6}(\rho + 3p) - \frac{2\sigma^2}{3} + \frac{\Lambda}{3} + \frac{\zeta\theta}{2}, \\ \frac{d\theta}{dt} &= \Lambda - 2\sigma^2 - \frac{1}{2}(\rho + 3p) - \frac{\theta^2}{3} + \frac{3\zeta\theta}{2}. \end{aligned} \quad (2.22)$$

We observe that bulk viscosity and positive Λ drive the acceleration of the universe whereas active gravitational mass density and anisotropy are responsible for deceleration. Also, presence of bulk viscosity slows down the rate of decrease of volume expansion θ .

Again from (2.20) and (2.21), we obtain that

$$q = 2 - \frac{\rho - p + 2\Lambda + \zeta\theta}{2H^2} \quad (2.23)$$

implying that the presence of bulk viscosity lowers the value of deceleration parameter.

3. Solutions of the Field Equations and Discussion

We observe that (2.6)–(2.9), (2.16), and (2.17) are six equations in eight unknowns A , B , C , ρ , p , η , ζ , and Λ . We require two more equations to obtain a determinate solution. Following Schützhold [35, 36], we consider the decaying vacuum energy density

$$\Lambda = aH, \quad (3.1)$$

where a is a positive constant and non-vacuum component of matter to be stiff fluid

$$\rho = p. \quad (3.2)$$

Let

$$\Omega = \frac{\Lambda}{\rho} \quad (3.3)$$

be the ratio between the vacuum and matter densities. From (2.21) and (3.1), we get the following:

$$a = \frac{3H\Omega}{\Omega + 1} \left(1 - \frac{3\sigma^2}{\theta^2} \right), \quad (3.4)$$

which determines the precise value of a from the observed current values of H , Ω , and σ/θ . Equations (2.20) and (2.21) together with (3.1) and (3.2) give rise to

$$\dot{H} + 3H^2 - aH - \frac{3}{2}\zeta H = 0 \quad (3.5)$$

determining the time evolution of Hubble parameter. Equation (3.5) with the use of (2.17) assumes the form

$$\dot{H} + \left(3 - \frac{3}{2}\zeta_1 \right) H^2 - \left(a + \frac{3}{2}\zeta_0 \right) H = 0. \quad (3.6)$$

We integrate (3.6) to obtain that

$$H = \frac{b}{d(1 - e^{-bt})}, \quad (3.7)$$

where the integration constants are related to the choice of origin and $b = a + (3/2)\zeta_0$, $d = 3 - (3/2)\zeta_1$.

On integration, (3.7) yields

$$R^d = m(e^{bt} - 1), \quad (3.8)$$

where m is constant of integration. For the model, matter density ρ , vacuum density Λ , and cosmological parameters θ , σ , and q have the following expressions:

$$\rho = \frac{b(3b - ad) + abde^{-bt}}{d^2(1 - e^{-bt})^2} - \frac{k^2}{3\{m(e^{bt} - 1)\}^{(6+12\eta_0)/d}}, \quad (3.9)$$

$$\Lambda = \frac{ab}{d(1 - e^{-bt})}, \quad (3.10)$$

$$\theta = \frac{3b}{d(1 - e^{-bt})}, \quad (3.11)$$

$$\sigma = \frac{k}{\sqrt{3}\{m(e^{bt} - 1)\}^{(3+6\eta_0)/d}}, \quad (3.12)$$

$$q = de^{-bt} - 1. \quad (3.13)$$

We observe that the model has singularity at $t = 0$. It evolves with a big bang at $t = 0$, where ρ , Λ , θ , and σ all diverge. Matter density ρ varies as R^{-2d} whereas vacuum energy density Λ as R^{-d} . Thus, the matter density decays faster than the vacuum energy density. Bulk viscosity component ζ_1 arrests the rate of decay. Energy density associated with anisotropy decays as $R^{-6-12\eta_0}$ contributing to matter production. Therefore, the presence of shear viscosity accelerates the decay of anisotropy energy. At early times $\Omega \approx 0$ and $q \approx 2 - (3/2)\zeta_1$ concluding that matter density is dominant in the beginning of the universe and deceleration of the expanding universe is reduced by the presence of bulk viscous component ζ_1 . In the limit of large times, we obtain that

$$\begin{aligned} \rho &\approx \frac{b(3b - ad)}{d^2}, & \theta &\approx \frac{3b}{d}, & \Lambda &\approx \frac{ab}{d}, \\ q &\approx -1, & \Omega &\approx \frac{ad}{3b - ad}, & \frac{\sigma}{\theta} &\approx 0. \end{aligned} \quad (3.14)$$

We observe that the presence of bulk viscosity does not allow the matter density to become zero in infinitely far future. Also, at late times cosmological term Λ tends to a genuine cosmological constant. Anisotropy σ/θ in the model vanishes asymptotically. The model evolves with decelerating expansion followed by accelerating one at late times. Initial anisotropy of the model dies out asymptotically. We also observe that our solution does not exactly tend to a deSitter universe due to bulk viscosity.

As particular cases of the above solution, when only one component ζ_0 or ζ_1 of the bulk viscosity is present, we can obtain the physical quantities of the resulting models by replacing b by a when $\zeta_0 = 0$, $\zeta_1 \neq 0$ and d by 3 when $\zeta_0 \neq 0$, $\zeta_1 = 0$ in (3.8) and (3.13). In case only ζ_0 is nonzero, we see that matter density varies as R^{-6} in accordance with the standard model whereas vacuum energy density decays as R^{-3} . In this case $q \approx 2$ at the initial epoch. When only ζ_1 is nonvanishing component of bulk viscosity ζ , matter density ρ decays as R^{-2d} and vacuum density Λ as R^{-d} . Since $d = 3 - (3/2)\zeta_1 < 3$, the presence of bulk viscosity component ζ_1 slows down the rate of decay. In this case $q \approx 2 - (3/2)\zeta_1$ at early times implying that deceleration of the expanding universe goes down by ζ_1 .

Figure 1: Variation of vacuum energy Λ with cosmic time t .

We observe that initial epoch of the universe is dominated by matter density, and it decays faster than vacuum energy density. Time t_Λ for which the cosmological term Λ turns out to be of the order of matter energy density is given by the condition $\Lambda = \rho$. For this condition, we obtain from (3.4) and (3.7)

$$t_\Lambda = \frac{1}{(3/2)\zeta_0 + (3H_0/2)(1 - \sigma_0^2/3H_0^2)} \ln \left\{ \frac{2d}{2d - 3 + \sigma_0^2/H_0^2 - 3\zeta_0/H} \right\}. \quad (3.15)$$

In the presence of bulk viscosity, the ratio $\Omega = \Lambda/\rho$ tends to a constant in the limit of large times. We also observe that the model starts with decelerating expansion, and expansion in the model changes from decelerating phase to accelerating one. Equating $q = 0$, we can calculate the time t_q when expansion changes from decelerating to accelerating phase. We use (3.4) and (3.13) to get that

$$t_q = \frac{\ln d}{(3/2)\zeta_0 + (3H_0\Omega_0/(\Omega_0 + 1))(1 - \sigma_0^2/3H_0^2)}. \quad (3.16)$$

Therefore, due to bulk viscosity, accelerating phase occurs earlier than in the case of perfect fluid. Time evolution of some cosmological parameters is shown graphically in Figures 1, 2, 3, and 4.

4. Conclusion

We have examined the possibility of viscous stiff matter distribution in the background of homogeneous anisotropic Bianchi type I space-time with a cosmological term Λ which scales with Hubble parameter H . Coefficient of shear viscosity η is assumed to vary as expansion scalar θ whereas bulk viscosity ζ is taken of the form $\zeta = \zeta_0 + \zeta_1(\dot{R}/R)$, ζ_0, ζ_1 being constants. Exact solutions of Einstein's field equations have been obtained. The resulting model evolves with decelerating expansion in the initial epoch followed by a late time accelerated expansion

Figure 2: Variation of matter energy density ρ with cosmic time t .

Figure 3: Variation of deceleration parameter q with cosmic time t .

Figure 4: Variation of expansion anisotropy σ/θ with cosmic time t .

presenting an appropriate expansion history of the universe consistent with observations [37–39]. We observe that the model approaches isotropy for large values of t , and presence of shear viscosity accelerates the isotropization process. Cosmological term Λ being very large at initial times relaxes to a genuine cosmological constant asymptotically. In the absence of bulk viscosity, the model results in a deSitter universe for large values of t with $H = \sqrt{\Lambda/3} = a/3$. We find that the initial phase is dominated by matter density which tends asymptotically to a constant. The ratio Ω between the vacuum and matter densities tends asymptotically to a constant. Presence of bulk viscosity prevents matter density to vanish for large t . We also observe that bulk viscous component ζ_1 slows down the rate of decrease of volume expansion. Due to bulk viscosity, accelerating phase occurs earlier than the perfect fluid case.

Acknowledgment

Authors are thankful to Inter University Centre for Astronomy and Astrophysics, Pune, India for providing facility where the part of this work was completed.

References

- [1] Ya. B. Zel'dovich, "A hypothesis, unifying the structure and the entropy of the Universe," *Monthly Notices of the Royal Astronomical Society*, vol. 160, p. 1, 1972.
- [2] J. D. Barrow, "Quiescent cosmology," *Nature*, vol. 272, pp. 211–215, 1978.
- [3] R. Bali and K. Sharma, "Tilted Bianchi Type I stiff fluid magnetized cosmological model in general relativity," *Astrophysics and Space Science*, vol. 283, pp. 11–22, 2003.
- [4] R. Bali, M. Ali, and V. C. Jain, "Magnetized stiff fluid cylindrically symmetric universe with two degrees of freedom in general relativity," *International Journal of Theoretical Physics*, vol. 47, no. 9, pp. 2218–2229, 2008.
- [5] M. K. Mak and T. Harko, "Exact dissipative cosmologies with stiff fluid," *Europhysics Letters*, vol. 56, pp. 762–767, 2001.
- [6] M. K. Mak and T. Harko, "Full causal dissipative cosmologies with stiff matter," *International Journal of Modern Physics D*, vol. 13, pp. 273–280, 2004.
- [7] C. Eckart, "The thermodynamics of irreversible processes, III. Relativistic theory of the simple fluid," *Physical Review*, vol. 58, pp. 919–924, 1940.
- [8] T. Padmanabhan and S. M. Chitre, "Viscous Universe," *Physics Letter A*, vol. 120, no. 9, pp. 433–436, 1987.
- [9] V. B. Johri and R. Sudarshan, in *Proceedings of the Proceedings of International Conference on Mathematical Modelling in Science and Technology*, World Scientific, Singapore, 1988.
- [10] V. Romano and D. Pavón, "Causal dissipative Bianchi cosmology," *Physical Review D*, vol. 47, pp. 1396–1403, 1993.
- [11] B. Saha, "Bianchi type I universe with viscous fluid," *Modern Physics Letters A*, vol. 20, no. 28, pp. 2127–2143, 2005.
- [12] B. Saha, "Nonlinear spinor field in Bianchi type-I cosmology: Inflation, isotropization, and late time acceleration," *Physical Review D*, vol. 74, Article ID 124030, p. 1685, 2006.
- [13] B. Saha, "Anisotropic cosmological models with spinor field and viscous fluid in the presence of Lambda term: qualitative solutions," *Journal of Physics A*, vol. 40, no. 46, pp. 14011–14027, 2007.
- [14] C. P. Singh, S. Kumar, and A. Pradhan, "Early viscous universe with variable gravitational and cosmological " constants", " *Classical and Quantum Gravity*, vol. 24, no. 2, pp. 455–474, 2007.
- [15] V. Sahni and A. Starobinsky, "The case for a positive cosmological Λ -term," *International Journal of Modern Physics D*, vol. 9, no. 4, pp. 373–443, 2000.
- [16] P. J. E. Peebles and B. Ratra, "The cosmological constant and dark energy," *Reviews of Modern Physics*, vol. 75, no. 2, pp. 559–606, 2003.
- [17] R. Bali and A. Pradhan, "Bianchi type-III string cosmological models with time dependent bulk viscosity," *Chinese Physics Letters*, vol. 24, no. 2, pp. 585–588, 2007.

- [18] R. Bali and P. Kumawat, "Bulk viscous L.R.S. Bianchi type V tilted stiff fluid cosmological model in general relativity," *Physics Letters B*, vol. 665, no. 5, pp. 332–337, 2008.
- [19] R. Bali, "Bianchi type V magnetized string dust bulk viscous fluid cosmological model with variable magnetic permeability," *International Journal of Theoretical Physics*, vol. 48, no. 2, pp. 476–486, 2009.
- [20] Ya. B. Zel'dovich, "The cosmological constant and the theory of elementary particles," *Soviet Physics Uspekhi*, vol. 11, no. 3, article 381, 1968.
- [21] J. Dreitlein, "Broken symmetry and the cosmological constant," *Physical Review Letters*, vol. 33, pp. 1243–1244, 1974.
- [22] L. M. Krauss and M. S. Turner, "The cosmological constant is back," *General Relativity and Gravitation*, vol. 27, pp. 1137–1144, 1995.
- [23] J. D. Barrow and D. J. Shaw, "The value of the cosmological constant," *General Relativity and Gravitation*, vol. 43, no. 10, pp. 2555–2560, 2011.
- [24] A. D. Linde, "Is the Lee constant a cosmological constant?" *JETP Letters*, vol. 19, p. 183, 1974.
- [25] O. Bertolami, "Time-dependence cosmological term," *Nuovo Cimento B*, vol. 93, p. 36, 1986.
- [26] A. Beesham, "Variable- G cosmology and creation," *International Journal of Theoretical Physics*, vol. 25, no. 12, pp. 1295–1298, 1986.
- [27] M. S. Berman, "Cosmological models with variable gravitational and cosmological "constants"," *General Relativity and Gravitation*, vol. 23, no. 4, pp. 465–469, 1991.
- [28] G. P. Singh and K. Desikan, "A new class of cosmological models in Lyra geometry," *Pramana*, vol. 49, no. 2, pp. 205–212, 1997.
- [29] Abdussattar and R. G. Vishwakarma, "Some FRW models with variable G and Λ ," *Classical and Quantum Gravity*, vol. 14, no. 4, pp. 945–953, 1997.
- [30] K. A. Bronnikov, A. Dobosz, and I. G. Dymnikova, "Nonsingular vacuum cosmologies with a variable cosmological term," *Classical and Quantum Gravity*, vol. 20, no. 16, pp. 3797–3814, 2003.
- [31] R. Bali and J. P. Singh, "Bulk viscous Bianchi type I cosmological models with time-dependent cosmological term Λ ," *International Journal of Theoretical Physics*, vol. 47, no. 12, pp. 3288–3297, 2008.
- [32] S. Ram and M. K. Verma, "Bulk viscous fluid hypersurface-homogeneous cosmological models with time varying G and Λ ," *Astrophysics and Space Science*, vol. 330, pp. 151–156, 2010.
- [33] J. P. Singh and R. K. Tiwari, "An LRS Bianchi type-I cosmological model with time-dependent Λ term," *International Journal of Modern Physics D*, vol. 16, no. 4, pp. 745–754, 2007.
- [34] X. H. Meng and J. Ren, "Cosmology with extended Chaplygin gas media and $w = -1$ crossing," *Communications in Theoretical Physics*, vol. 47, no. 2, pp. 379–384, 2007.
- [35] R. Schützhold, "Small cosmological constant from the QCD trace anomaly?" *Physical Review Letters*, vol. 89, Article ID 081302, 2002.
- [36] R. Schützhold, "On the cosmological constant and the cosmic coincidence problem," *International Journal of Modern Physics A*, vol. 17, p. 4359, 2002.
- [37] A. G. Riess, L.-G. Strolger, J. Tonry et al., "Type Ia supernova discoveries at $z > 1$ from the Hubble space telescope: evidence for past deceleration and constraints on dark energy evolution," *The Astrophysical Journal*, vol. 607, no. 2, p. 665, 2004.
- [38] R. A. Knop, G. Aldering, R. Amanullah et al., "New constraints on Ω_M , Ω_Λ , and w from an independent set of 11 high-redshift supernovae observed with the Hubble Space Telescope," *The Astrophysical Journal*, vol. 598, no. 1, p. 102, 2003.
- [39] M. Tegmark, M. A. Strauss, M. R. Blanton et al., "Cosmological parameters from SDSS and WMAP," *Physical Review D*, vol. 69, no. 10, Article ID 103501, 2004.